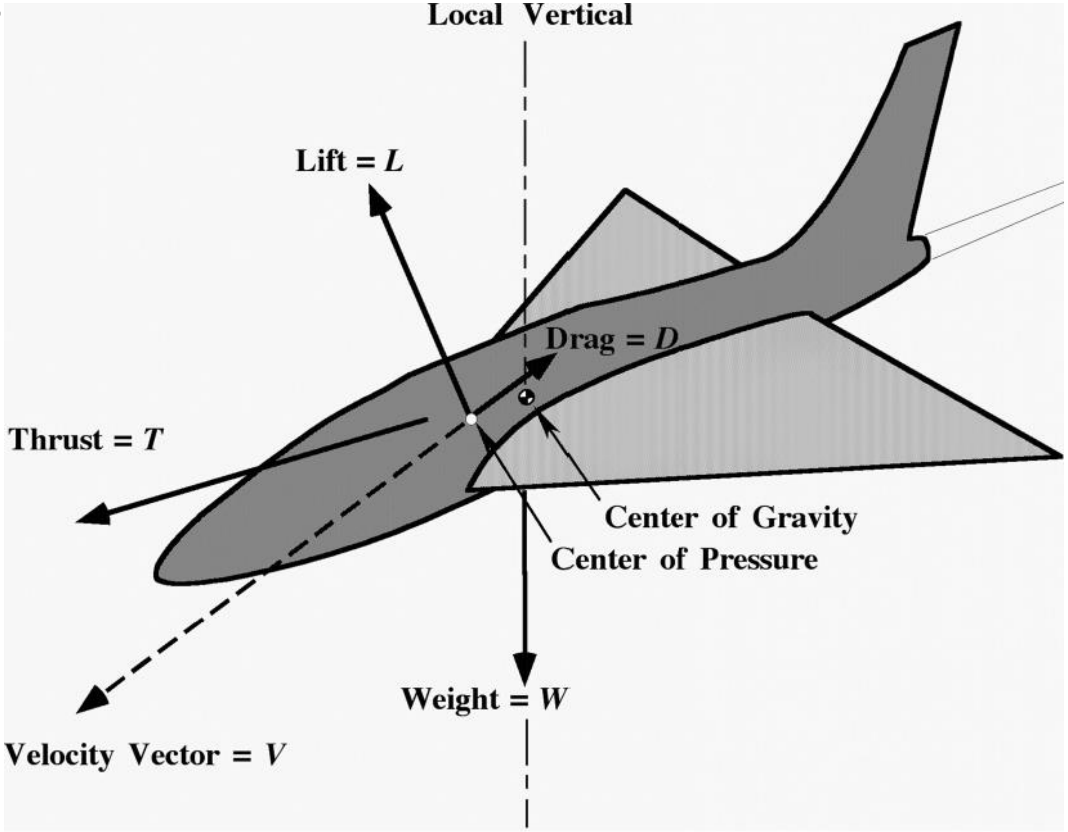
Chapter 1: Generalities

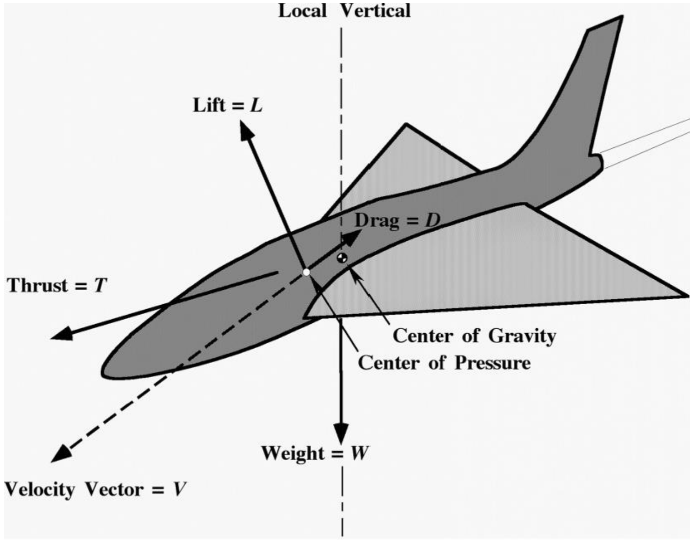
The system of aerodynamic forces

- Aerodynamic forces arise from airflow-body interactions



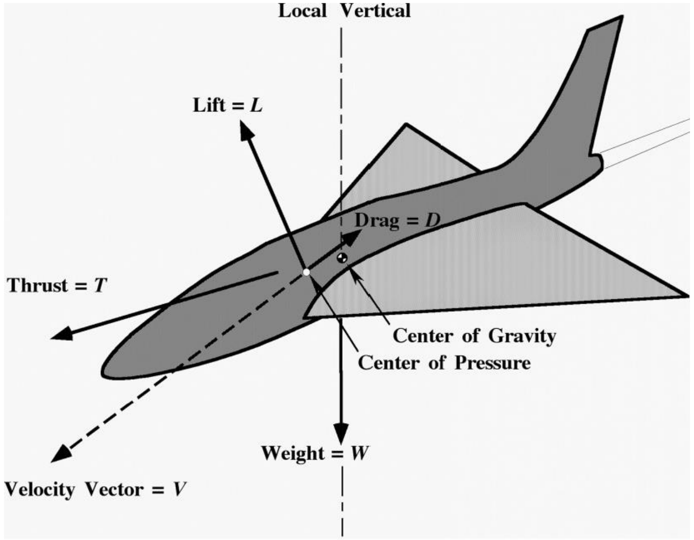
The system of aerodynamic forces

- Lift and drag represent the **integrated effect** of a continuous distribution of pressure and viscous forces acting on all exposed surfaces
- Lift and drag are always defined wrt the velocity of the vehicle

