Fila A

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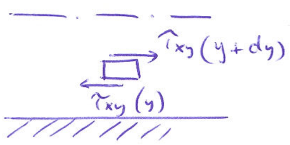
1



$$0 = \left[\tau_{xy}(y+dy) - \tau_{xy}(y)\right] dx + \left[P(x) - P(x+dx)\right] dy$$

$$\frac{d\tau_{xy}}{dy} = \frac{dP}{dx}$$

$$\frac{\partial}{\partial x}\vec{u} = 0 \; ; \; \frac{\partial}{\partial t} = 0$$

$$\tau_{xy} = \mu \frac{du}{dy} \; , \; u = u(y) \text{ solamente}$$

$$P = P(x) \text{ solamente}$$

$$\mu = \text{cnt.} \rightarrow \mu \frac{d^2u}{dy^2} = \frac{dP}{dx} = \text{cnt.}$$

$$\frac{du}{dy} = \frac{1}{\mu}\frac{dP}{dx} y + A \qquad u = \frac{1}{2\mu}\frac{dP}{dx} y^2 + Ay + B$$

$$y = \pm h \rightarrow u = 0 \implies A = 0 \; , \; B = -\frac{1}{2\mu}\frac{dP}{dx} h^2$$

$$\boxed{u = \frac{1}{2\mu}\frac{dP}{dx}\left(y^2 - h^2\right)}$$

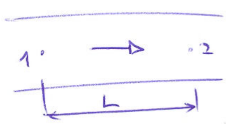
$$\boxed{u_{max} = u(0) = -\frac{1}{2\mu}\frac{dP}{dx} h^2}$$

$$\boxed{u_{media} = \frac{1}{2h}\int_{-h}^{h} \frac{1}{2\mu}\frac{dP}{dx}(y^2 - h^2)\, dy = -\frac{1}{3\mu}\frac{dP}{dx} h^2 = \frac{2}{3} u_{max}}$$

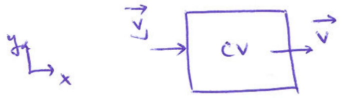
$$u^* = \frac{u}{u_{media}} \qquad y^* = \frac{y}{h} \qquad \rightarrow \boxed{u^* = \frac{3}{2}\left(1 - y^{*2}\right)}$$

$$h_L = \frac{\Delta P}{\rho g} \; ; \qquad u_{media} = +\frac{1}{3\mu}\frac{\Delta P}{L} h^2 \qquad \left(-\frac{dP}{dx} = \frac{P_1 - P_2}{L} = \frac{\Delta P}{L}\right)$$

$$\boxed{h_L = \frac{\Delta P}{\rho g} = \frac{3\mu\, u_{media}\, L}{h^2 \rho g}}$$



2



La velocità del getto relativa al carrello vale $v_J - v$

La portata d'acqua che entra nel carrello è: $\dot{m}_{H_2O} = \rho A (v_J - v)$

$$m_{H_2O} = \int_0^t \dot{m}_{H_2O}\, dt = \int_0^t \rho A (v_J - v)\, dt = \rho A v_J t - \rho A \int_0^t v\, dt$$

= massa d'acqua nel carrello al tempo t

$$\sum \vec{F} = \frac{d}{dt}(m_{tot}\vec{v}) + \cancel{\sum_{out} \beta \dot{m} \vec{v}} - \sum_{in} \beta \dot{m} \vec{v} \qquad \beta \simeq 1$$

Proiettando su x:

$$0 = \frac{d}{dt}\left[(m_{carrello} + m_{H_2O})\, v\right] - \dot{m}_{H_2O} v_J =$$

$$= (m_c + m_{H_2O}) \frac{dv}{dt} + \dot{m}_{H_2O}(v - v_J) =$$

$$\boxed{m_c \frac{dv}{dt} + \rho A v_J t \frac{dv}{dt} - \rho A \left(\int_0^t v\, dt\right) \frac{dv}{dt} + \rho A (v_J - v)(v - v_J) = 0}$$

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$C_D = f(Re)$, si adotta la similitudine di Reynolds

$$Re_{modello} = \frac{\rho_{aria} U_{modello} D_{modello}}{\mu_{aria}} = \frac{\rho_{acqua} U_{prototipo} D_{prototipo}}{\mu_{acqua}} = Re_{prototipo}$$

$$U_{modello} = U_{prototipo} \frac{\rho_{acqua}}{\rho_{aria}} \frac{\mu_{aria}}{\mu_{acqua}} \frac{D_{prototipo}}{D_{modello}} = 0{,}560 \frac{999{,}1}{1{,}184} \frac{1{,}849 \times 10^{-5}}{1{,}138 \times 10^{-3}}\, 24$$

$$= 184 \text{ m/s}$$

Tale velocità corrisponde ad un Mach $\simeq$ 0,55 ; nel modello sono presenti dei fenomeni d'onda che aumentano la resistenza rispetto al caso incomprimibile.

