OPTIMAL DISTURBANCES:
A SEXY AND USELESS CONCEPT?

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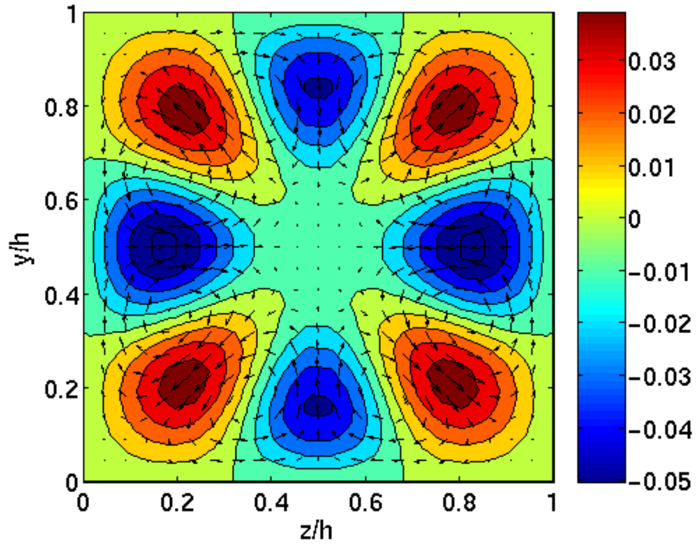
Goals:

- identify stable/unstable, *non-linear recurrent patterns* in square ducts
- find a link between *optimal disturbances* and *NLRP*



Based on joint work with:
D. Biau and H. Soueid

Why *recurrent patterns*?

Current wisdom holds that a “small” set of *recurrent patterns* are sufficient to develop a predictive tool for non-equilibrium turbulent flows.

This idea has roots in the *prehistory* of chaos theory!

Lorenz attractor

(***J. Atmos. Sci.*** 1963)

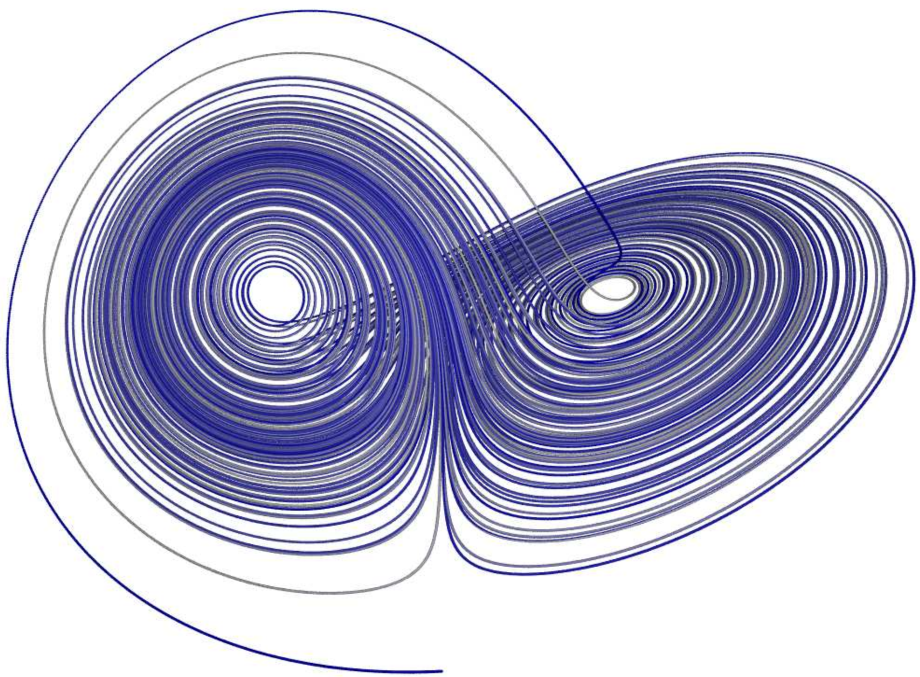
No steady states

No limit cycles

Sensitive dependence on IC

↓

Local unpredictability



If turbulence can be interpreted as the wandering of the flow system's trajectory in phase space among mutually repelling states (Cvitanović refers to this as *Hopf theory of chaos*) it may be possible to

1. identify the set of *recurrent patterns* pertinent to each flow configuration and Reynolds number, &
2. Compute sensible global averages (⇒ global predictability) possibly retaining only the more *meaningful* patterns (i.e. the least unstable ones?)

Both tasks are difficult …

(Lan & Cvitanović, ***Phys. Rev E*** 2003, had some success with the 1D Kuramoto-Sivashinsky equation)

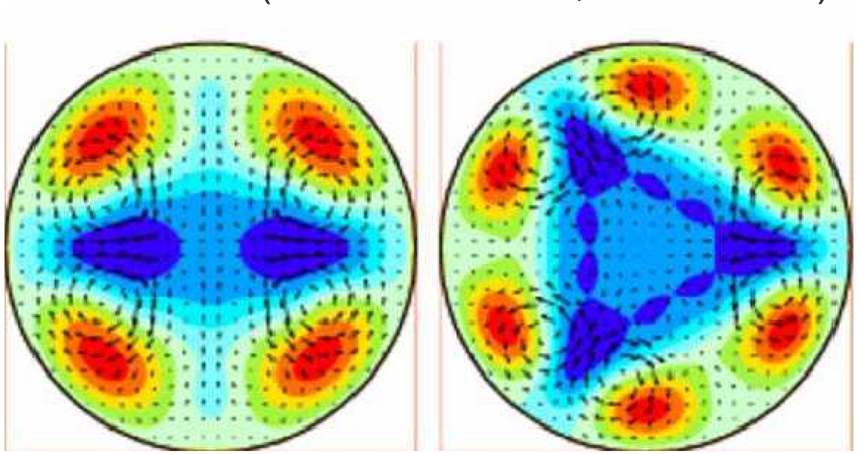
What are *optimal perturbations* good for?