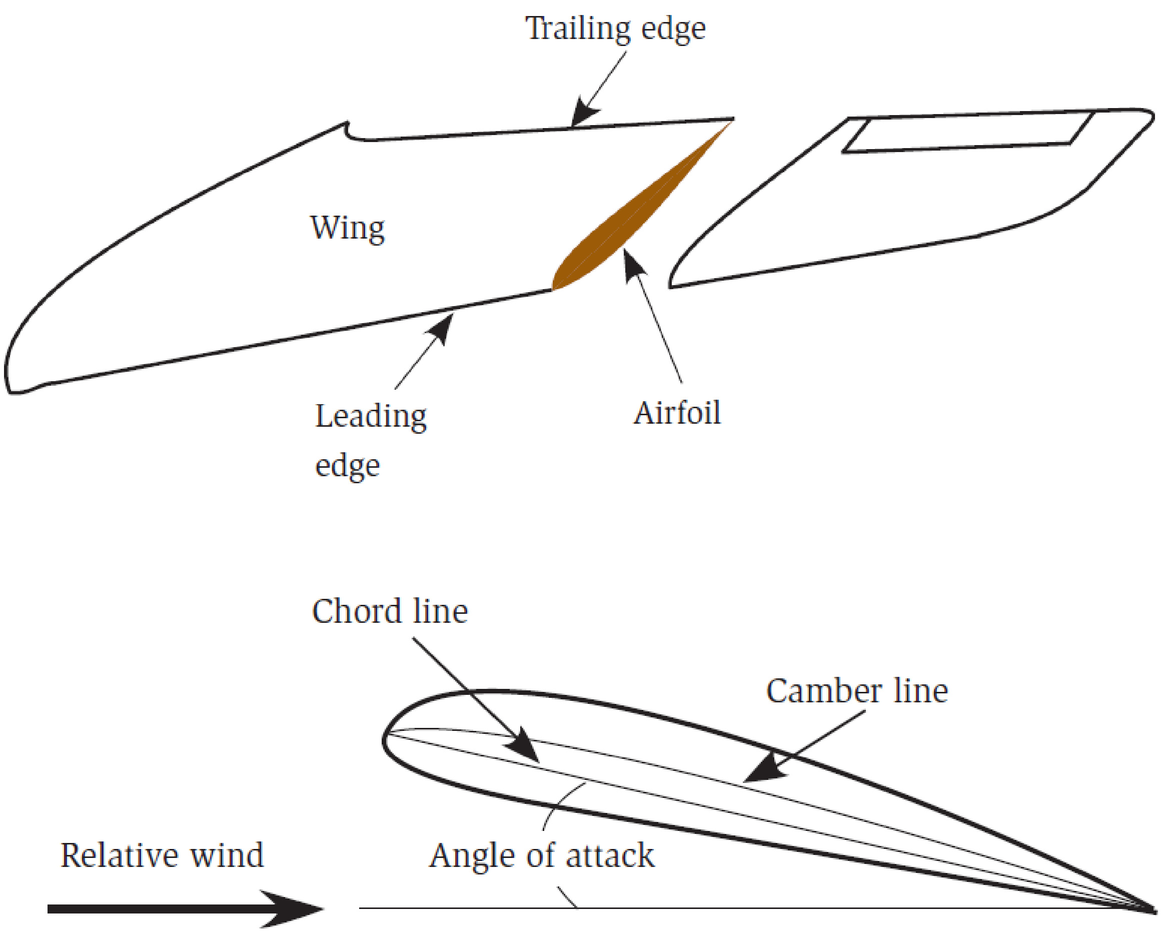


The system of aerodynamic forces

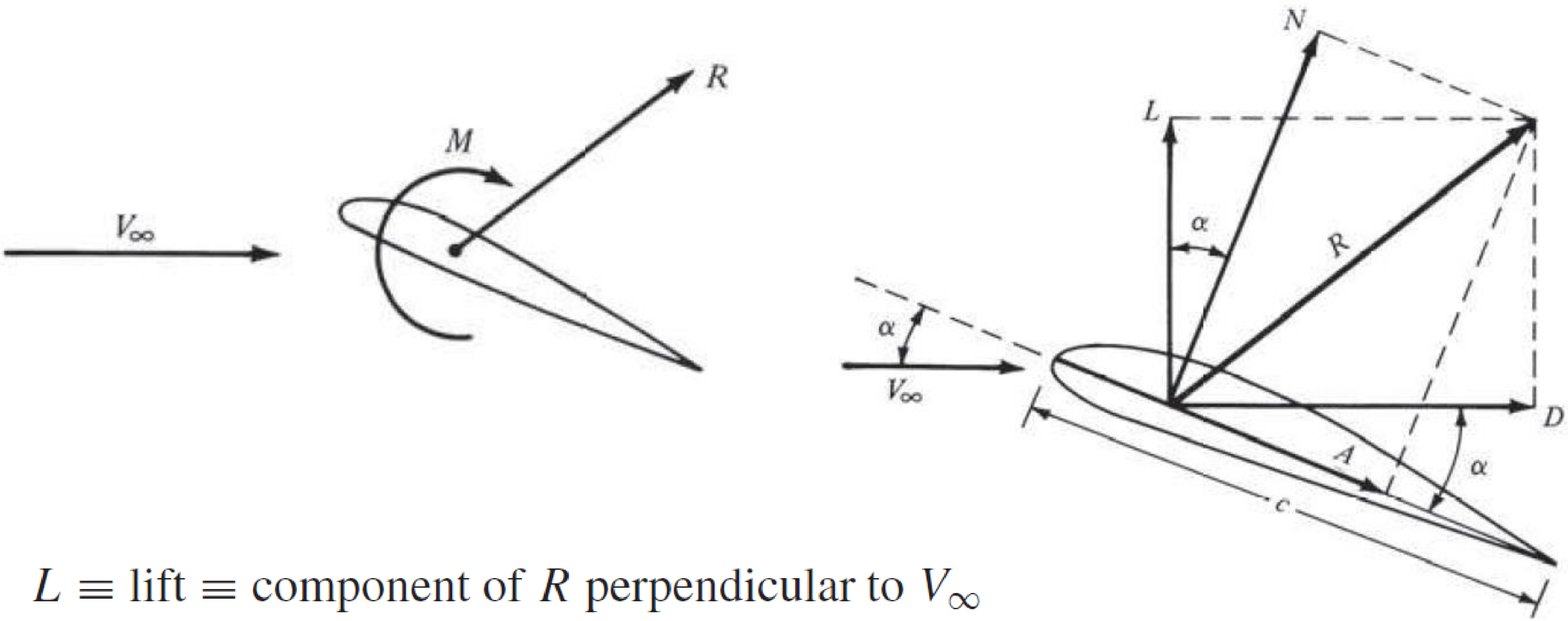
- Focus on
  - How are these forces generated?
  - Where are they applied?
  - What is and where is the center of pressure?
  - The system of forces is independent of the choice of coordinate frame: we often apply a **Galilean transformation** in steady flight



Wings and airfoils



The system of aerodynamic forces



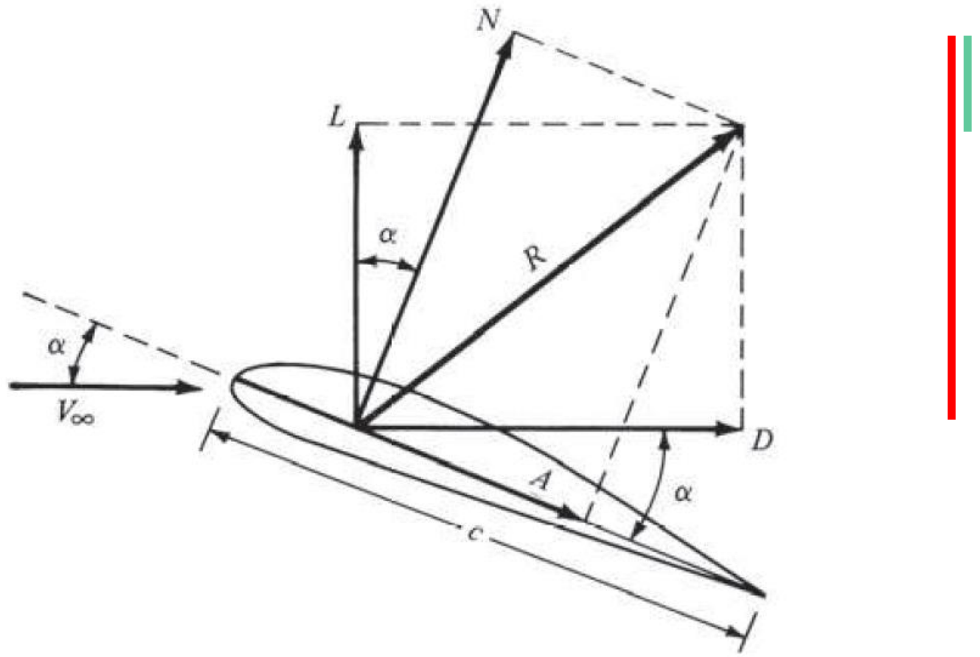
$L \equiv$ lift $\equiv$ component of R perpendicular to V_∞

