BIOSKINS:
L'analogie macroscopique

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Toulouse, 14 Septembre 2017

WHY THIS TITLE?

"A mathematician is a person who can find analogies between theorems;
a better mathematician is one who can see analogies between proofs
and the best mathematician can notice analogies between theories.
One can imagine that the ultimate mathematician is one
who can see analogies between analogies."
S. Banach

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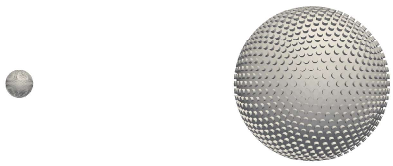

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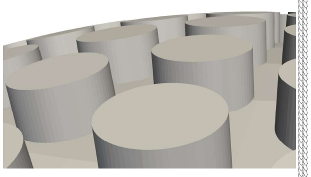
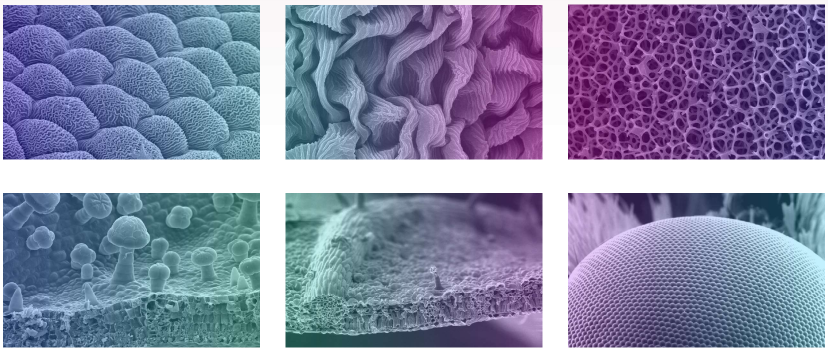


WHAT DOES THIS ANALOGY CONSIST OF (and why do I like multiscales)?

THE CHALLENGE: apply homogenization to non-homogeneous phenomena



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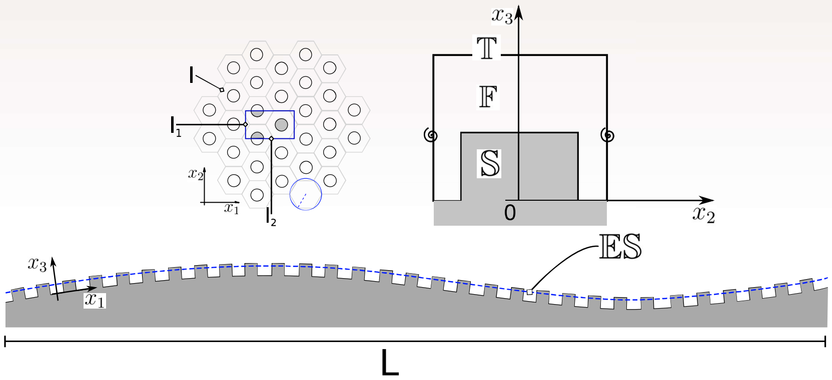
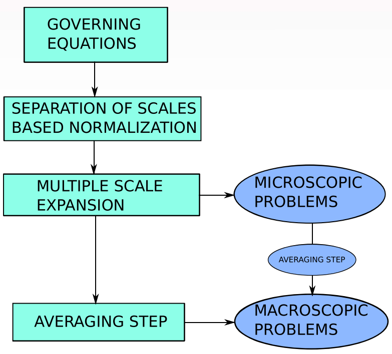


Rough surface under investigation, composed of a hexagonal periodic lattice:

- $\mathbb{FS}$ microscopic cell ($l_1 \times l_2 \times l_3 \simeq l$)
- porosity $\vartheta = |\mathbb{F}|/|\mathbb{FS}|$
- $\mathbb{ES}$ fictitious equivalent surface

