COHERENT STATES IN TRANSITIONAL PIPE FLOW

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Abstract A numerical simulation of the early nonlinear stages of transition in a pipe flow, for which the base profile presents a small defect, reveals the formation of coherent states reminiscent of the recently found non-linear travelling waves.

Keywords: transition in pipe flow, coherent structures, streamwise vortices and streaks.

1. INTRODUCTION

The causes of transition in pipe flow have been debated for a long time and have been considered to be unrelated to the linear stability of the underlying, parabolic base flow. It is, in fact, accepted that Hagen–Poiseuille flow is stable to all infinitesimal disturbances. Finite amplitude disturbances are necessary to provoke transition, which is generally believed to take place for a Reynolds number Re around 2000. The observed transition “point” moves to higher Re when the magnitude of the inflow disturbance field decreases, pointing to the role of the *receptivity* in deciding the fate of the flow. Since transition is eventually observed in any experimental set-up (including those with exceptional low level of flow disturbances), and since all eigenmodes of the linearized stability operator are damped for all values of Re, there must be a mechanism for the amplification of ambient noise, leading to the subsequent breakdown of the motion. Via such a mechanism the system filters the environmental disturbances and transforms them into instability waves.

Current interpretation of the results of linear stability theory points in the direction of transient growth of disturbances as the likely candidate for the initial phase of transition (Schmid and Henningson, 2001). The argument goes that in subcritical conditions the initial/inflow disturbance field can be amplified (transiently) to such a level that eventually nonlinear phenomena kick in causing transition of the flow, thus superseding the asymptotic modal behaviour of exponential decay. The weak point of the argument appears to be the – as yet unclearly defined – generic nonlinear mixer responsible for maintaining large amplitude disturbances during transition. It has been argued that such a generic mixer, which is needed to turn streamwise streaks into streamwise vortices in the self-sustaining cycle of near-wall turbulence, begins with the secondary, inviscid and modal instability of the streaks (Waleffe, 2003). Recent results demonstrate that a streaky base flow can support a strong algebraic amplification of perturbations (Hoepffner et al., 2004) so that a scenario of transition is emerging based on a succession of transient phenomena (see also Grossmann, 2000). There is as yet little experimental/numerical evidence to show what precisely these transients states are, and how they follow one another (i.e. what the phase-space trajectory is, in dynamical systems terminology). The influence of the environment in deciding the states which prevail and their space-time evolution is undoubtedly crucial.

More recent work has focussed on the possible presence of defects in the base flow (eventually caused and/or maintained by the transient growth) which can give rise to exponential amplification of perturbations (Dubrulle and Zahn, 1991; Bottaro et al., 2003; Gavarini et al., 2004). Here the argument goes that defects of small (albeit finite) amplitude would cause a distorted base flow that is linearly unstable. The unlimited growth provided by exponential amplification represents an initial stage of transition, which does not require any speculation on subsequent processes (the generic nonlinear mixing) for a high level of disturbances to be maintained. Furthermore, the existence of defects does not hamper the possibility of transient growth, which remains unaltered. It is hence likely, as argued by Biau and Bottaro (2004), that transient growth and flow defects cooperate in defining the initial stages of transition in shear flows of the Poiseuille, Hagen–Poiseuille or Couette type.

As far as the nonlinear stages are concerned, there has been some excitement recently over the discovery of unstable travelling wave solutions which could constitute the "skeleton" of states around which transitional and perhaps turbulent flows organize. Such solutions have been identified theoretically for pipe flow by Faisst and Eckhardt (2003) and Wedin and Kerswell (2004) with the help of continuation techniques; for example, Faisst and Eckhardt imposed an initial body force on the momentum balance equations, capable of generating streamwise vortices in the pipe cross-section. The amplitude of the driving force was then decreased (while simultaneously increasing Re) until eventually a finite amplitude solution (with vanishing body force) was reached. The chosen domain was streamwise periodic, implying that the receptivity of the flow was unaccounted (and unaccountable) for. The excitement arose when experimenters in Delft observed coherent states in a pipe very similar to the computed ones (Hof et al., 2004). The experiments were carried out by injecting fluid through a hole in the pipe for a short time, and observing the puffs generated by the injection as it passed through the observation window 150 diameters downstream of the injection point. The temporal observations of the puff were then translated into spatial observations by the use of Taylor's hypothesis, showing that at $Re = 2000$ the puff is about $12R$ long (with R the cylinder radius), with three high speed streaks near the wall at the downstream (leading) edge of the puff, that transform into six near-wall high speed streaks at the upstream edge of the puff. At the center of the pipe a large low speed streak was observed, with arms stretching radially towards the walls. Not only did the experimental flow states present a remarkable similarity to some of the computed ones, also features of the solutions such as the amplitudes, wavelengths and phase speeds were in good agreement, supporting a scenario that describes transition to turbulence through the self-organization of the flow around some dominant travelling waves.