$D \equiv$ drag $\equiv$ component of R parallel to V_∞

$N \equiv$ normal force $\equiv$ component of R perpendicular to c

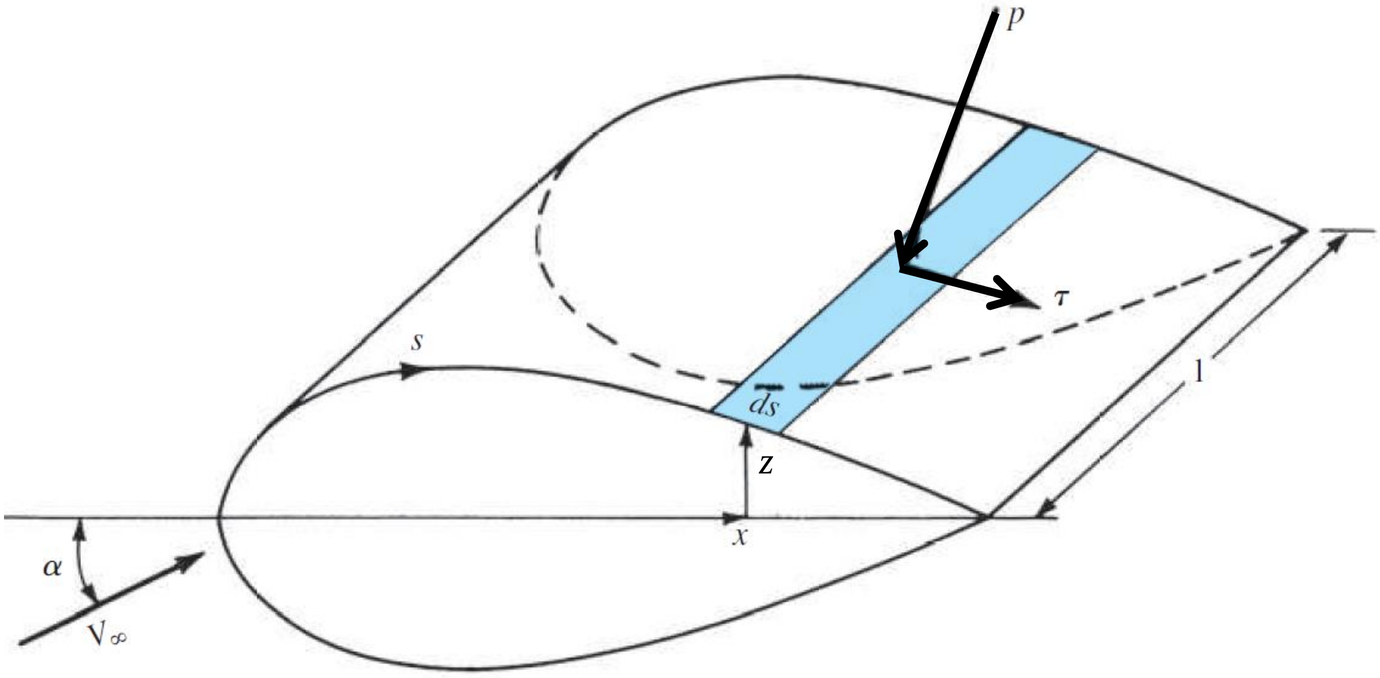
$A \equiv$ axial force $\equiv$ component of R parallel to c

The system of aerodynamic forces

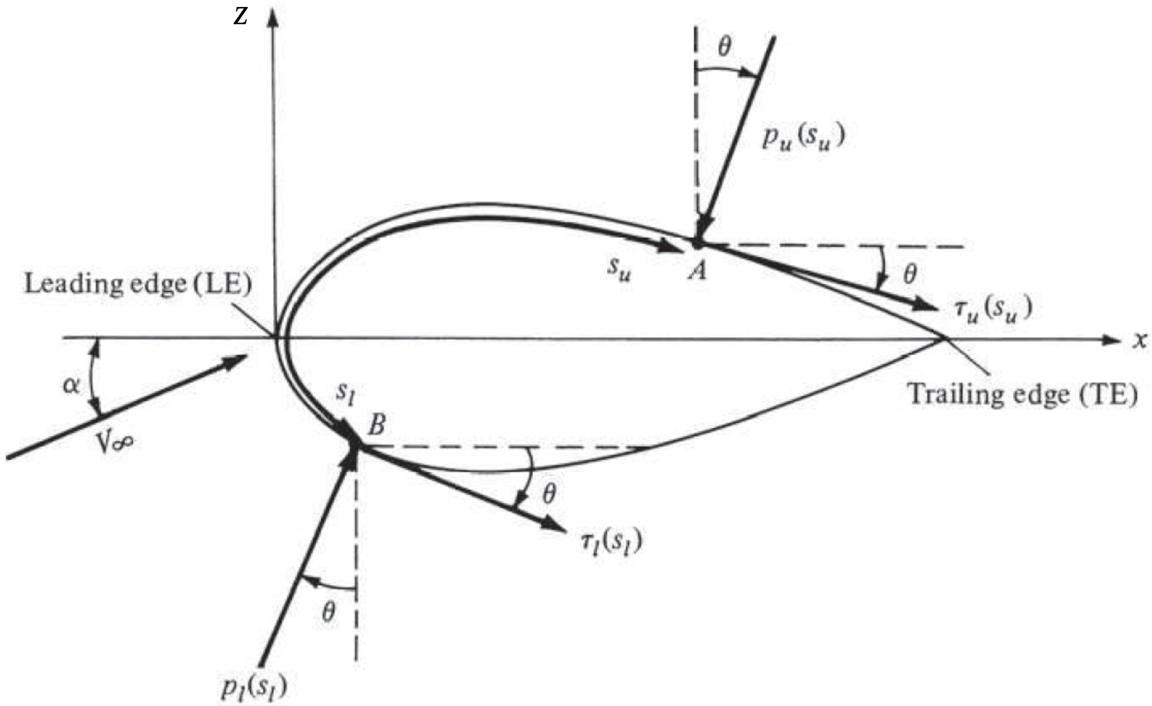


$$L = N\cos\alpha - A\sin\alpha$$
$$D = N\sin\alpha + A\cos\alpha$$

The system of aerodynamic forces

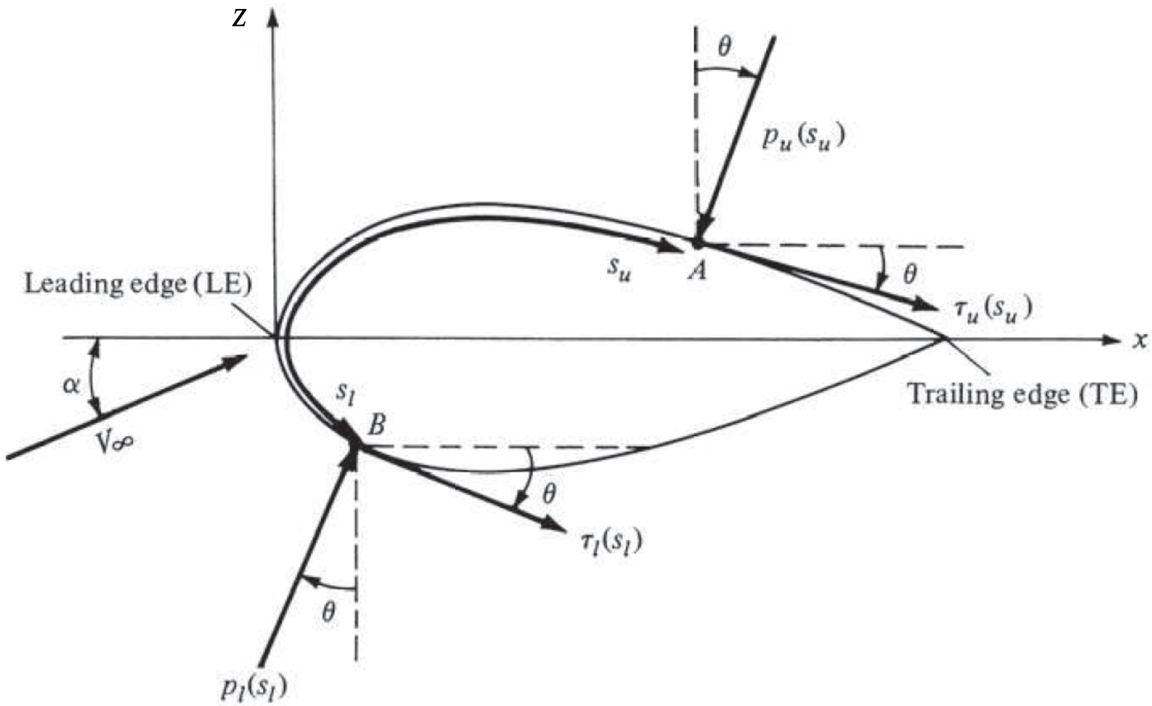


The system of aerodynamic forces



Convention: θ is positive when measured clockwise from the vertical line to the direction of p

The system of aerodynamic forces



$$dN'_u = -p_u ds_u \cos\theta - \tau_u ds_u \sin\theta \qquad dN'_l = p_l ds_l \cos\theta - \tau_l ds_l \sin\theta$$

$$dA'_u = -p_u ds_u \sin\theta + \tau_u ds_u \cos\theta \qquad dA'_l = p_l ds_l \sin\theta + \tau_l ds_l \cos\theta$$