THE METHOD: standard homogenization



$$\epsilon = \frac{l}{L} \ll 1$$

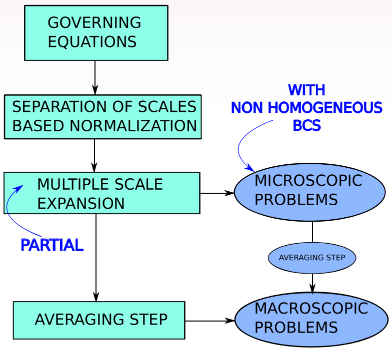
We introduce $x, x' = \epsilon x$

$$\mathbf{u}(x, x', t) = \sum_{n=0}^{N} \epsilon^n \mathbf{u}^{(n)}(x, x', t)$$

$$p(x, x', t) = \sum_{n=0}^{N} \epsilon^n p^{(n)}(x, x', t)$$

$$\langle f \rangle := \frac{1}{|\mathbb{FS}|} \int_{\mathbb{F}} f \, dV.$$

THE METHOD: modified homogenization



$$\epsilon = \frac{l}{L} \ll 1$$

$$x = (x_1, x_2, x_3), x' = \epsilon(x_1, x_2, 0)$$

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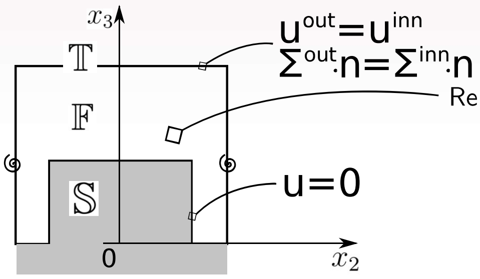
THE METHOD



$$\text{Re}\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=-\frac{\partial p}{\partial x_i}+\nabla^2 u_i$$

$$\frac{\partial u_i}{\partial x_i}=0$$

Further hypotheses:

- up to $\text{Re}=U^{out}L/\nu=\mathcal{O}(1/\epsilon)$
- every kind of microscopic structure

After some algebra...

THE RESULTING MODEL

$$\langle u_i^{(0)} \rangle = -\epsilon^2 \mathrm{Re}\, \mathcal{K}_{ij} \frac{\partial p^{(0)}}{\partial x'_j} + \epsilon\, \mathcal{L}_{ilk} \left(\frac{\partial u_l^{out}}{\partial x'_k} + \frac{\partial u_k^{out}}{\partial x'_l} \right) |_{\mathbf{x}' \in \mathbb{ES}},$$

$$\langle p^{(1)} \rangle = -\mathcal{A}_j \frac{\partial p^{(0)}}{\partial x'_j} + \mathcal{B}_{lk} \left(\frac{\partial u_l^{out}}{\partial x'_k} + \frac{\partial u_k^{out}}{\partial x'_l} \right) |_{\mathbf{x}' \in \mathbb{ES}},$$

$\mathcal{K}_{ij} = \langle K_{ij} \rangle$, $\mathcal{L}_{ijk} = \langle L_{ijk} \rangle$, $\mathcal{A}_j = \langle A_j \rangle$, $\mathcal{B}_{lk} = \langle B_{lk} \rangle$,

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$$\begin{cases} \dfrac{\partial A_j}{\partial x_i} - \nabla^2 K_{ij} = \delta_{ij}, & j = 1, 2, \ \forall\, i \\ \dfrac{\partial K_{ij}}{\partial x_i} = 0 & j = 1, 2, \ \forall\, i, \\ K_{ij} = 0 \quad \text{on } \partial \mathbb{S}, \\ \dfrac{\partial K_{jl}}{\partial x_3} + \dfrac{\partial K_{3l}}{\partial x_j} = 0, x_3 \in \mathbb{T} & l = 1, 2, \ \forall\, j, \\ K_{ij} \quad \text{periodic along } x_1 \text{ and } x_2, \end{cases}$$

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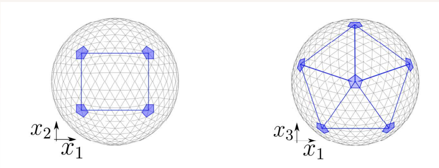
cf. Beavers & Joseph (1967) and Navier (1823).

HOW TO USE IT...

Once the phenomenon to be analyzed is choosen, we need to:

- select the microscopic cell and solve the microscopic problems for K_{ij} and L_{ijk}
- compute the averaged values $\mathcal{K}_{ij} = \langle K_{ij} \rangle$ and $\mathcal{L}_{ijk} = \langle L_{ijk} \rangle$
- put them in the model equation
- use the model equation as a boundary condition for the macroscopic simulation

HOW TO USE IT... flows past rough spherical particles



Rough spherical particles completely defined by 3 free parameters:

- ξ, linked to the number of vertices of the quasi-regular icosahedron
- ϑ, the porosity
- h, the height of each protrusion