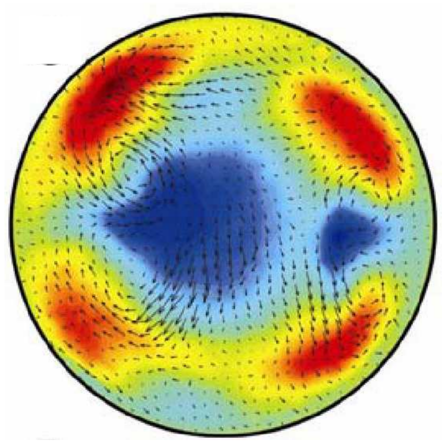
Traditional argument: OP elicit the largest response, thus they are good candidate solutions to study by-pass transition …

<u>Success stories</u> for NLRP in the context of the Navier-Stokes equations

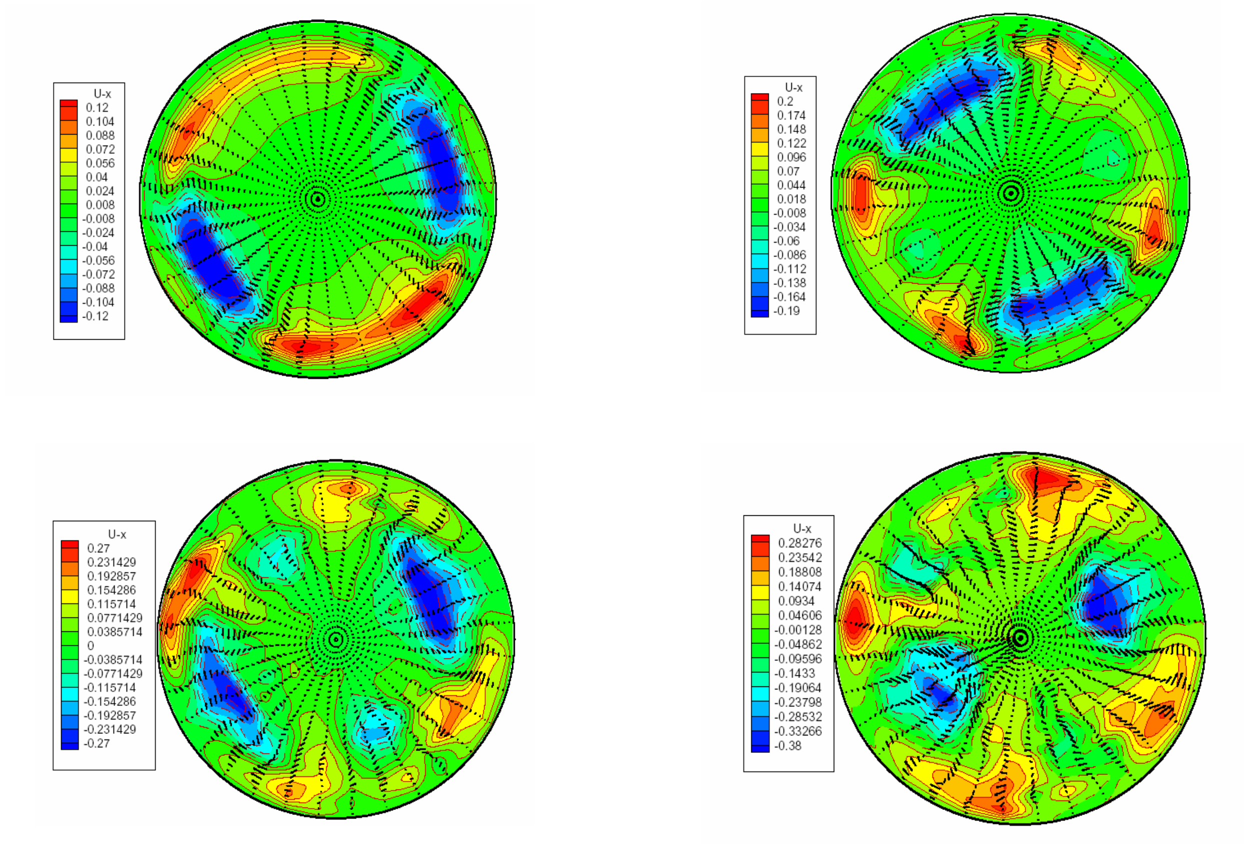
(chronological and incomplete …)

1. Nagata (***JFM*** 1990) — 3D finite amplitude solutions in plane Couette flow
2. Ehrenstein & Koch (***JFM*** 1991) — Poiseuille flow
3. Cherhabili & Ehrenstein (***JFM*** 1997) — plane Couette flow
4. Waleffe (***JFM*** 2001, ***PoF*** 2003) — plane Couette & Poiseuille flows, self-sustaining process
5. Kawahara & Kida (***JFM*** 2001) — first numerical evidence of the existence of unstable recurrent patterns in Couette flow at Re=400
6. Faisst & Eckhardt (***PRL*** 2003)
   Wedin & Kerswell (***JFM*** 2004)
   Hof *et al*. (***Science*** 2004, ***PRL*** 2005) — TW in pipe flow





Further confirmation as to the presence of recurrent patterns in pipe flow: Gavarini, Bottaro & Nieuwstadt, ***JFM***, 2004 & *IUTAM Symposium*, Bristol, 2004



MINIMAL TURBULENT UNIT:

Vortices, streaks and *TW* sustain one another against viscous decay

Possibly, the square duct is “nicer” because of the presence of geometrical symmetries that constrain the flow …

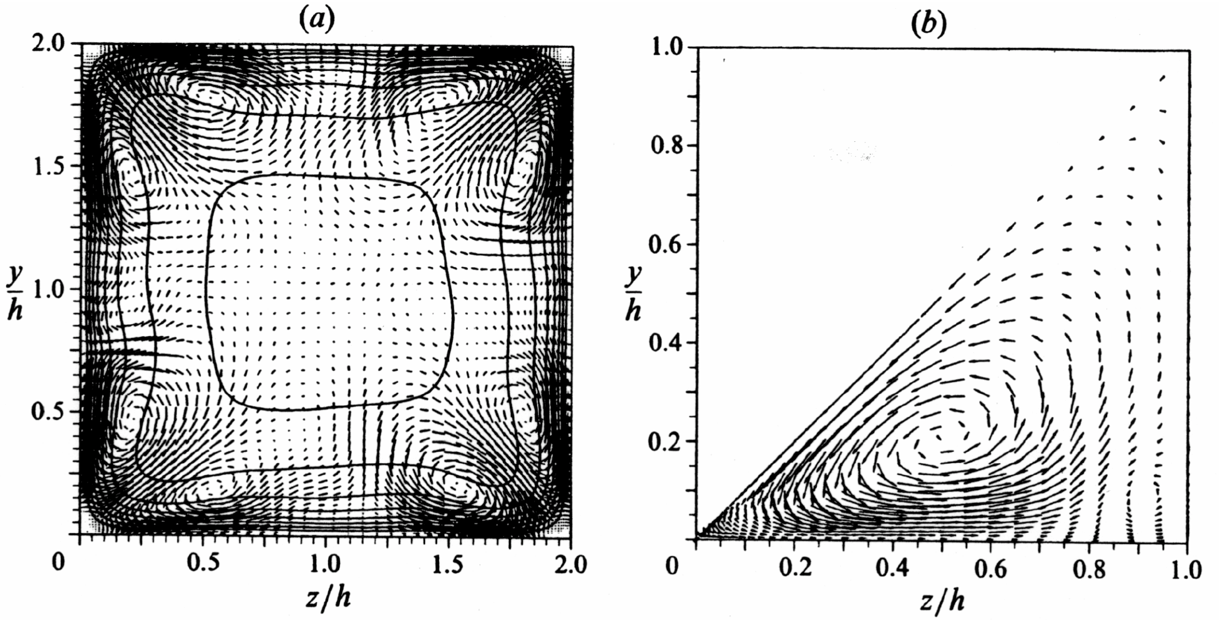


FIGURE 6. (a) Mean secondary velocity vectors and mean streamwise flow contours. The contour increment is $4u_\tau$ with the lowest value contour being nearest to the duct walls representing $4u_\tau$ units. (b) Vector field in (a) averaged over all octants. Only half the vectors in each direction are shown.