The system of aerodynamic forces

$$N' = -\int_{\mathrm{LE}}^{\mathrm{TE}} (p_u \cos\theta + \tau_u \sin\theta)\, ds_u + \int_{\mathrm{LE}}^{\mathrm{TE}} (p_l \cos\theta - \tau_l \sin\theta)\, ds_l$$

$$A' = \int_{\mathrm{LE}}^{\mathrm{TE}} (-p_u \sin\theta + \tau_u \cos\theta)\, ds_u + \int_{\mathrm{LE}}^{\mathrm{TE}} (p_l \sin\theta + \tau_l \cos\theta)\, ds_l$$

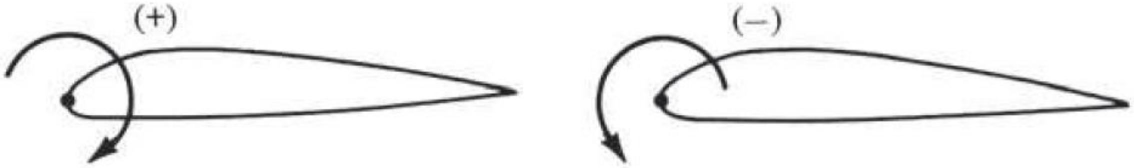
Note: when "primes" are used we mean "per unit span"

The system of aerodynamic forces

The moment (taken **about the LE**) is positive when α tends to increase (*pitch up*), negative otherwise.

$$dM'_u = (p_u \cos\theta + \tau_u \sin\theta)x \; ds_u + (-p_u \sin\theta + \tau_u \cos\theta)z \; ds_u$$

$$dM'_l = (-p_l \cos\theta + \tau_l \sin\theta)x \; ds_l + (p_l \sin\theta + \tau_l \cos\theta)z \; ds_l$$



Lifting airfoils have typically negative M'

The system of aerodynamic forces

The moment (taken **about the LE**) is positive when α tends to increase (*pitch up*), negative otherwise.

$$dM'_u = (p_u \cos\theta + \tau_u \sin\theta)x\ ds_u + (-p_u \sin\theta + \tau_u \cos\theta)\, z\ ds_u$$

$$dM'_l = (-p_l \cos\theta + \tau_l \sin\theta)x\ ds_l + (p_l \sin\theta + \tau_l \cos\theta)\, z\ ds_l$$

$$M'_{\mathrm{LE}} = \int_{\mathrm{LE}}^{\mathrm{TE}} [(p_u \cos\theta + \tau_u \sin\theta)x - (p_u \sin\theta - \tau_u \cos\theta)z]\, ds_u$$

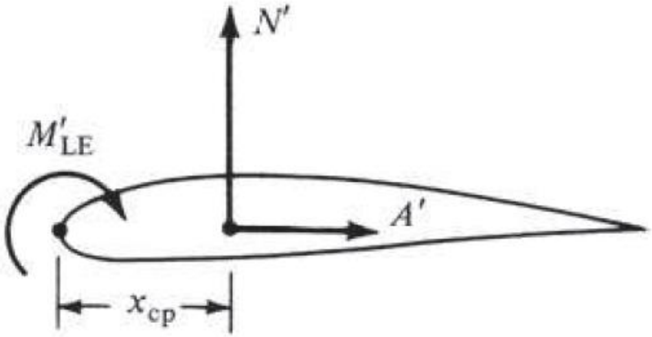
$$+ \int_{\mathrm{LE}}^{\mathrm{TE}} [(-p_l \cos\theta + \tau_l \sin\theta)x + (p_l \sin\theta + \tau_l \cos\theta)z]\, ds_l$$

The system of aerodynamic forces

Goal of aerodynamics: calculate $p(s)$ and $\tau(s)$ for a given body shape and free-stream conditions

The system of aerodynamic forces

Where are the **resultant forces** N' and A' (or L' and D') positioned on the airfoil? They must be placed at a point where the distributed load produces the same moment as the resultant force. If A' is on chord line:



$$M'_{\mathrm{LE}} = -(x_{\mathrm{cp}})N'$$

$$x_{\mathrm{cp}} = -\frac{M'_{\mathrm{LE}}}{N'}$$

The system of aerodynamic forces

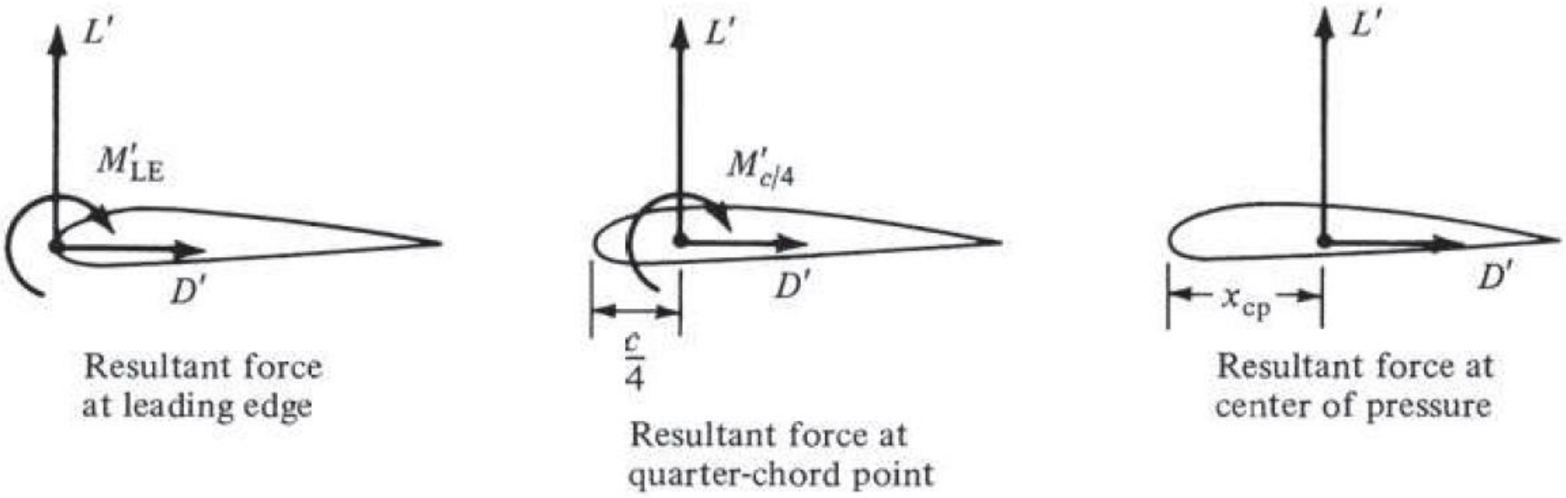
When the angle of attack α is small: $L' \approx N'$

$$x_{\mathrm{cp}} \approx -\frac{M'_{\mathrm{LE}}}{L'}$$

and as the forces N' (and L') decrease the CP moves to infinity → CP is not necessarily a convenient point where to place aerodynamic forces, it moves with α ...