Recent direct numerical simulations (in a streamwise periodic domain) by Priymak and Miyazaki (2004) support the existence of equilibrium puffs at Re as low as 2200, propagating in the direction of the mean flow at a speed of the order of the bulk velocity, while maintaining their spatial downstream length equal to about 40 cylinder radii. The disagreement in the puff's length between experiments and simulations could perhaps be explained by the difficulty in identifying properly the leading and trailing edges of the puff, by the characteristics of the forcing employed to trigger the puff, and by the fact that simulations in streamwise periodic domains (even very long ones, as in the present case where a domain $50R$ long was employed) can only mimic the real, spatially developing situation. When periodic conditions are employed, perturbations exiting the domain are continuously fed into the inflow plane thus providing energy for the sustainment of the puff.

The present paper aims at presenting further evidence for the existence of flow states such as those computed by Faisst and Eckhardt (2003) and Wedin and Kerswell (2004). It has been chosen here to focus on a transition scenario with *spatially developing* disturbances and to capture both linear and nonlinear stages through a numerical computation in which the fast initial development of perturbations is caused by the linear instability of a mildly distorted Hagen–Poiseuille flow. Hence, the present simulation does not describe the evolution of a puff induced by a Dirac-like perturbation, rather it describes the patterns produced when transition is triggered and sustained by a permanent inflow forcing.

2. THE TRAVELLING WAVES

The existence of families of finite-amplitude coherent states in shear flows has been known for some time (Nagata, 1990; Clever and Busse, 1997; Waleffe, 2001). However, the symmetry properties of these families were such that no straightforward connection could be established with the coherent structures

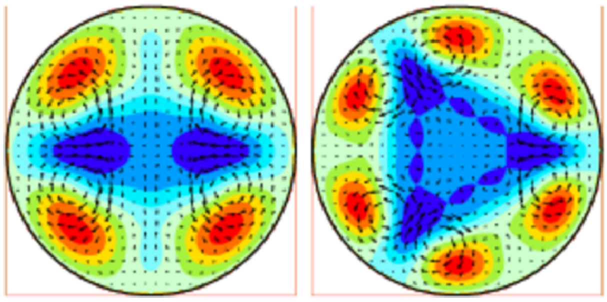

Figure 1. Downstream averaged travelling waves of the C_2 and C_3 families (after Faisst and Eckhardt).

observed in turbulent shear flows. In pipe flow the solutions found by Faisst and Eckhardt (2003) and Wedin and Kerswell (2004) are travelling waves (TW), moving downstream at wave speeds larger than the bulk velocity. Although the stability/instability of such states is currently not known, they are thought to be unstable solutions of the Navier–Stokes equations. It is speculated that they form a chaotic repellor with the system's trajectory wandering in phase space among mutually repelling solutions; thus the flow would remain in the vicinity of a given TW state for a while before going elsewhere. No formal mathematical justification has ever been given for this behaviour, although Christiansen et al. (1997) have shown results on the one-dimensional Kuramoto–Sivashinsky system endorsing the so-called "Hopf's description of chaos", with a dynamics based on unstable recurrent patterns. Just as the theory of finite amplitude TW was being developed, the above speculative picture of the early stages of transition received experimental support by the measurements conducted by Hof et al. (2004) and van Doorne (2004).