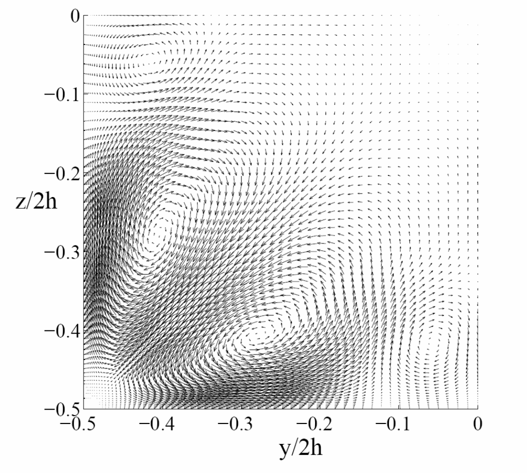
DNS, Gavrilakis, ***JFM*** 1992

Problem being investigated since Nikuradse (*Ph.D. Thesis*, Göttingen, 1926)

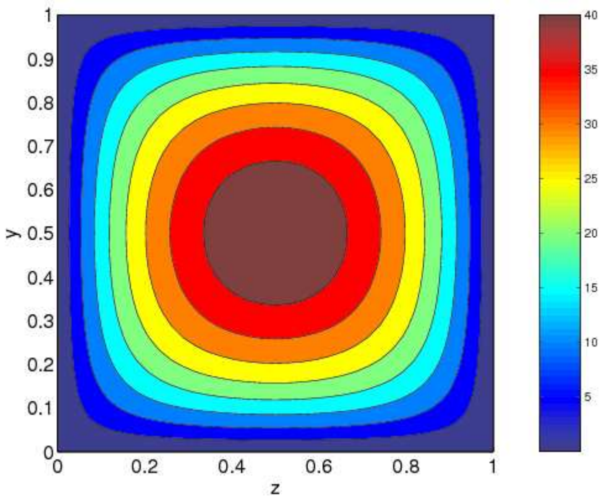
Experiments:	Brundett & Baines (1964), Gessner (1973)
Reynolds-averaged simulations:	Launder & Ying (1973), Demuren & Rodi (1984) Mompean (1998)
DNS/LES:	Madabhushi & Vanka (1991), Gavrilakis (1992), Huser & Biringen (1993)

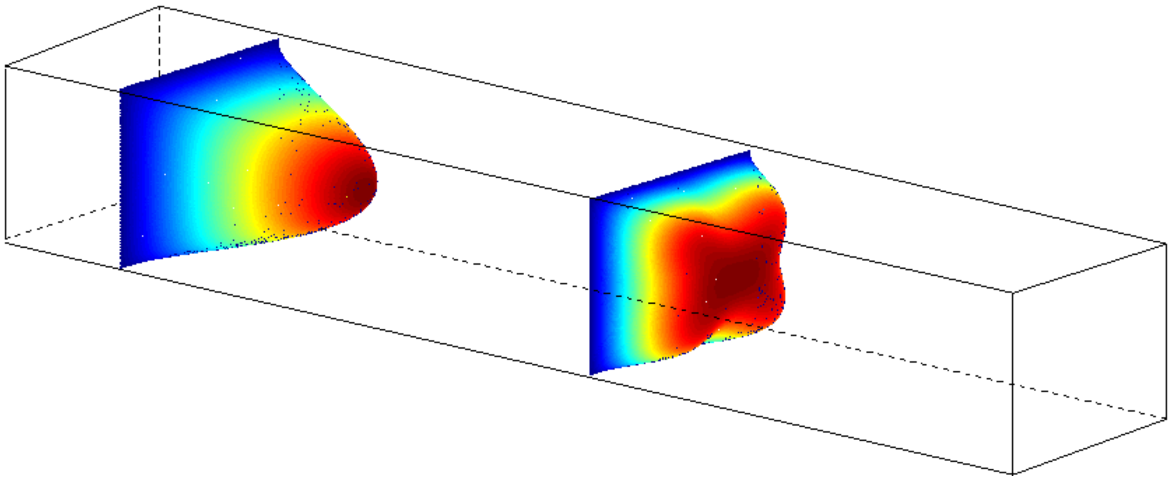
- Secondary flows near corners induced by anisotropic turbulent fluctuations
- Second-order closure underpredicts secondary vortices, possibly because of inadequate modeling of secondary shear stress components
- "*A theory of the flow structures that give rise to the observed mean flow is not yet available*"

Qualitative picture of the mean secondary flow:



Transition to turbulence





dp/dx = <u>const</u>.

<u>characteristic length</u> = *h*

[channel height]

<u>characteristic speed</u> = u_τ

[$u_\tau^2 = -(h/4\rho)\ dp/dx$]

$Re_\tau = 150$

(*MARGINAL* VALUE)

Governing equations:

$$\begin{cases} u_x + v_y + w_z = 0, \\ u_t + uu_x + vu_y + wu_z = -p_x + \frac{1}{Re}\Delta u + 4, \\ v_t + uv_x + vv_y + wv_z = -p_y + \frac{1}{Re}\Delta v, \\ w_t + uw_x + vw_y + ww_z = -p_z + \frac{1}{Re}\Delta w, \end{cases}$$

Linearized disturbance equations:

$$\begin{cases} i\alpha u + v_y + w_z = 0 \\ u_t + i\alpha U u + v U_y + w U_z = -i\alpha p - \alpha^2 u + u_{yy} + u_{zz} \\ v_t + i\alpha U v = -p_y - \alpha^2 v + v_{yy} + v_{zz} \\ w_t + i\alpha U w = -p_z - \alpha^2 w + w_{yy} + w_{zz} \end{cases}$$

Traditional eigenvalue analysis shows that all eigenmodes are damped (Tatsumi & Yoshimura, *JFM* 1990)