The system of aerodynamic forces

Equivalent points of view for aerodynamic forces and pitching moment:



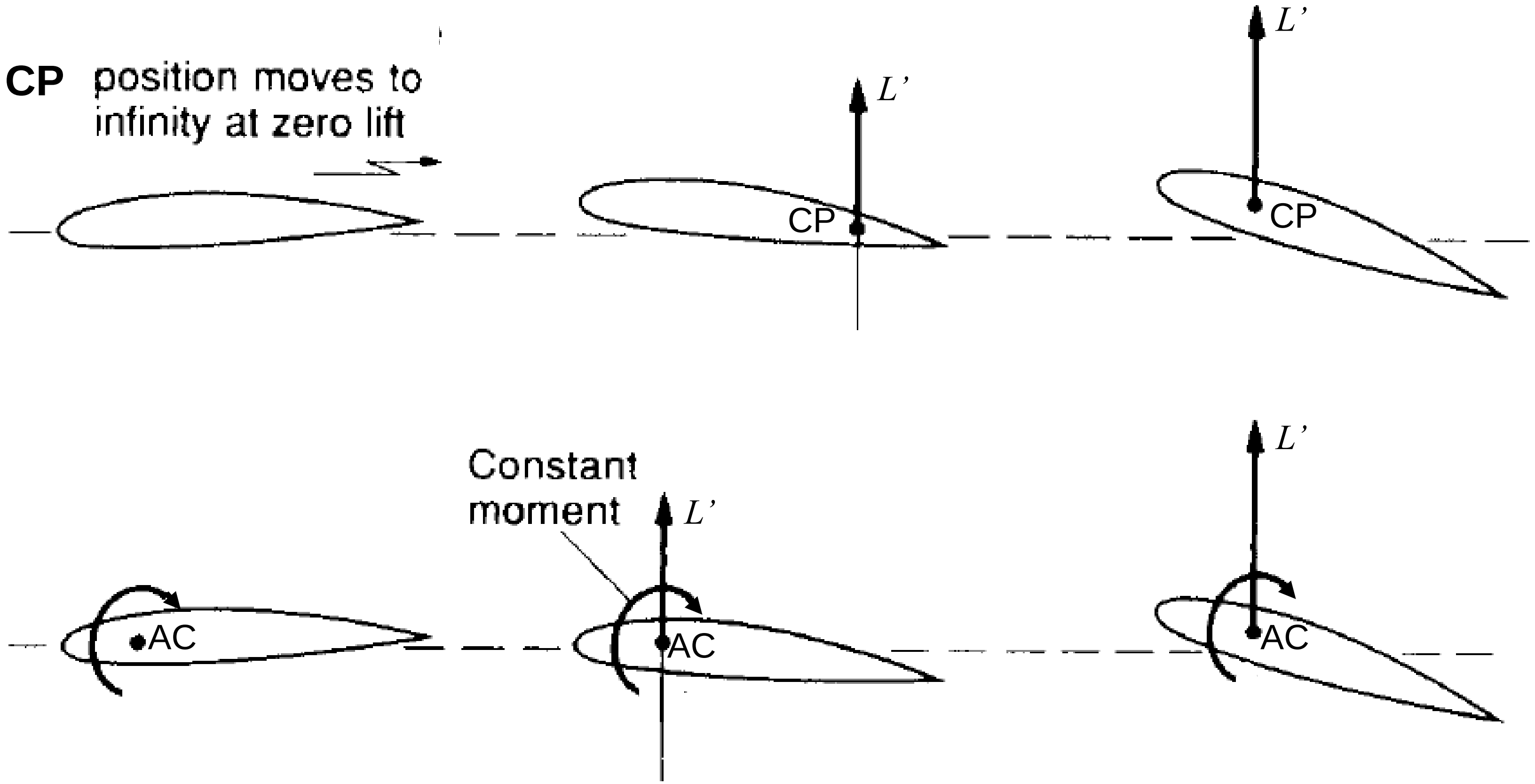
Aerodynamic moment is zero (by definition) when measured about CP

The AC

The aerodynamic center (AC) is that point wrt which the pitching moment becomes independent of aerodynamic forces, i.e. of the angle of attack.

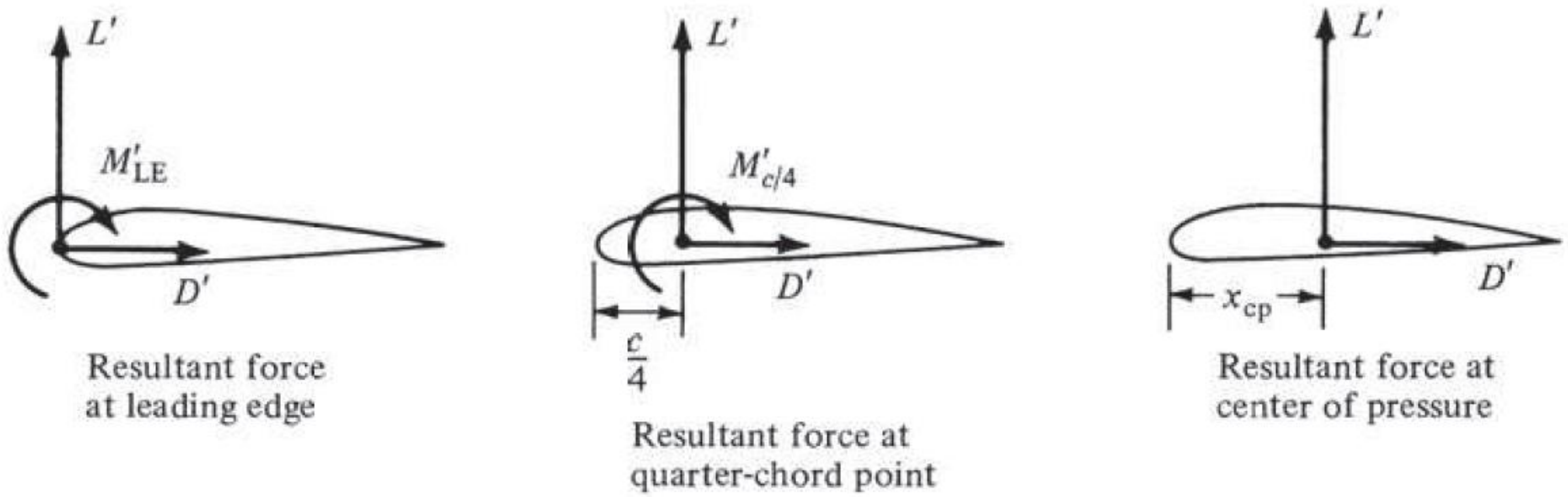
For thin airfoils the aerodynamic center (AC) is at quarter chord from LE.

The AC



CP position moves to infinity at zero lift
CP
L'
CP
L'
Constant moment
L'
AC
AC
L'
AC

The system of aerodynamic forces



$$M'_{\text{LE}} = -\frac{c}{4}L' + M'_{c/4} = -x_{\text{cp}}L'$$

The aerodynamic coefficients

Dimensional analysis easily yields

$$C_L = f_{10}(\mathrm{Re}, M_\infty, \alpha)$$

$$C_D = f_{11}(\mathrm{Re}, M_\infty, \alpha)$$

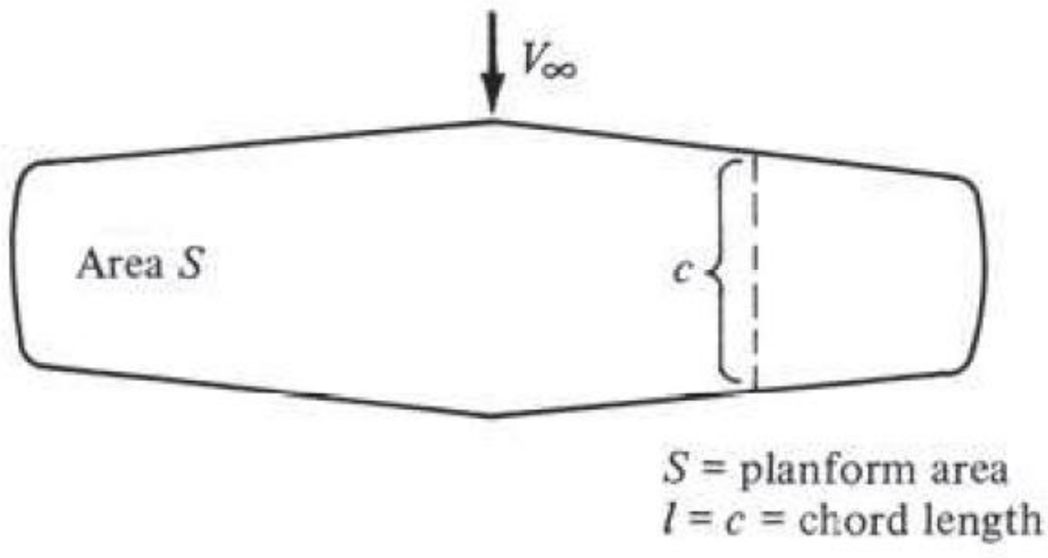
$$C_M = f_{12}(\mathrm{Re}, M_\infty, \alpha)$$

for a given body shape. The coefficients for a wing are:

$$C_L \equiv \frac{L}{q_\infty S} \qquad C_M \equiv \frac{M}{q_\infty S l}$$

$$C_D \equiv \frac{D}{q_\infty S} \qquad q_\infty \equiv \tfrac{1}{2}\rho_\infty V_\infty^2$$