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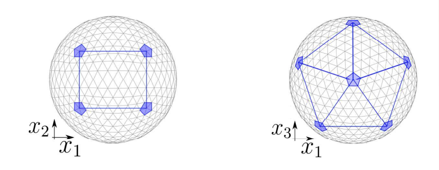

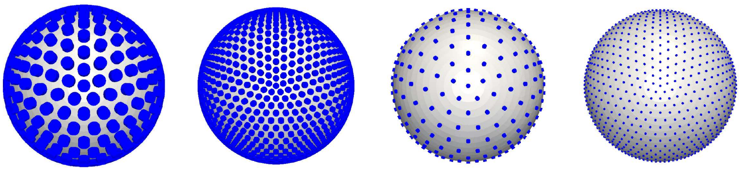

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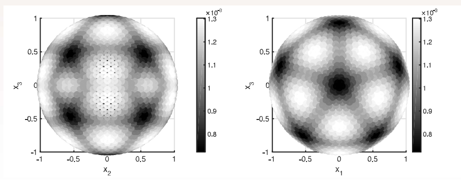

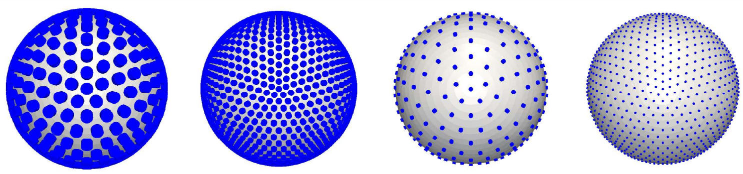

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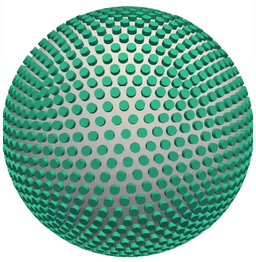


Free uniform unidirectional flow $(U^{out}, 0, 0)$ past a spherical particle such that:

- $\vartheta = 0.60$
- $\xi = 12 \implies 1440$ protrusions

With this configuration we will:

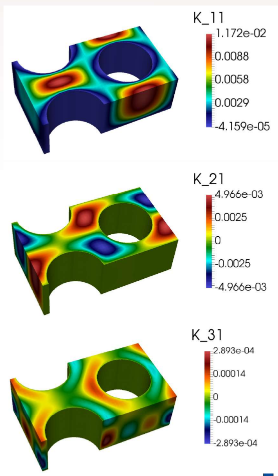
- validate the new boundary condition (by comparisons with DNS)
- study the dynamics of the rough sphere
- study the thermal boundary layer around the rough sphere

MICROSCOPIC PROBLEMS: the wettability tensor $\mathcal{K}_{ij}$

$$\begin{cases} \dfrac{\partial A_j}{\partial x_i} - \nabla^2 K_{ij} = \delta_{ij} \\ \dfrac{\partial K_{ij}}{\partial x_i} = 0 \\ K_{ij} = 0 \quad \text{on } \partial\mathbb{S} \\ \dfrac{\partial K_{jl}}{\partial x_3} + \dfrac{\partial K_{3l}}{\partial x_j} = 0, x_3 \in \mathbb{T} \\ K_{ij} \quad x_1, x_2\text{-periodic} \end{cases}$$

$\mathcal{K}_{ij} = 0 \quad i \neq j$ by symmetry
$\mathcal{K}_{33} = 0$ by axis orientation

weak anisotropy: $\mathcal{K}_{11} \neq \mathcal{K}_{22}$



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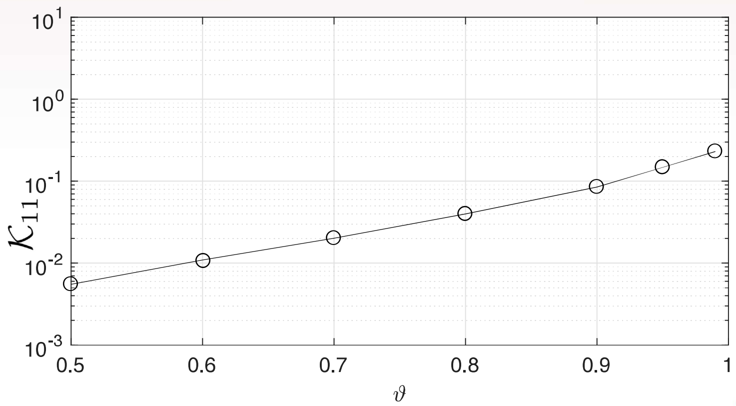