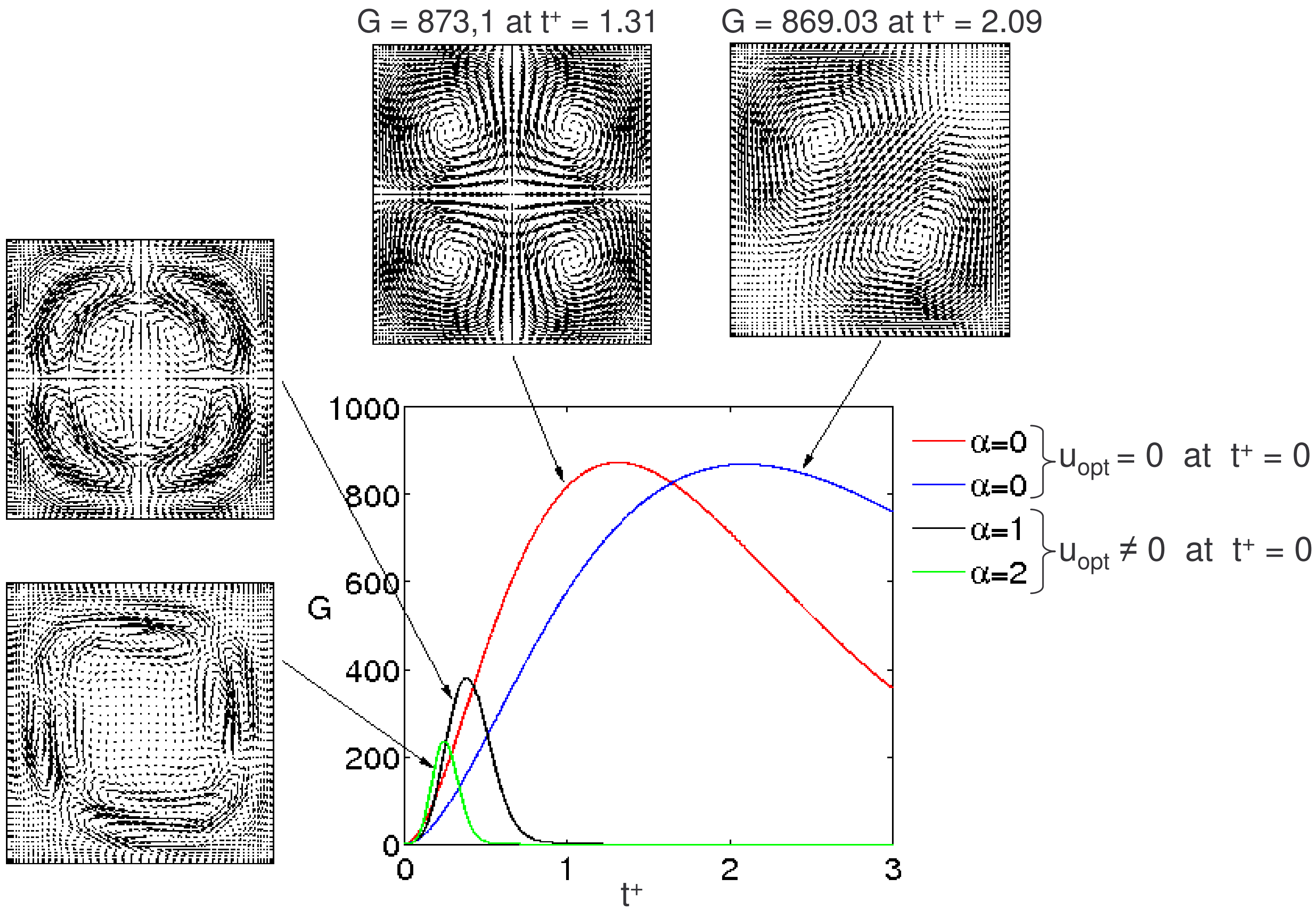
OPTIMAL PERTURBATIONS: Traditional functional optimization with adjoints.

$$G(T) = \frac{E(T)}{E(0)}, \quad with \quad E = \frac{1}{2} \int_y \int_z (\bar{u}u + \bar{v}v + \bar{w}w) \quad dy\, dz$$

The adjoint problem reads:

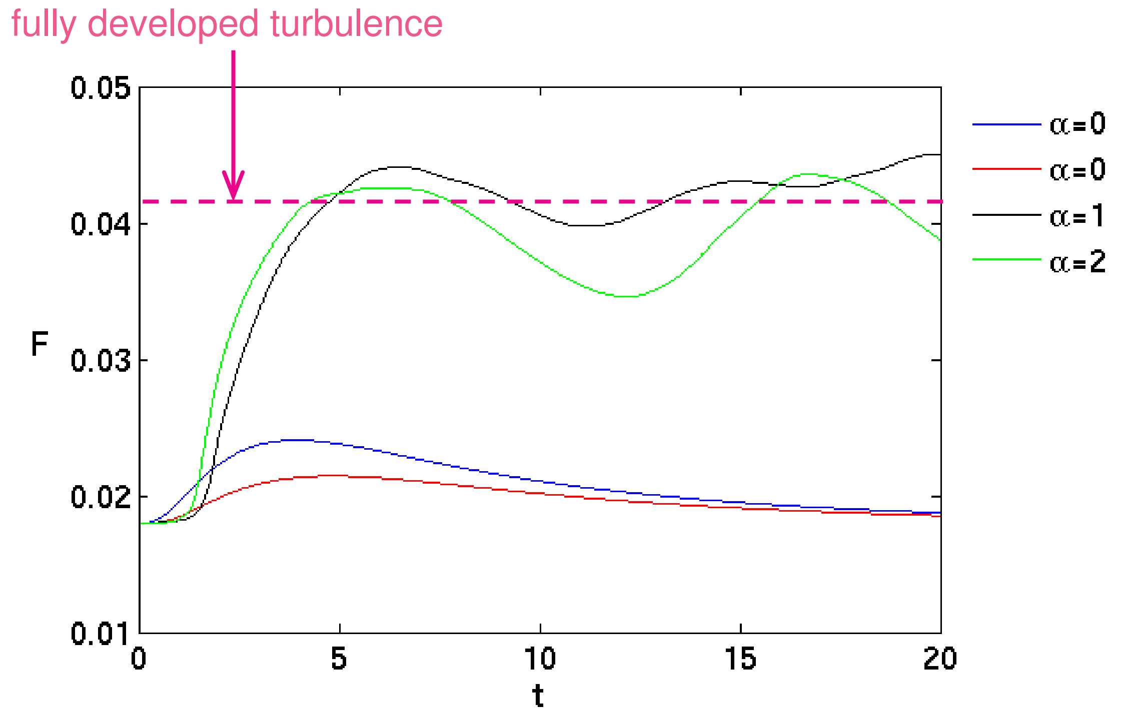
$$\begin{cases} i\alpha u^\dagger + v_y^\dagger + w_z^\dagger = 0 \\ -u_t^\dagger - i\alpha U u^\dagger = -i\alpha p^\dagger - \alpha^2 u^\dagger + u_{yy}^\dagger + u_{zz}^\dagger \\ -v_t^\dagger - i\alpha U v^\dagger + u^\dagger U_y = -p_y^\dagger - \alpha^2 v^\dagger + v_{yy}^\dagger + v_{zz}^\dagger \\ -w_t^\dagger - i\alpha U w^\dagger + u^\dagger U_z = -p_z^\dagger - \alpha^2 w^\dagger + w_{yy}^\dagger + w_{zz}^\dagger \end{cases}$$