The aerodynamic coefficients



The aerodynamic coefficients

For an airfoil:

$$c_l \equiv \frac{L'}{q_\infty c} \quad c_d \equiv \frac{D'}{q_\infty c} \quad c_m \equiv \frac{M'}{q_\infty c^2}$$

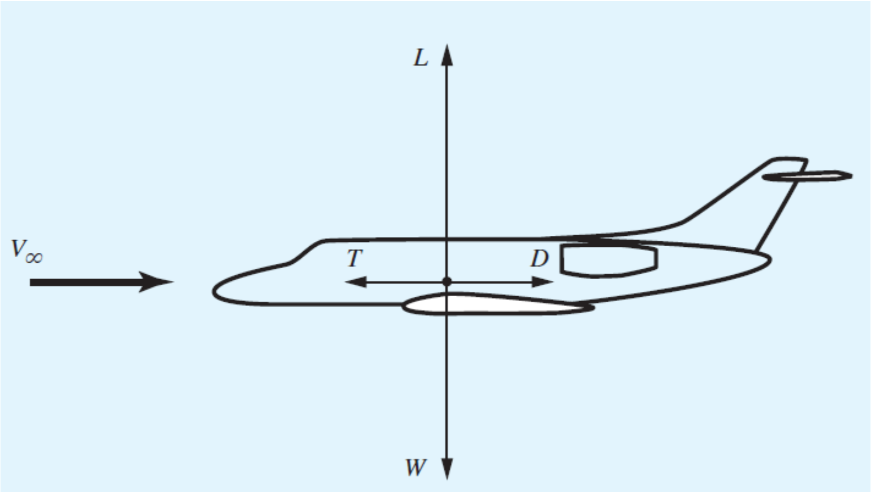
with reference area $S = c(1) = c$.

Furthermore:

$$c_l = c_n \cos\alpha - c_a \sin\alpha$$
$$c_d = c_n \sin\alpha + c_a \cos\alpha$$

Final considerations on the coefficients

Level flight at constant speed

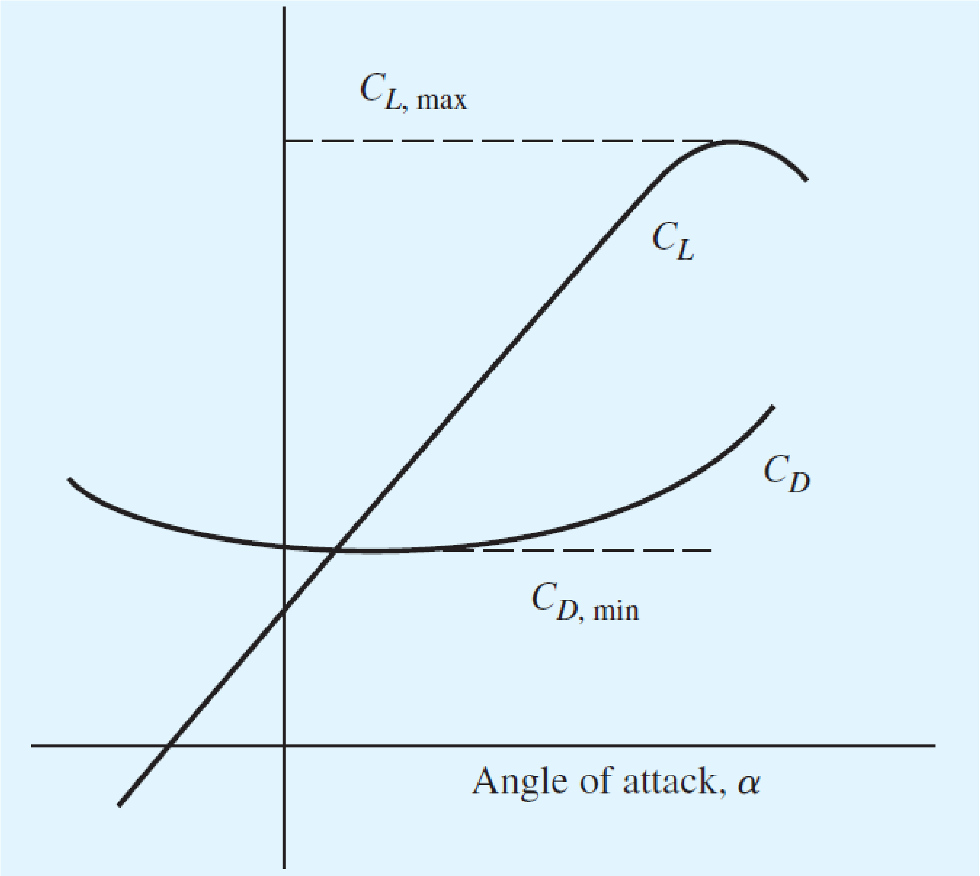


$L = W$ and $T = D$

Typically, for conventional cruising flights, the *aerodynamic efficiency* is $L/D \approx 15$ to 20.

Final considerations on the coefficients

Given aircraft shape, Mach and Reynolds numbers:



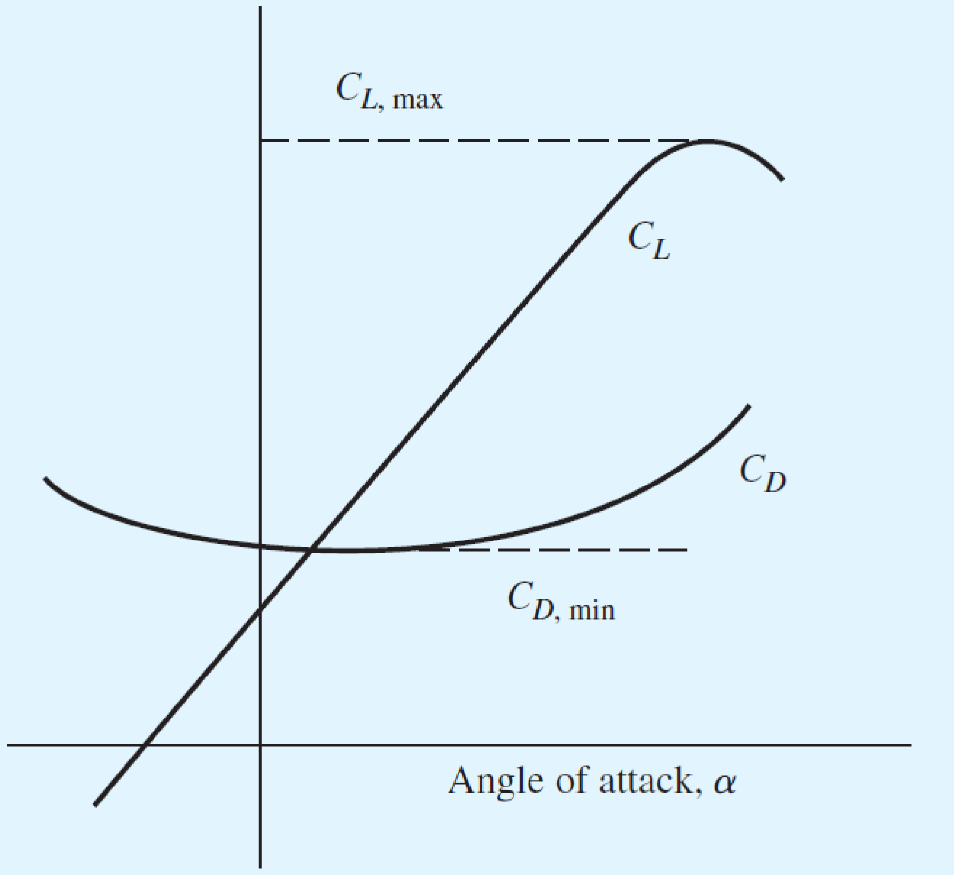
Final considerations on the coefficients