The TWs that appear earliest (in terms of the Reynolds number) are those denoted as C_2 and C_3 by Faisst and Eckhardt, where the subscripts 2 and 3 refer to the azimuthal rotation symmetry, for example a C_3 state is invariant under rotation around the pipe axis by an angle $2\pi/3$. Streamwise averaged C_2 and C_3 states are displayed in Figure 1; they appear for Re as low as 1250. Notable characteristics of such states (at their saddle-node bifurcation points) are the streamwise wavelengths (scaled with the pipe radius), which equal 4.19 and 2.58 for the C_2 and C_3 states, respectively, and the phase speeds (scaled with $U_{\max}$, the maximum velocity on the pipe axis in the laminar case), which equal 0.71 and 0.64, also for the C_2 and C_3 states, respectively. Furthermore, the streamwise velocity disturbance (with respect to the laminar state) is found to be one order of magnitude larger than the transverse velocity, and the dimensionless peaks are found at 0.19 (C_2) and 0.17 (C_3), and 0.017 (C_2) and 0.023 (C_3). Although such peak disturbance values increase with increasing of Re, the qualitative picture of the states changes little, with rather static high speed streaks near the wall (in red) and low speed streaks (in blue) which change shape and position near the center of the cross-section.

3. THE PRESENT SIMULATION

A direct numerical simulation has been conducted for the flow in a pipe of streamwise extent equal to 80 cylinder radii, with inflow and outflow boundaries and at a Reynolds number $Re = 3000$. The incompressible Navier–Stokes equations written in cylindrical coordinates are discretized by a second order finite volume technique (for details see Eggels et al., 1994); after numerical tests it has been decided that a resolution of $64 \times 32 \times 640$ grid points in the radial, azimuthal and axial directions, respectively, is adequate for our purposes. The underlying axisymmetric base motion consists of the Hagen–Poiseuille flow plus a *minimal defect* (Gavarini et al., 2004), which is forced via an appropriate source term in the governing equations throughout the whole length of the pipe. We do not dwell on the possible physical origins of the imposed mean flow deflection; we simply assume that it is due to environmental effects. Due to the presence of the defect an axisymmetric mode of the linear stability operator becomes unstable. Such a mode – with a given initial amplitude – is prescribed at the inflow section of the pipe; it grows exponentially and is responsible for the early stages of transition. For further details on the choice of the defect and its stability characteristics refer to Gavarini et al. (2004).

In Figure 2 we have plotted the energy of the various modes produced by nonlinear interactions against the streamwise distance x. The modes are indicated be a number pair (m, n) with m the azimuthal wave number of the disturbance and n denoting its frequency ω_n. The exponentially unstable disturbance is noted as (0, 2) in figure 2. We further observe that small amplitude random noise has been introduced at $x = 0$ to permit rapid growth of other modes due to the subharmonic instability of the primary axisymmetric pattern. The subharmonic mode labelled (2, 1) dominates the spectrum around $x \approx 50$.

It is precisely the (2, 1) mode, and to some extent also the (0, 0) and the (4, 0) modes (the latter is shown by a thin solid line in Figure 2) which define the structure of the motion in downstream regions of the pipe, i.e. for x around 60. In Figure 3, the pipe has been unfolded in a plane, and an instantaneous plan view at $r = 0.7$ is shown. It should be noticed that axisymmetry is gradually broken and that the subharmonic disturbance (2, 1) dominates until $x \approx 60$, from which point on the mean flow correction, i.e. mode (0, 0), becomes more energetic and individual high-frequency, high-wavenumber structures become more blurred. The length of each individual Λ structure is about 5 pipe radii (in the x-range where they are visible; this is quite close to the