$$\begin{array}{ccc} \mathbf{q}(t=0) & \xrightarrow{q_t = Lq} & \mathbf{q}(t=T) \\ \uparrow & & \downarrow \\ \mathbf{a}(t=T) & \xleftarrow{-a_t = L^\dagger a} & \mathbf{a}(t=0) \end{array}$$



G = 873,1 at t⁺ = 1.31
G = 869.03 at t⁺ = 2.09
1000
800
600
400
200
0
G
0
1
2
3
t⁺
α=0
α=0
α=1
α=2
u_opt = 0 at t⁺ = 0
u_opt ≠ 0 at t⁺ = 0

Nonlinear evolution of the optimal disturbances (+ random noise)
(streamwise periodic duct of length 4π)



F = skin friction factor (F = 0.0415 from DNS at Re_τ = 150 (Re_b = 2084))

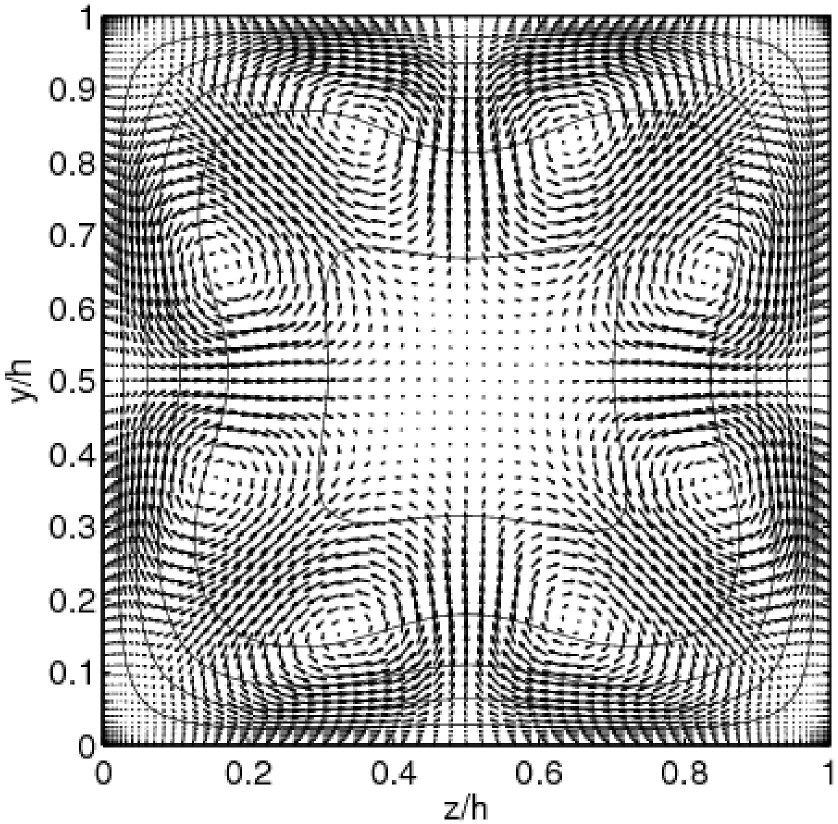
At t = 0:

$\alpha = 0$ $E_0 = 10^{-1}$!!

$\alpha = 1$ $E_0 = 7.8 \times 10^{-3}$

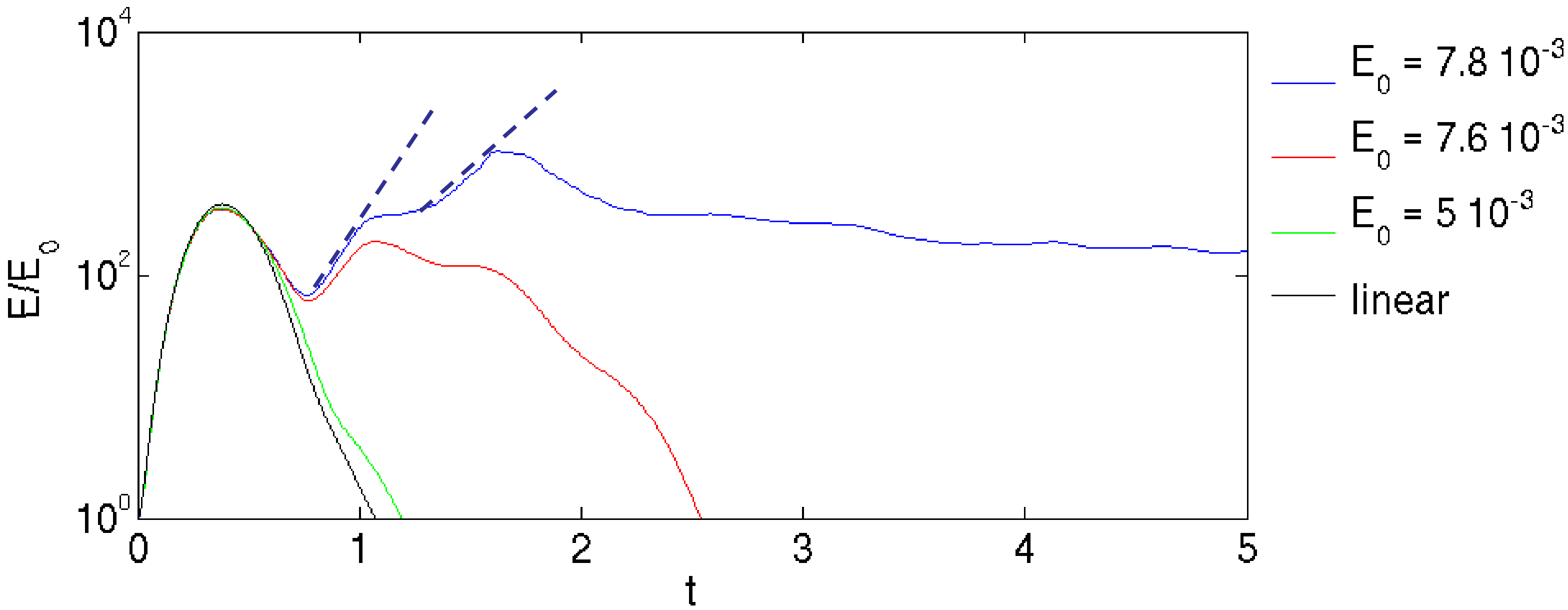
$\alpha = 2$ $E_0 = 4.4 \times 10^{-3}$

Streamwise/time averaged turbulent flow



Excellent agreement with Gavrilakis, *JFM* 1992.