Level flight

$$C_L = \frac{L}{q_\infty S} = \frac{W}{q_\infty S} = \frac{2W}{\rho_\infty V_\infty^2 S}$$

W/S : **wing loading**

$$C_D = \frac{D}{q_\infty S} = \frac{T}{q_\infty S} = \frac{2T}{\rho_\infty V_\infty^2 S}$$



Final considerations on the coefficients

The minimum speed at which the aircraft can maintain level flight is the *stall speed*

$$V_{\text{stall}} = \sqrt{\frac{2W}{\rho_\infty S C_{L,\max}}}$$

High lift devices (flaps, slats, slots …) on the wings are used by the pilot to increase $C_{L,\max}$ and decrease the stall speed (they are usually deployed for take-off or landing)

Final considerations on the coefficients

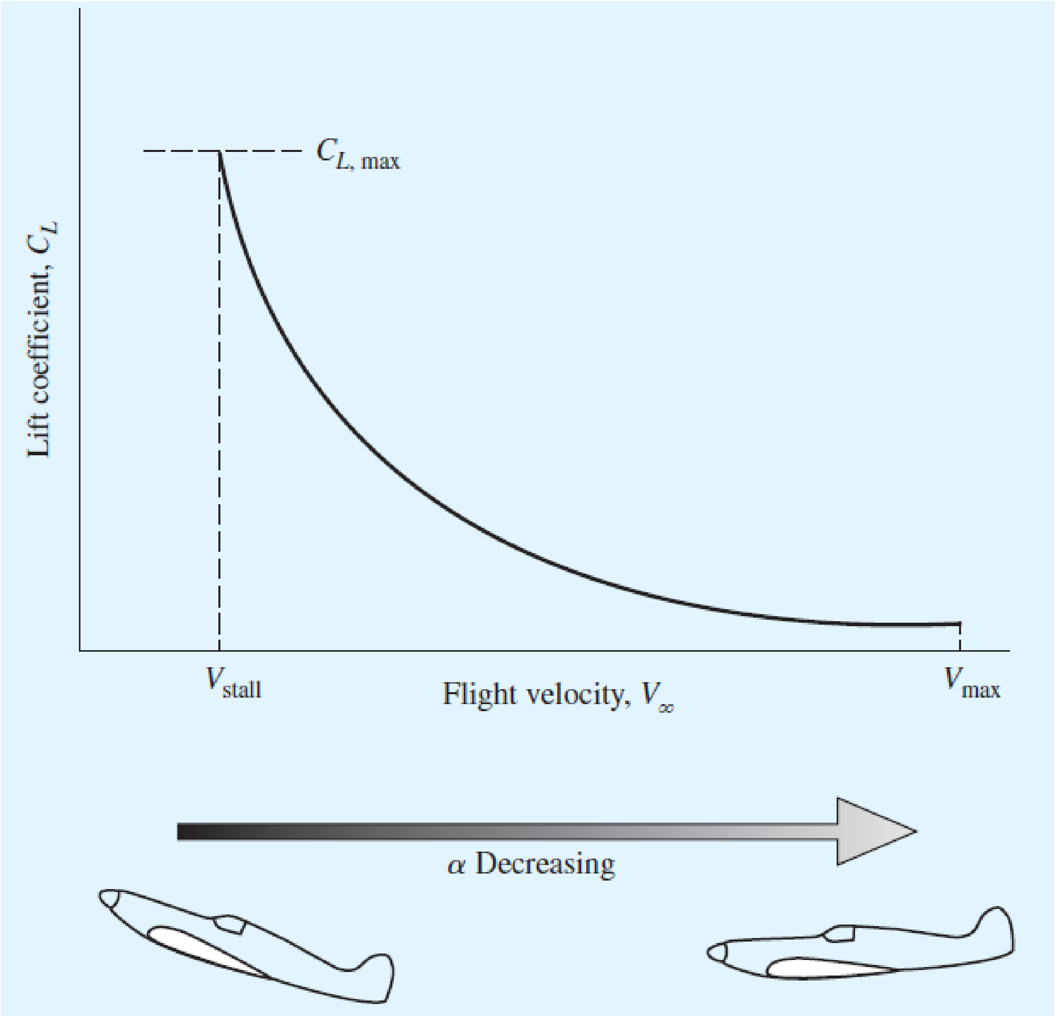
The maximum speed of the aircraft, for given maximum thrust, $T_{\max}$, corresponds to flight at $C_{D,\min}$.

$$V_{\max} = \sqrt{\frac{2T_{\max}}{\rho_\infty S C_{D,\min}}}$$

Thus, aerodynamic coefficients $C_{L,\max}$ and $C_{D,\min}$ dictate the performance and design of airplanes.

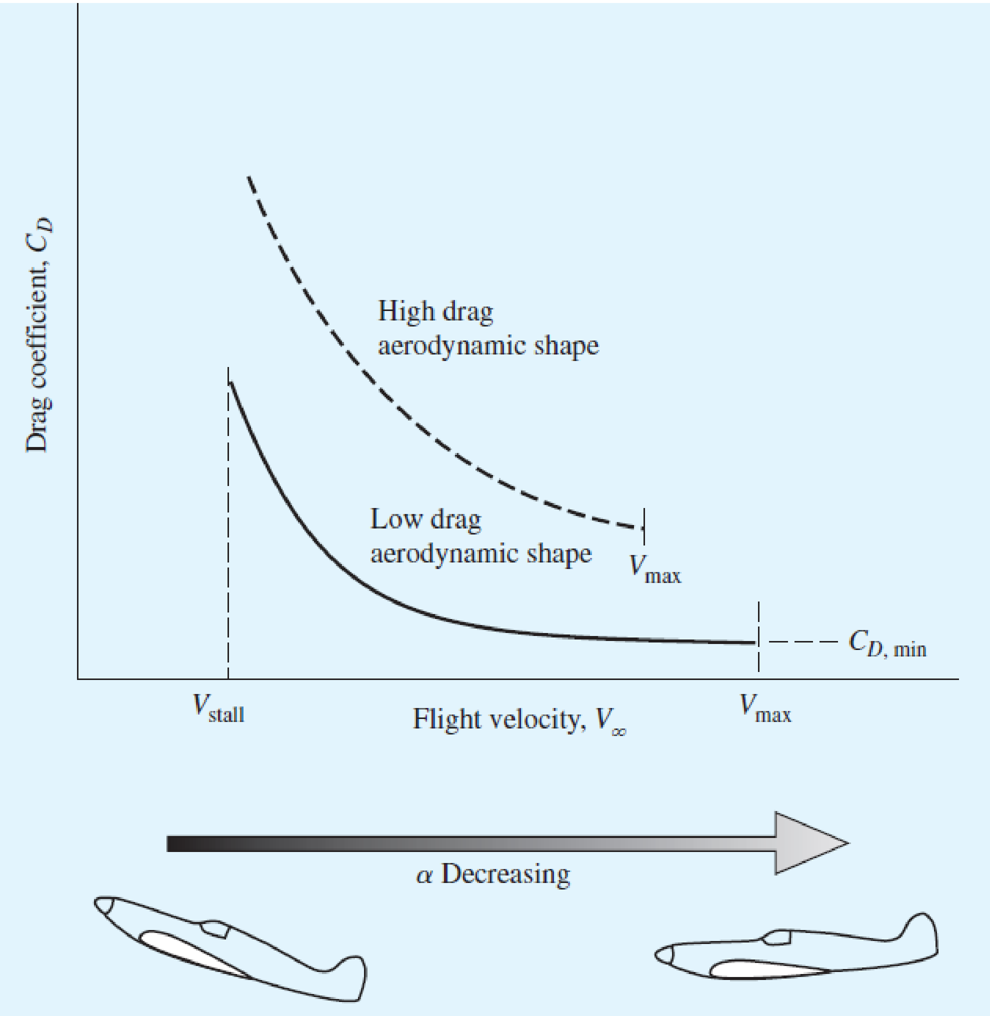
Lift coefficient

Sketch of C_L needed to maintain level flight over a range of velocities. The aircraft designer must design the airplane to achieve these values for an airplane of given weight and wing area.