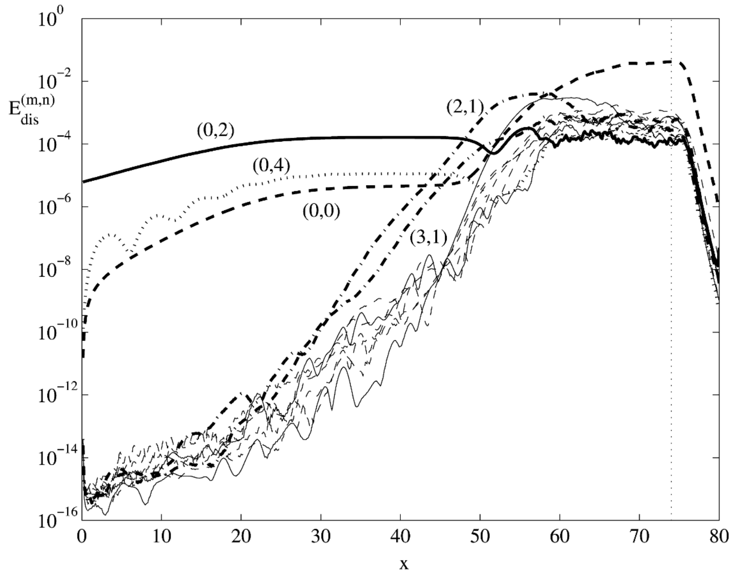

Figure 2. Evolution of the different Fourier components of the disturbance energy density with x. The number pair (m,n) next to each line defines the azimuthal wavenumber m of the mode and its frequency ω_n. For example, the initial condition, which consists only of the $(0, 2)$ mode, aside from small amplitude random noise, is axisymmetric since $m = 0$ and is characterised by a frequency number $n = \pm 2$ (that is $\omega_2 = 1$); similarly, the notation $(2, 1)$ denotes the mode with $m = 2$ (a wave with two periods along the circumference) and $n = \pm 1$ (that is $\omega_1 = 0.5$, the fundamental frequency). The dotted vertical line at $x \approx 75$ indicates the start of the fringe region near the outflow plane, from which point on the equations are gradually rendered parabolic.

optimal wavelength found in the C_2 case by Faisst and Eckhardt), which translates to a phase speed of 0.4 (in units of $U_{\max}$) for $\omega = 0.5$.

Aside from the C_1 state which exists only at values of Re exceeding 3000 (Wedin and Kerswell, 2004), the theory predicts that as Re increases past 1250 successively new TWs with C_n symmetries make their appearance (the index n increasing monotonically with Re), i.e. finer and finer scales should emerge downstream in our spatial simulation. In the late transitional and turbulent regimes the problem becomes that of discerning each repelling state from one another in every given experimental/numerical data set, a task which could possibly be pursued by wavelet transform or by POD analysis.

In the qualitative approach pursued here, we will satisfy ourselves with showing that in the cross-section of the pipe flow structures exist resembling those found theoretically. In Figure 4, the instantaneous flow patterns at $x = 54$ and 56.6 are shown. Large scale streaks similar to those of the C_2 state are present near the pipe walls: the largest transverse velocity is around 0.07,

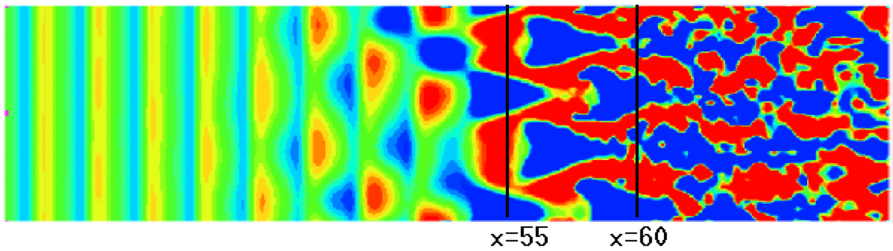

Figure 3. Instantaneous streamwise disturbance velocity in a $(x, r\theta)$ plane, with $r = 0.7$ The azimuthal modulation of the flow and the formation of staggered arrays of Λ vortices is strongly reminiscent of the so-called *H-type* transition in the flat plate boundary layer. The axes are not to scale; the horizontal axis spans from $x = 35$ to $x = 70$.