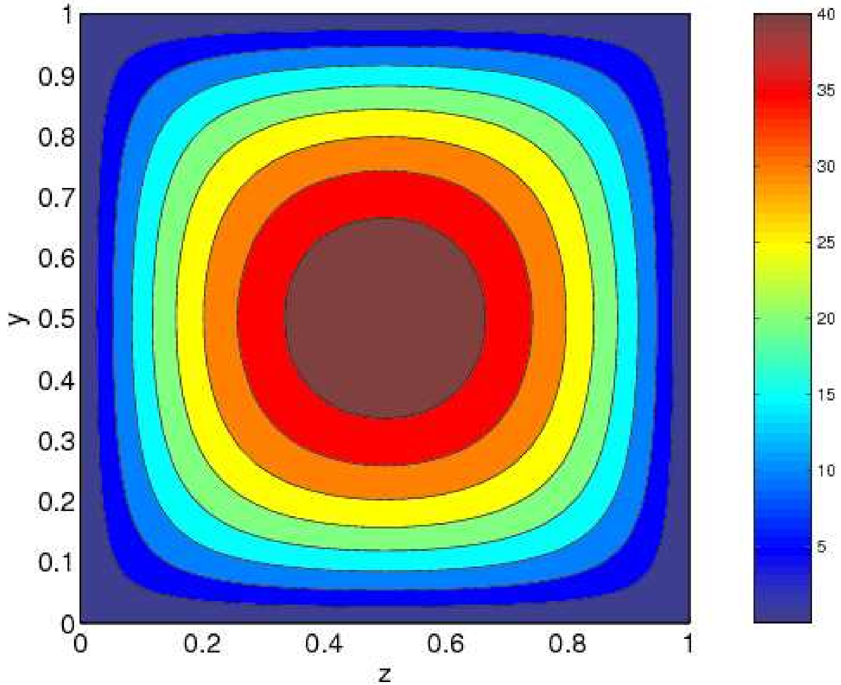
The α = 1 mode



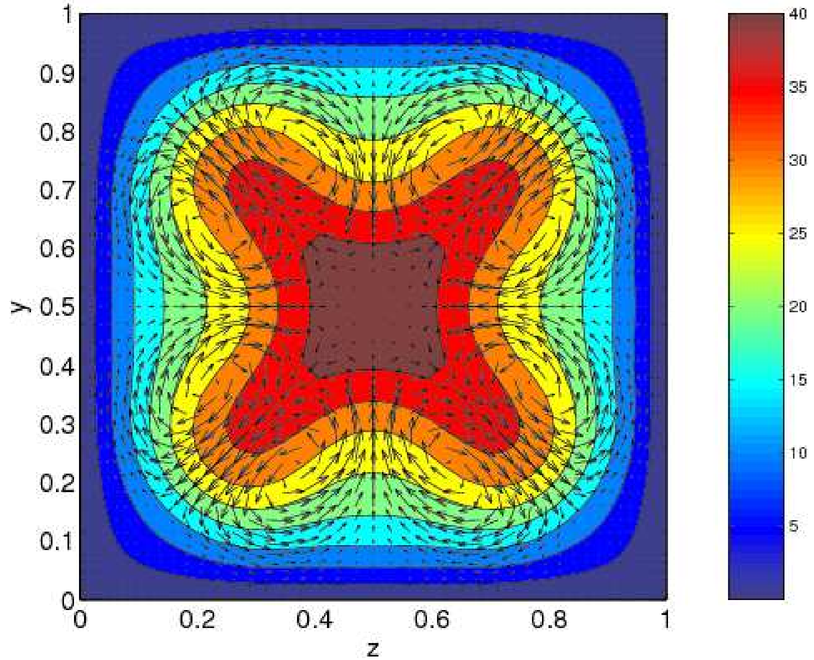
The threshold value is $E_0 = 7.8 \times 10^{-3}$. In fact the optimal perturbation for α = 1 is NOT important. What matters is the distorted field which emerges at t ~ 0.8. Such a field is linearly unstable and can grow exponentially.

The α = 1 mode

unperturbed base flow



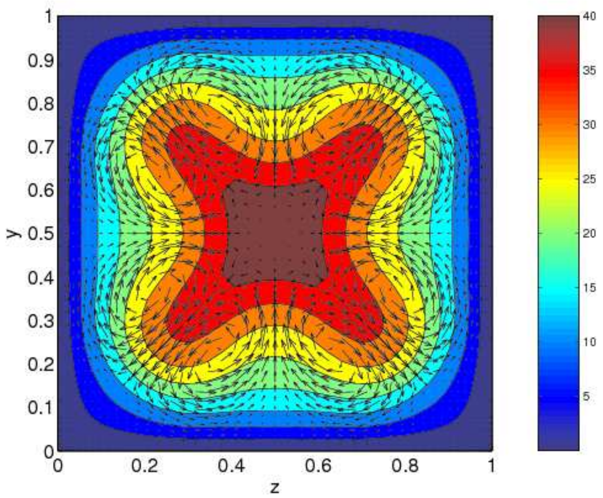
0-Fourier mode at t ~ 0.8



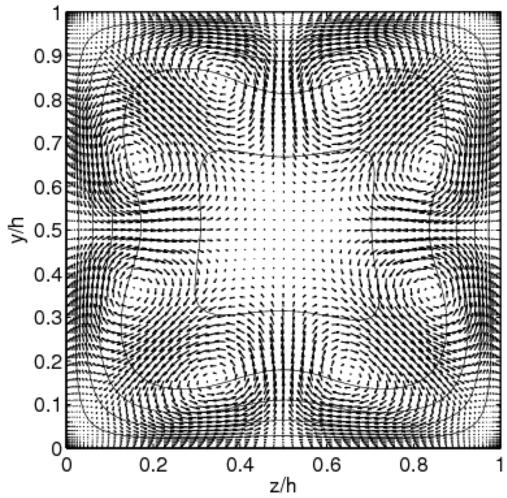
Quadratic interactions produce a distorted base flow (which could be generated by just about any initial condition with streamwise structure) which satisfies the conditions for the growth and sustainement of turbulence. The distortion is of rather large amplitude and inviscidly unstable (*cf*. with theory of minimal defects, Bottaro et al, *JFM* 2003, Biau & Bottaro, *PoF* 2004)).