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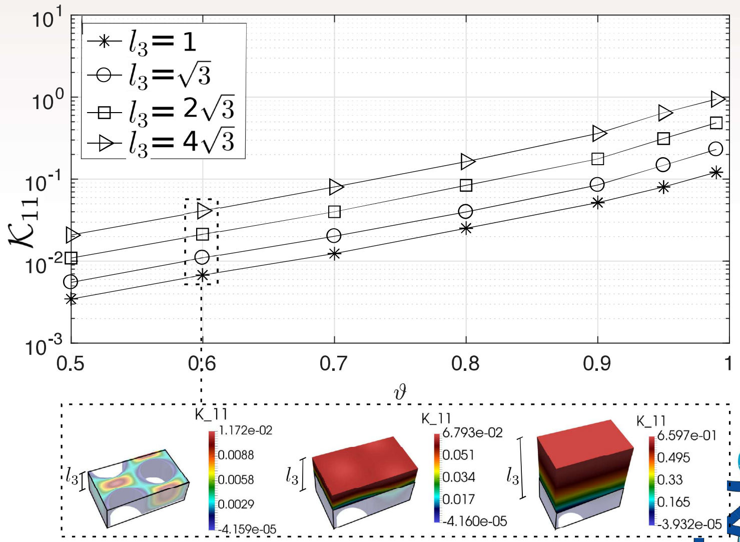

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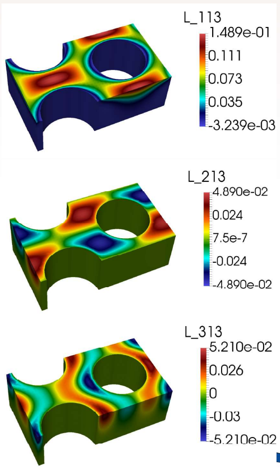
$\mathcal{L}_{ii3} \neq 0 \quad i = 1, 2$ by symmetry and axis orientation

weak anisotropy: $\mathcal{L}_{113} \neq \mathcal{L}_{223}$



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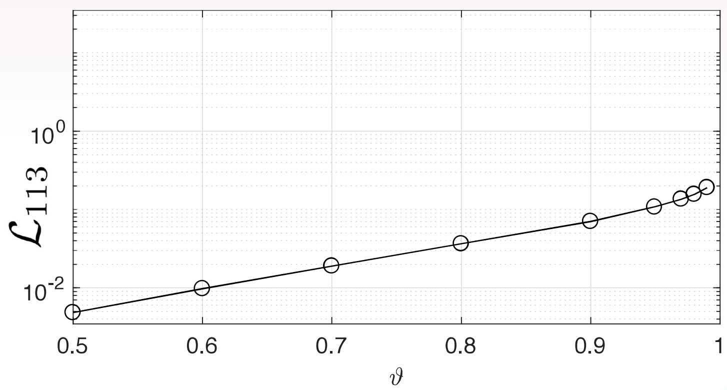

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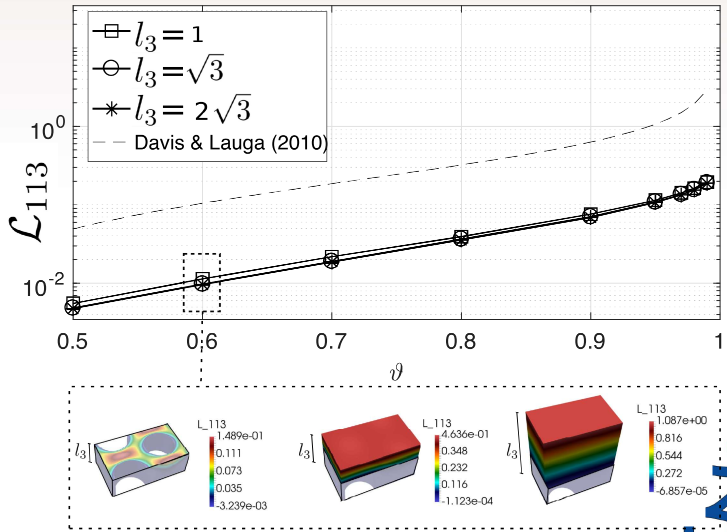
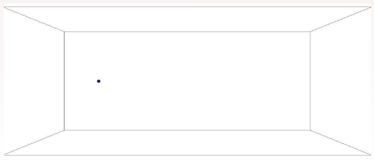
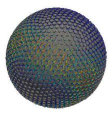