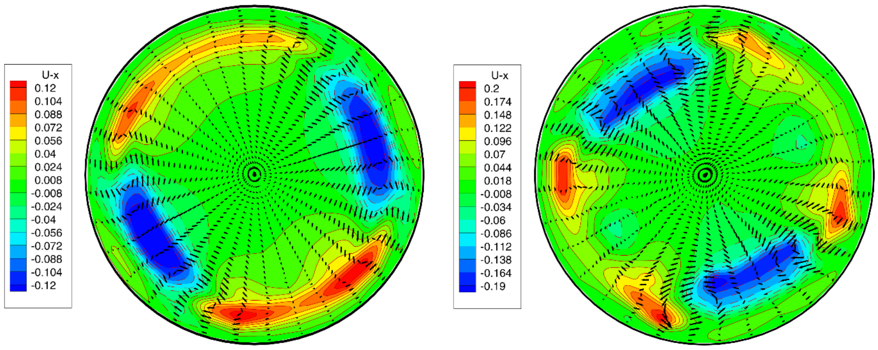

Figure 4. Instantaneous isocontours of the streamwise disturbance velocity, and velocity vector plots of the secondary flow, at $x = 54$ (left) and $x = 56.6$. The states resemble the travelling wave solution with C_2 symmetry. The colour scale to the left of each figure refers to the streamwise disturbance speed. The full range of disturbance velocity values is given only for the figure on the right.

whereas the streamwise disturbance velocity peaks at 0.2. The latter value is in line with theory (at a smaller Re), whereas the secondary speed is four times as large. The discrepancy could be attributed to a number of factor, e.g. to the presence of many harmonics in the flow. In fact, when we superpose only the three dominant modes present at $x = 56.6$, i.e. modes $(2, 1)$, $(0, 0)$ and $(4, 0)$, the resulting solution (Figure 5) is much less energetic than the full solution, besides displaying a remarkable similarity to the C_2 state.

In the streamwise interval shown in Figure 4 (which corresponds to roughly half a wavelength) the slow streaks (shown in blue) appear to have rotated half a wavelength in the azimuthal direction. In reality this is not the case, since in the two cases we are at x-positions centered on different (and staggered)

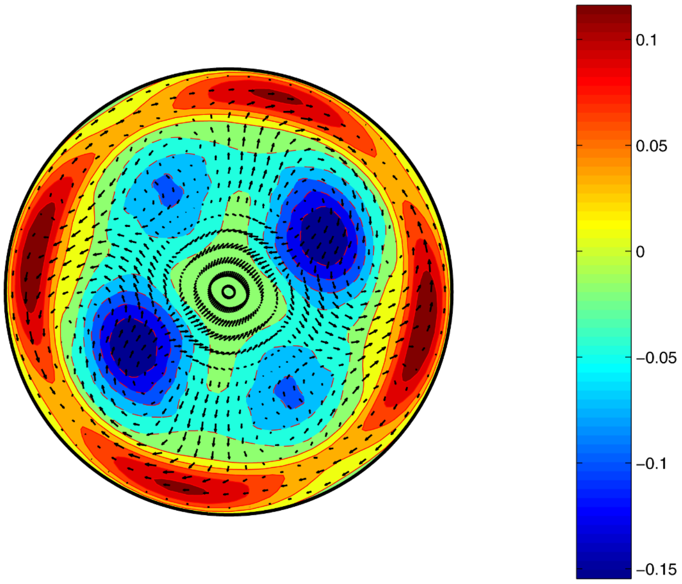

Figure 5. Sum of the three dominant Fourier modes at $x = 56.6$.

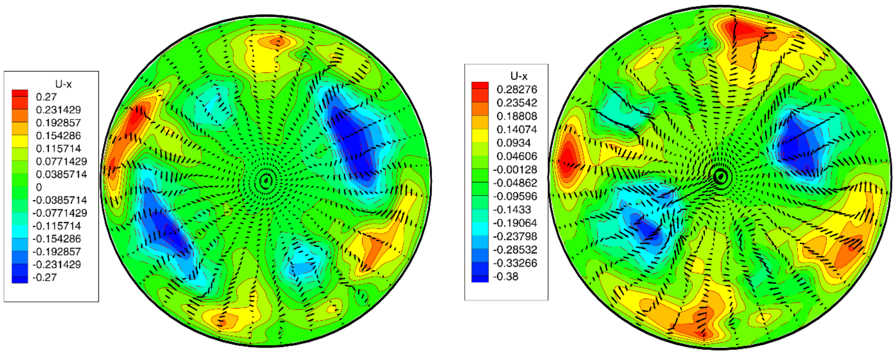

Figure 6. Instantaneous isocontours of the streamwise disturbance velocity, and velocity vector plots of the secondary flow, at $x = 59$ (left) and $x = 60$. The full range of disturbance velocity values is given only for the figure on the right.