The α = 1 mode and turbulence

0-Fourier mode at t ~ 0.8



mean turbulent flow



The <u>similarity</u> between the zeroth-order mode at t ~ 0.8 and the streamwise/time averaged flow field in the fully developed regime suggests that this is the end of the story, *i.e.* one could easily imagine that the state on the left is an *unstable limit cycle* around which the flow trajectory oscillates for a long time.

Or not?

Conclusions (midway through the work)

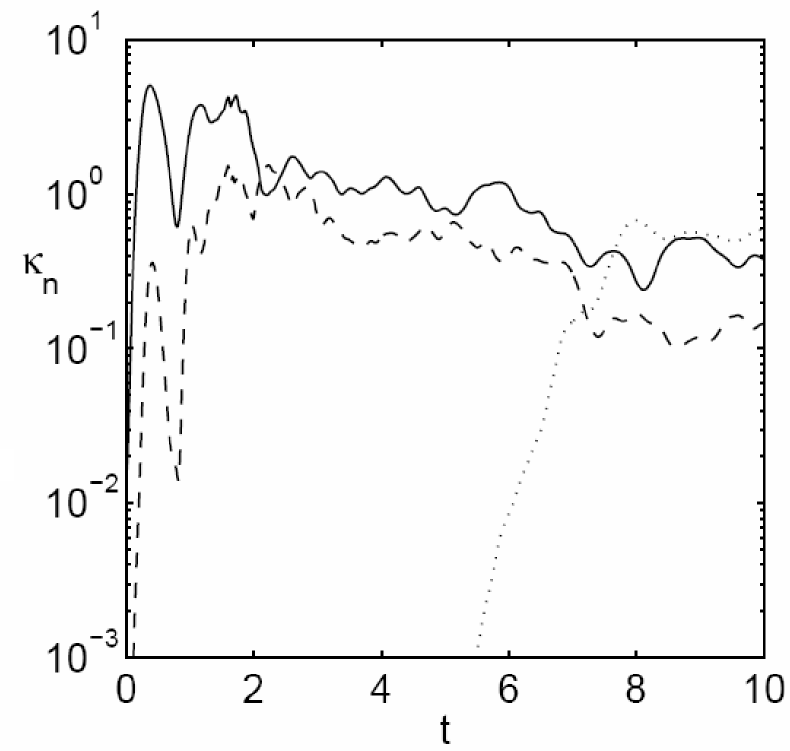
- Global optimal disturbances are interesting concepts, with little connection to transition.

- The key to transition is to set up a distorted base flow with certain features, in particular such a distorted base flow must be able to grow exponentially and resembles the mean fully developed turbulent flow (?)

- The distorted base flow can be set up efficiently by sub-optimal disturbances in the form of streamwise travelling waves. In fact, any initial condition in the form of a travelling wave of sufficiently large amplitude is capable to do it!

- Is there any scope for studying optimal perturbations?

What happens past t ~ 0.8?



α=0.5 : grows out of the random noise
α=1 : present at t = 0
α=2 : from quadratic interactions

with $$\kappa_n = \frac{1}{N^2} \int_{yz} (\tilde{\mathbf{u}}^* \tilde{\mathbf{u}})_n + (\tilde{\mathbf{u}}^* \tilde{\mathbf{u}})_{N-n} \, dydz$$

What happens past t ~ 0.8?

The secondary flow does not seem to oscillate around the 8-vortex state!!!

FILM …

A pair of opposite walls remains active for 50 + time units. Afterwards, the opposite pair of walls become active. The lifetime of the 4-vortex state is “infinite”.