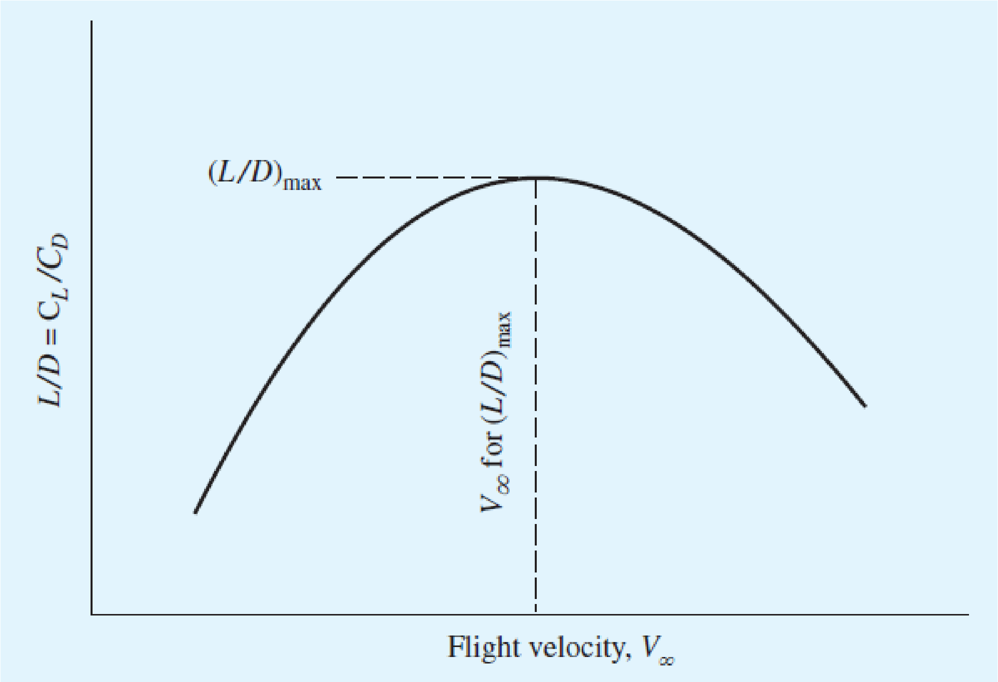
Drag coefficient

Sketch of C_D versus flight velocity. Comparison between high and low drag aerodynamic bodies, with the consequent effect on maximum velocity.



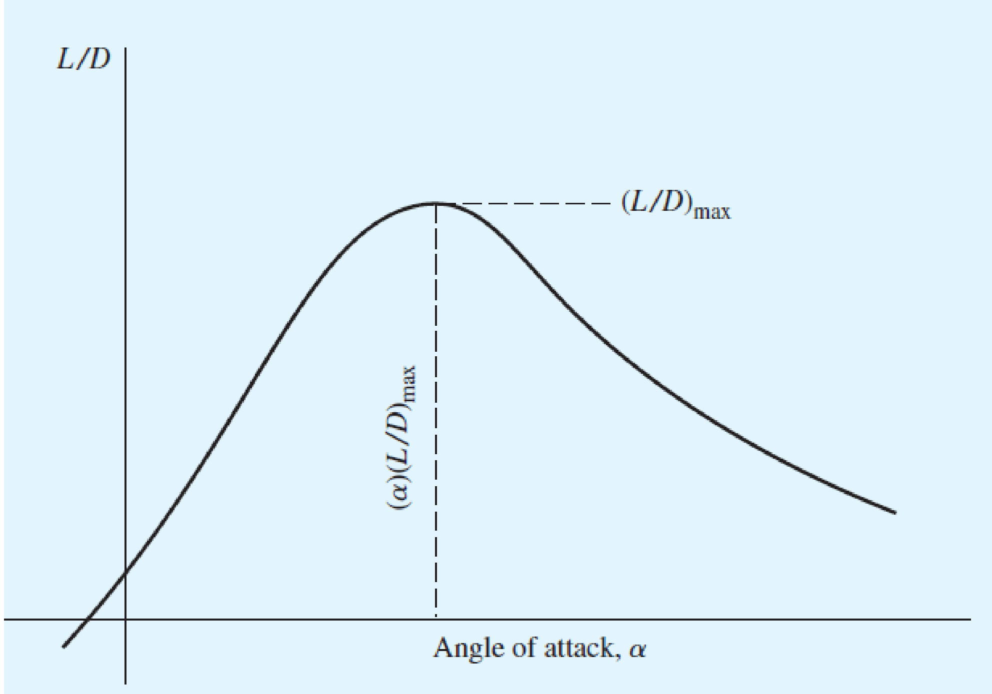
The aerodynamic efficiency

Lift-to-drag ratio: $$\frac{L}{D} = \frac{q_\infty S C_L}{q_\infty S C_D} = \frac{C_L}{C_D}$$



The aerodynamic efficiency

Lift-to-drag ratio: $$\frac{L}{D} = \frac{q_\infty S C_L}{q_\infty S C_D} = \frac{C_L}{C_D}$$



Exercise 1

A Cessna 560 in level flight at 10 000 m altitude (ρ = 0.4135 kg/m^3) is travelling at a cruise speed of 200 m/s. The planform area of the wing is S = 31.8 m^2 and the mass of the aircraft is 6800 kg.

1. How much is the wing loading? W/S = 2098 N/m^2
2. What is the value of C_L? C_L = 0.253
3. If C_D in cruise flight is 0.015, how much is the aerodynamic efficiency? L/D = 16.9
4. Let us now consider the same plane at take off at sea level (ρ = 1.225 kg/m^3). What is the value of $C_{L,\max}$ if the stall velocity is V_{stall} = 44.7 m/s (considering a fully loaded aircraft of mass equal to 7212 kg). $C_{L,\max}$ = 1.82

Exercise 2

Consider a Boeing 747 in level flight at V_∞ = 885 km/h at an altitude of 11 500 m (where T = 220 K and p = 0.2 atm). We wish to design a 1:50 scale model airplane in **complete dynamic similarity** and test it in a wind tunnel where temperature is maintained at 288 K. Evaluate the model's velocity and the air pressure in the wind tunnel assuming that both the dynamic viscosity μ and the speed of sound vary as $T^{1/2}$.

Note: assume that air is an ideal gas.

V_{model} = 1113 km/h

p_{model} = 13 atm (a *pressurized* wind tunnel is needed to simulate the proper free-flight Reynolds number!)

Exercise 3

A airfoil shaped as a flat plate of unit chord is invested by a uniform flow with an angle of attack $\alpha = 12^o$. It is found that $A' = 0.03$ N, $N' = 1.2$ N and $M'_{LE} = -0.14$ N m. Compute the aerodynamic efficiency of the flat plate and the coordinate of the center of pressure.

$L'/D' = 4.18$
$x_{cp} = 0.117$ m

Exercise 4

Let us consider a thin airfoil of chord c , and let x be the chord coordinate. The pressure coefficient is:

$$c_p \text{ suction side} = \begin{cases} 1 - 300\left(\frac{x}{c}\right)^2 & for\ \ 0 \le \frac{x}{c} \le 0.1 \\ -2.22\bar{7} + 2.2\bar{7}\left(\frac{x}{c}\right) & for\ \ 0.1 \le \frac{x}{c} \le 1 \end{cases}$$

$$c_p \text{ pressure side} = 1 - 0.95\left(\frac{x}{c}\right) \qquad for\ \ 0 \le \frac{x}{c} \le 1$$

Assuming that the angle of attack is very small and neglecting friction, compute c_l, $c_{m\,LE}$ and the position of the center of pressure.

c_l = 1.398, $c_{m\,LE}$ = -0.5301, x_{CP}/c = 0.379