VALIDATION: DNS of flow past rough spherical particles

- free uniform unidirectional flow $(U, 0, 0)$, Re $= 100$
- $200R \times 80R \times 80R$
- 15 to 25 millions cells
- OpenFOAM on CALMIP with up to 500 cores
- up to about 1.2×10^5 CPU hours for each DNS to reach the steady state at Re$= 100$





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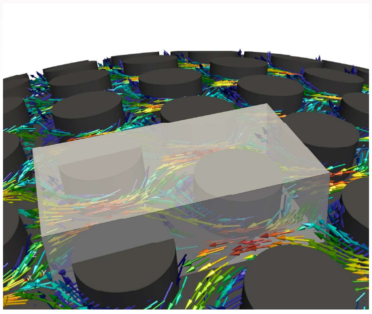
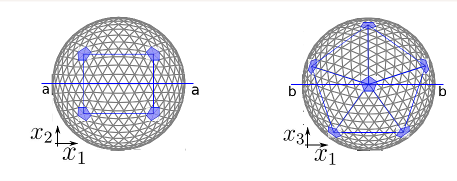
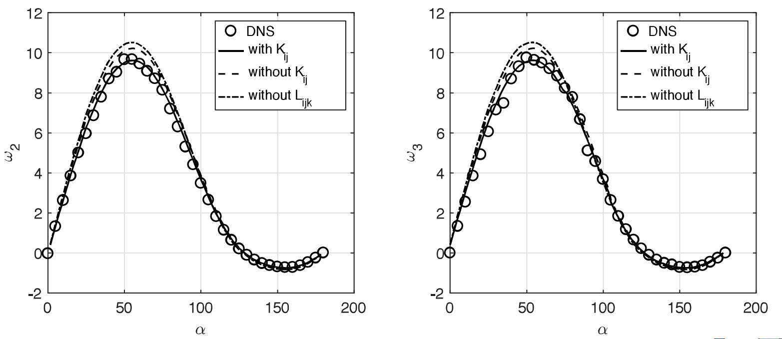


VALIDATION: local comparisons





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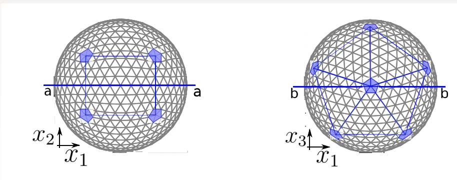

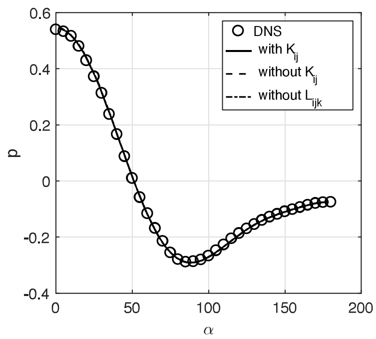

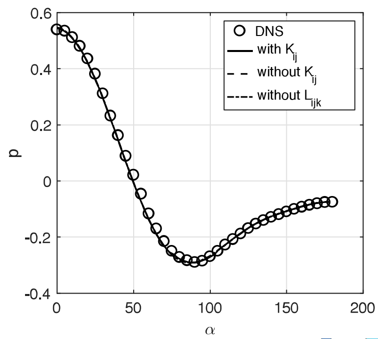

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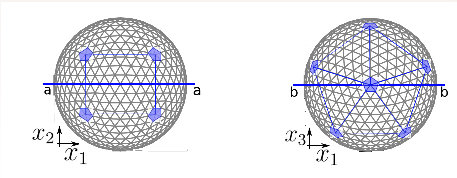
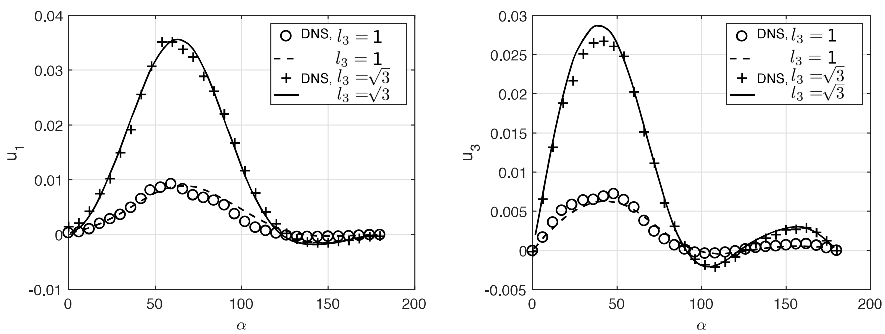
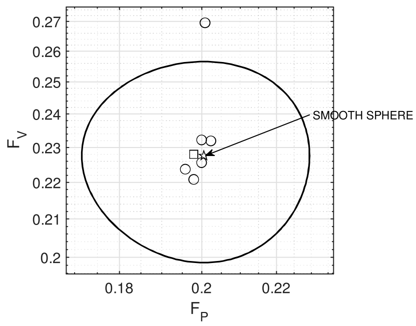