M. Uhlmann, A. Pinelli, G. Kawahara and A. Sekimoto (submitted *JFM*, 2007)

active walls

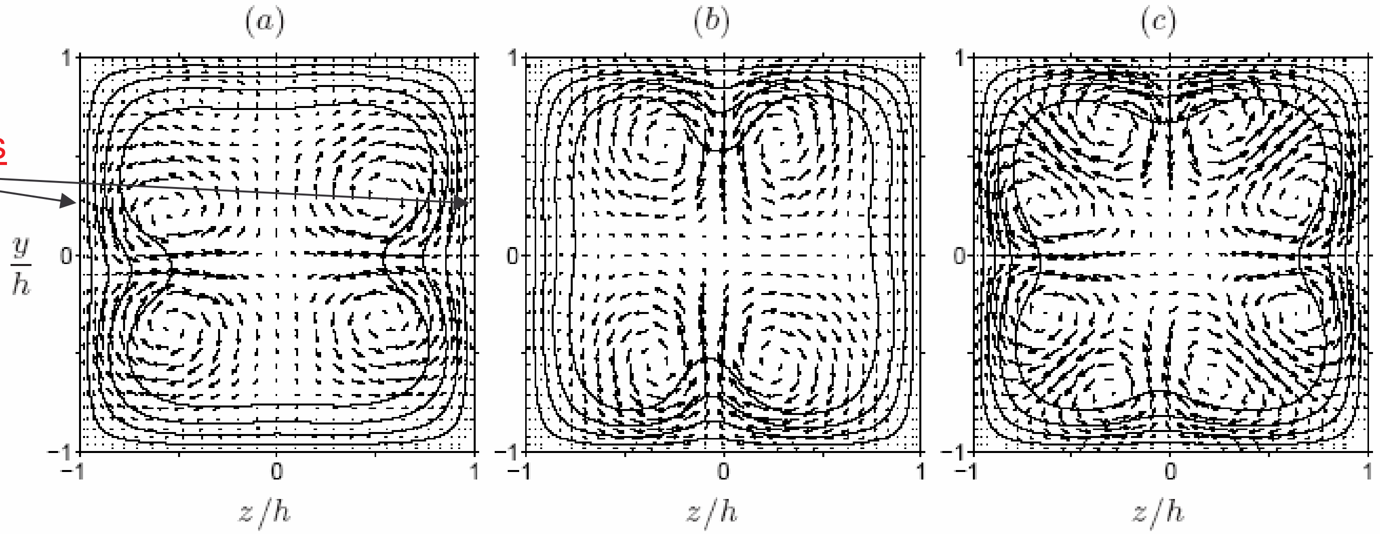


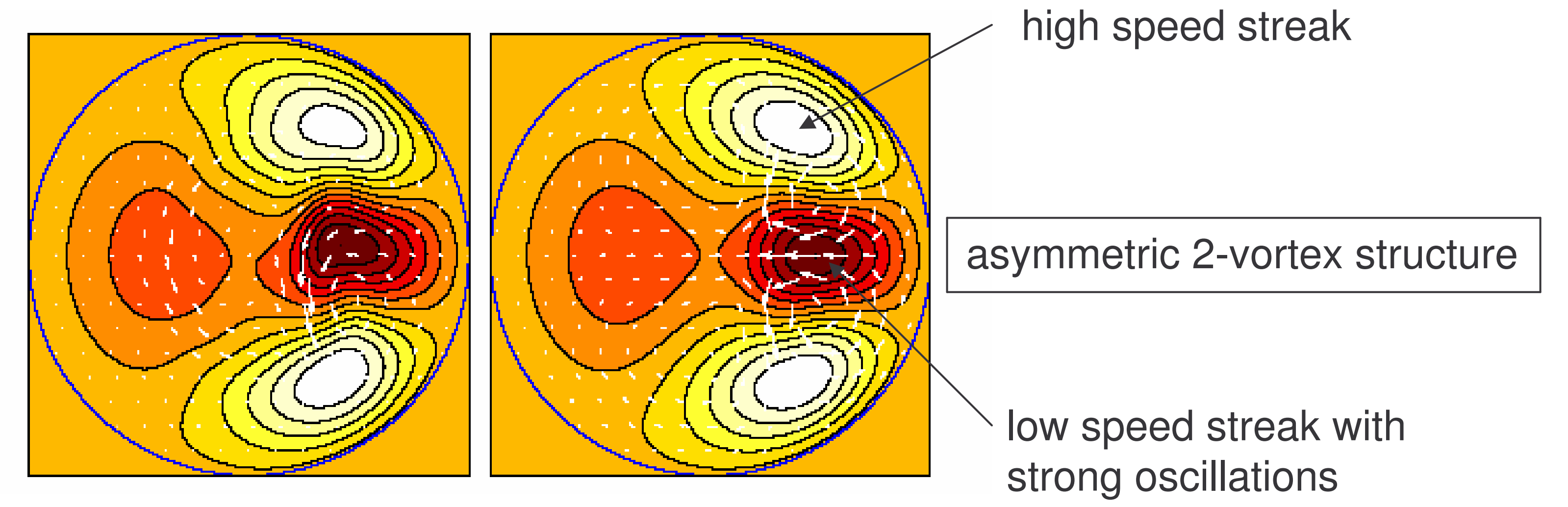
FIGURE 3. Contour lines of the primary mean flow $\langle u \rangle$ (with increment max$\langle u \rangle/5$) and vectors of the secondary mean flow $\langle v \rangle, \langle w \rangle$ for $Re_b = 1205$ and $L_x = 2\pi$: (*a*) averaging interval $771h/u_b$; (*b*) a different interval with length $482h/u_b$; (*c*) long-time integration including both former intervals ($1639h/u_b$). Vectors are shown for every third grid point.

4-vortex state in an "edge state"? (Eckhardt *et al.*, *ARFM* 2007)

Edge state = the flow state that sits on the separatrix between laminar and turbulent flow (*i.e.* that dividing surface in phase space separating initial conditions which remain laminar from those leading to turbulence)

Edge states in circular pipe flow have been recently computed by Pringle & Kerswell (arXiv:physics/0703210v1).

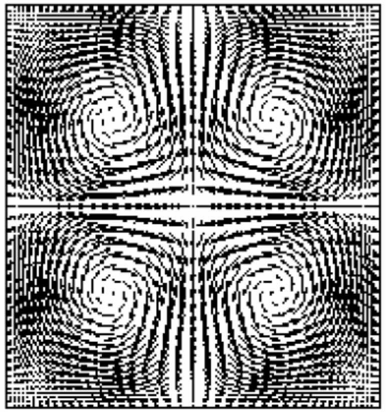
An example of “edge state” is shown below (instantaneous and time average)

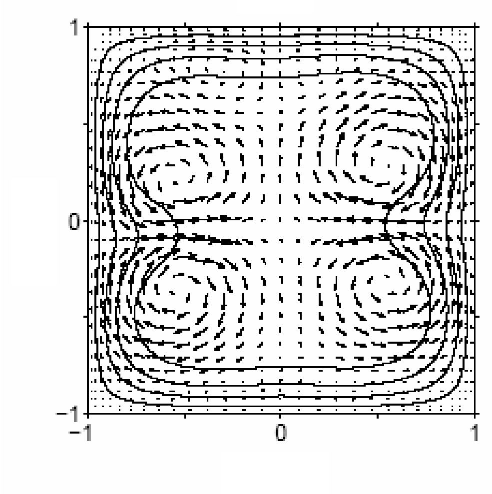


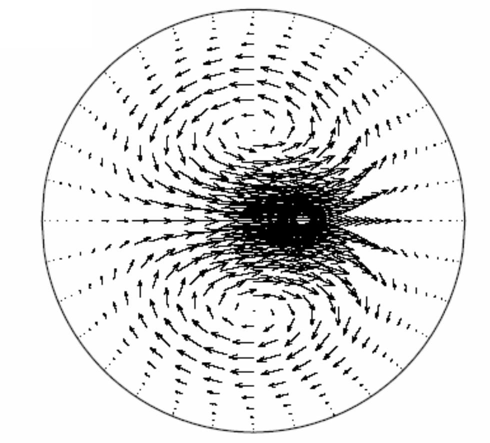
The same state has been found by Eckhardt *et al.*, *ARFM* 2007.

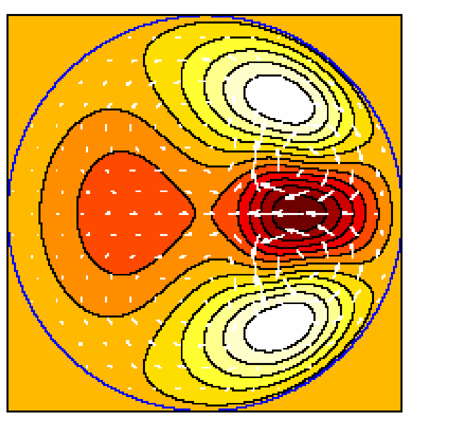
Global optimal perturbations

“Edge states”









Conclusions (final?)

- *Global* optimal disturbances are incapable to trigger transition to turbulence (unless the initial amplitude is unreasonably large).

- *Local* optimals (with $\alpha \neq 0$) can sep up a distorted base flow which undergoes transition, provided the initial condition has an amplitude superior to a given threshold. The key to transition is the nonlinearly distorted base flow (which here appears around $t \sim 0.8$), not the suboptimal initial condition.

- The distorted base flow presents a 8-vortex structure and resembles the mean fully developed turbulent flow in the square duct (any reason?)

- Global optimals (4-vortex structure) resemble “edge states” (just a coincidence?).

- There might still be scope for studying sexy optimal perturbations.