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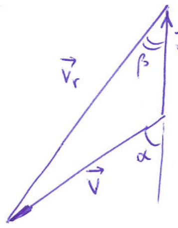
$$\boxed{\Delta \dot{E}_{mecc,\,fluido} = \dot{m} g h = 10 \times 9{,}81 \times 26 = 2550{,}6 \text{ W}}$$



$$\vec{V}_{cv} = \frac{2\pi n}{60} R \, \vec{e}_\theta$$

$$\vec{V}_r = \vec{V} - \vec{V}_{cv}$$

$$|\vec{V}_{cv}| = \frac{2\pi\, 300}{60}\, 0{,}5 = 15{,}708 \text{ m/s}$$

$$V_r \sin\beta = V \sin\alpha$$

$$V_r \cos\beta - V_{cv} = V \cos\alpha$$

$$\rightarrow \tan\alpha = 1{,}198 \rightarrow \alpha = 50{,}16°$$

$$\rightarrow \boxed{V = 22{,}79 \text{ m/s}}$$

$$\vec{M} = m \cancel{\frac{d}{dt}(\vec{R}\times\vec{v})} + \sum_{out} \dot{m}\, \vec{R}\times\vec{v} - \sum_{in} \dot{m}\, \vec{R}\times\vec{v}$$

$$\boxed{\vec{M} = 10 \times 0{,}5 \times 22{,}79 \times \sin(90 + 50{,}16)\, \vec{k} = 73{,}01 \text{ N m } \vec{k}}$$

$$\boxed{\dot{W}_{albero} = \omega M = \frac{2\pi n}{60} M = 2293{,}8 \text{ W}}$$

$$\boxed{\eta_{turbina} = \frac{\dot{W}_{albero}}{\Delta \dot{E}_{mecc,\,fluido}} = 0{,}899} \qquad \boxed{\eta_{globale} = \eta_{turb.} \times \eta_{alternatore} = 0{,}82}$$

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$$\dot{V} = V A_c = V \frac{\pi D^2}{4} = 3\, \frac{\pi\, 0{,}2^2}{4} = 0{,}0942 \text{ m}^3/\text{s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{1{,}225 \times 3 \times 0{,}2}{1{,}802 \times 10^{-5}} = 4{,}079 \times 10^4 \Rightarrow \text{flusso turbolento}$$

$$\epsilon/D = \frac{2\times 10^{-6}}{2\times 10^{-1}} = 10^{-5}$$

→ dal diagramma di Moody : $f \simeq 0{,}022$

(si può avere più accuratamente con Colebrook ...)

$$\Delta P_L = f \frac{L}{D} \frac{\rho V^2}{2} = 9{,}09 \text{ Pa}$$

$$\boxed{\dot{W}_{elettrica} = \frac{\dot{W}_{pompa,u}}{\eta_{globale}} = \frac{\dot{V}\Delta P_L}{0{,}62} = \frac{0{,}0942 \times 9{,}09}{0{,}62} = 1{,}4 \text{ W}}$$

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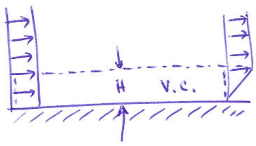

Lo strato limite si forma per la viscosità del fluido → aderenza del fluido alla parete.

V.C: rettangolare di altezza H

Conservazione della massa: $\rho U_0 H = \dot m_{out} + \rho \int_0^H U_0 \frac{y}{H}\, dy$

$$\to \boxed{\dot m_{out} = \rho U_0 H - \rho \frac{U_0 H}{2} = \rho U_0 \frac{H}{2}}$$ portata uscente dalla superf. orizzontale sup. del V.C.

Principio della quantità di moto:

$$\sum \vec F = \cancel{\frac{d}{dt}\int_{cv} \rho \vec v\, dV} + \int_{cs} \rho \vec v (\vec v_r \cdot \vec n)\, dA$$

Proiettando sull'asse x

$$-F_{Rx} = \underbrace{\rho U_0 \frac{H}{2} U_0}_{y=H} + \underbrace{\rho \int_0^H \left(U_0 \frac{y}{H}\right)^2 dy}_{x=L} - \underbrace{\rho U_0^2 H}_{x=0} = -\rho U_0^2 \frac{H}{6}$$

forza della parete sul fluido, in senso opposto al moto

Quindi: $\boxed{F_{Rx} = \rho U_0^2 \frac{H}{6}}$

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$$I = f(P, \rho, c_s)$$

$[W/m^2]$ $\left[\frac{N}{m^2}\right]$ $\left[\frac{kg}{m^3}\right]$ $[m/s]$

$= \left[\frac{kg}{s^3}\right]$ $= \left[\frac{kg}{m s^2}\right]$

$$\begin{array}{c|ccc} & kg & m & s \\ \hline P & 1 & -1 & -2 \\ \rho & 1 & -3 & 0 \\ c_s & 0 & 1 & -1 \end{array}$$

det $A = 0$!

Scelgo 2! G.F.: $\boxed{\rho \text{ e } c_s}$

$\pi_1 = f(\pi_2)$ $\boxed{\pi_1 = \frac{I}{\rho c_s^3}}$ $\boxed{\pi_2 = \frac{P}{\rho c_s^2}}$

$$\boxed{\pi_1 = \frac{(L/c_s)^3}{\rho L^3 / I} = \frac{T^3_{acustico}}{T^3_{intensità}}}$$

$$\boxed{\pi_2 = \frac{(L/c_s)^2}{(\rho L^2/P)} = \frac{T^2_{acustico}}{T^2_{dinamico}}}$$