VALIDATION: integral comparisons

$$F_{TOT} = F_P + F_V$$
$$F_{TOT}^{smooth} = F_{TOT}^{rough} + \mathcal{O}(\epsilon)$$



in agreement with homogenization and previous theoretical results

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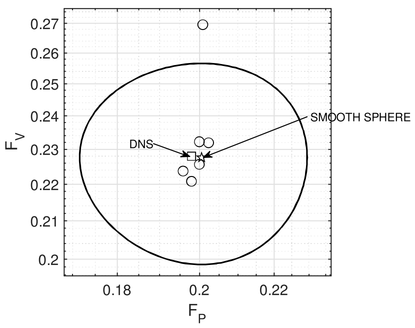

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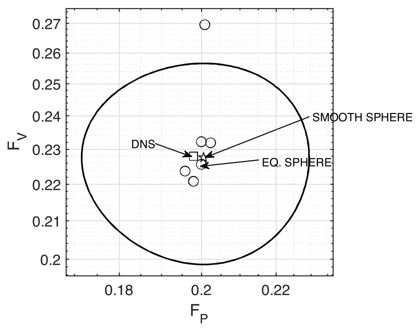

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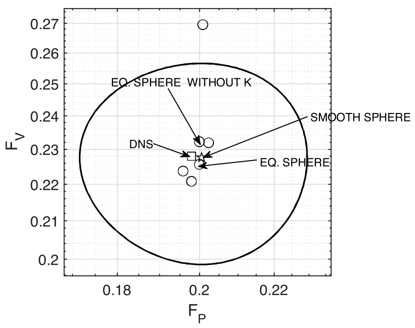

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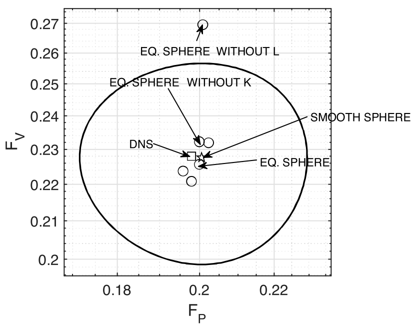

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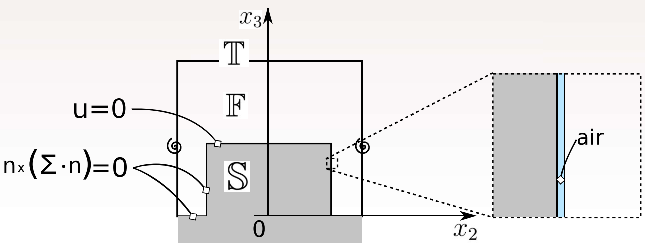
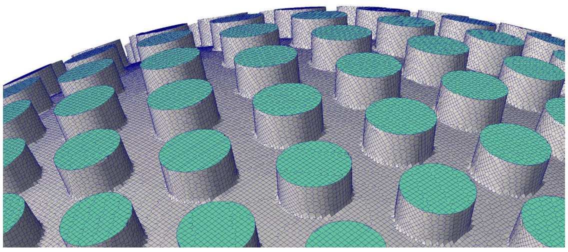
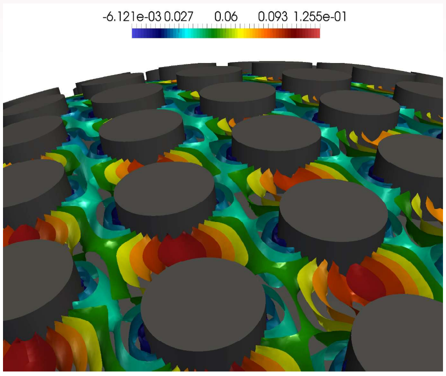
TOWARDS DRAG REDUCTION