Λ vortices (cf. Figure 3). As we proceed downstream (Figure 6), high and low speed streaks are intensified (the latter more, cf. the colour scale for the figure corresponding to $x = 60$), although the qualitative picture remains that of Figure 1 (left frame).

Even further downstream, the picture in Figure 3 would suggest that coherence is almost lost. However, inspection of the flow at the cross-sections $x = 74$ and 75 (the latter value already in the fringe region), reveal a configur-

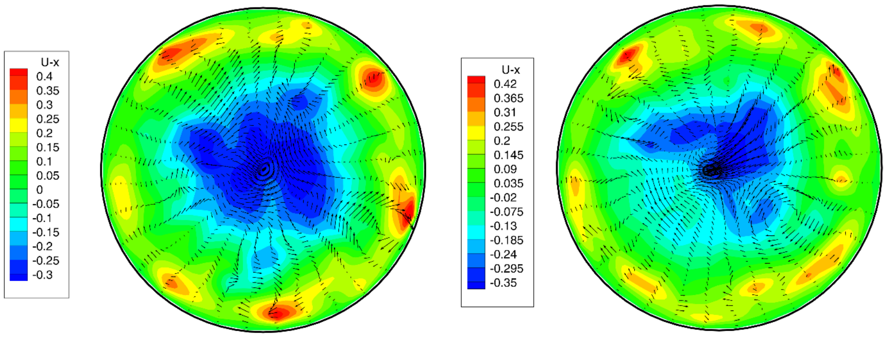

Figure 7. Instantaneous isocontours of the streamwise disturbance velocity, and velocity vector plots of the secondary flow, at $x = 74$ (left) and $x = 75$.

ation which matches well both the experimental observation of a puff by Hof et al. (2004) and theoretical predictions In particular, we observe in Figure 7 the presence of several high speed streaks near the wall, likely to result in increased values of the friction factor. At these streamwise positions the disturbance velocity peaks at $0.5U_{\max}$, with a maximum transverse speed exceeding $0.1U_{\max}$. The states depicted resemble the TW solution with C_3 symmetry.

4. CONCLUSIONS

A qualitative analysis of the instantaneous flow patterns observed in a transitional pipe flow at $Re = 3000$ has been presented. In contrast to previous numerical studies, transition has been triggered by the exponential amplification of small disturbances, evolving in a mildly distorted base flow, which have been followed in their spatial evolution. The path to transition considered is not necessarily that which is universally followed by all pipe flow experiments, it is just a plausible scenario which displays sufficiently generic features. Other scenarios exist (Gavarini et al., 2004). In the present case, central to transition is the formation of large Λ vortices which form staggered arrays, before small scale structures grow enough to destroy the coherence of such vortices. It can be argued that Λ structures constitute the basic units of transition; within them, low speed fluid is contained so that, in a cross-section, two large scale slow speed streaks appear. This corresponds to the C_2 travelling state of the theory. Along the flanks of the Λ structures (blue regions in Figure 3) high speed streaks of smaller dimensions can be observed. Downstream of the position where the Λ vortices break down, the cross-sectional picture of the flow displays a large patch of slow velocity fluid at the center of the pipe, with several small scale high-speed streaks near the wall. This picture is highly suggestive of the C_3 state of Faisst and Eckhardt.

Although these results are preliminary, they are sufficiently promising to warrant a more detailed analysis of the available data base. In particular, it would be interesting to decompose the results of the DNS to extract the states – bound to satisfy specific azimuthal symmetry constraints – which maximise the rate of dissipation energy, following the lead of recent results in this direction by Plasting and Kerswell (2005).

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