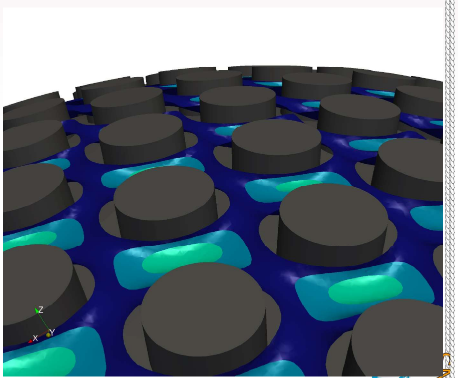


TOWARDS DRAG REDUCTION

mixed configuration



no-slip everywhere



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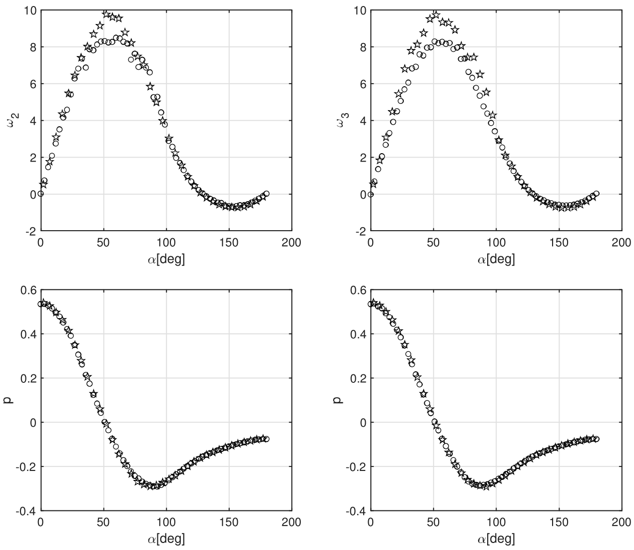
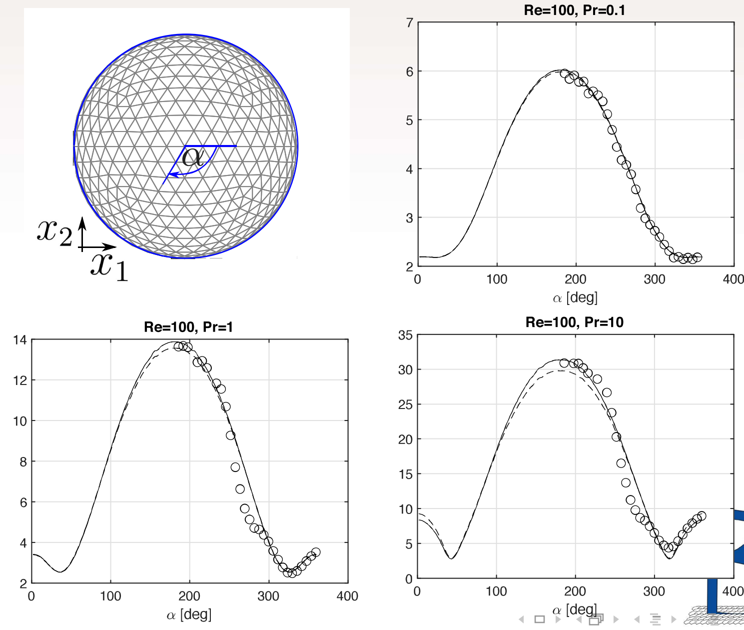


drag reduced by about 20% with respect to the no-slip case
(cf. Gruncell et al. 2013 for different configurations).

THERMAL BOUNDARY LAYER: $Nu = -\partial T/\partial n$



CONCLUSIONS and more

A robust BC for incompressible fluid flows above rough (and smooth?) surfaces has been developed and validated in a non trivial case (showing a good agreement also when the homogeneity is weakened)

- go further in the ϵ-expansion to develop a strategy to better locate the equivalent surface in the macroscopic analogy
- extend the procedure in a stochastic direction to analyze irregular surfaces without a fixed shape of the protrusions
- following the path of drag reduction, homogenization of two phase flows is necessary
- study the dynamics of Janus-like spheres optimizing distribution and shape of the protrusions to enhance their dynamic performances
- perform homogenization for the heat problem to explore different kinds of boundary conditions for T