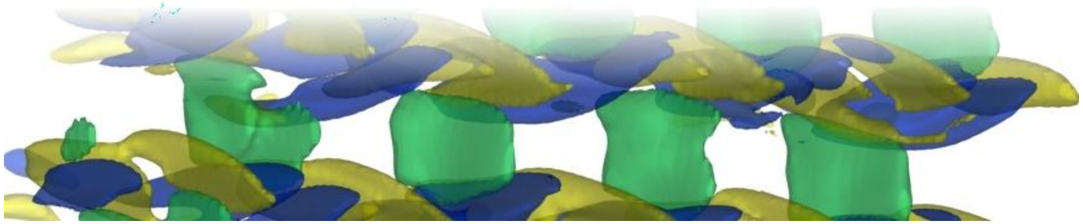


Localized, nonlinear optimals

i.e. the *minimal seed* of transition to turbulence in a boundary layer

Stefania Cherubini e Pietro De Palma

Politecnico di Bari

Jean-Christophe Robinet

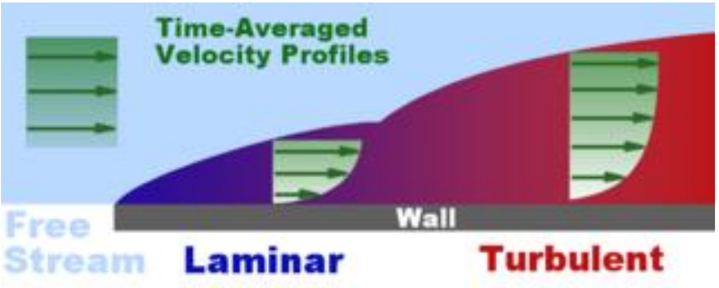
Arts et Métiers, ParisTech

Alessandro Bottaro

Università di Genova

Transition, a burning question for 100+ years …

What happens/why?



http://en.wikipedia.org/wiki/Boundary_layer_transition

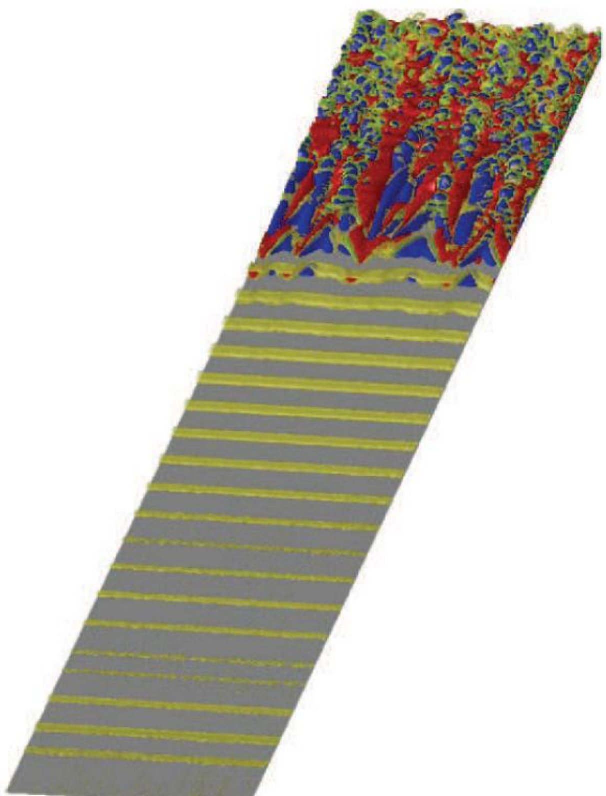
'… the concept of boundary layer transition is a complex one and still lacks a complete theoretical exposition.'

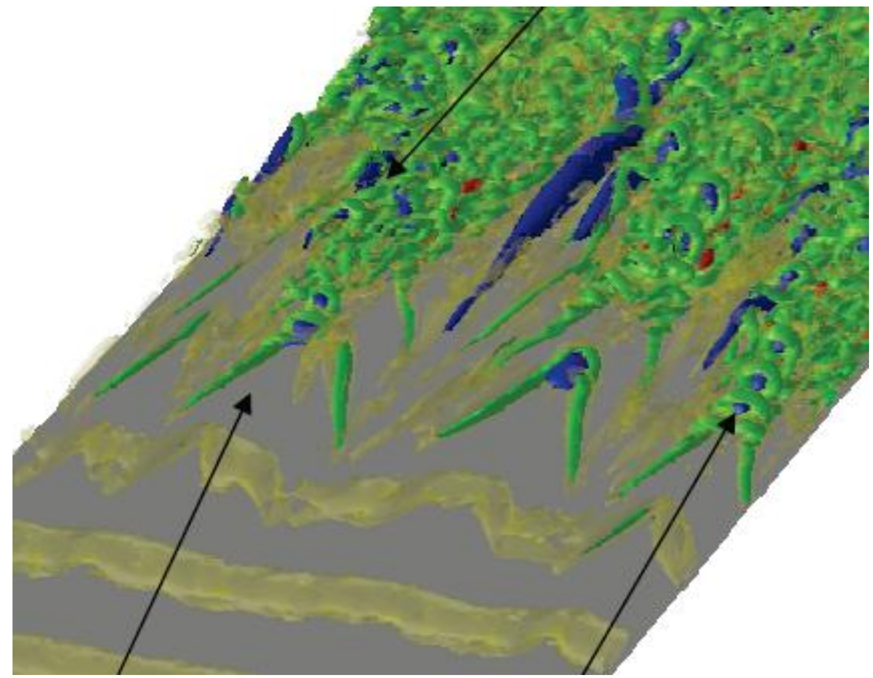
What we know already

- 2D TS waves

SUPERCRITICAL TRANSITION

(for ‘small’ disturbance levels)

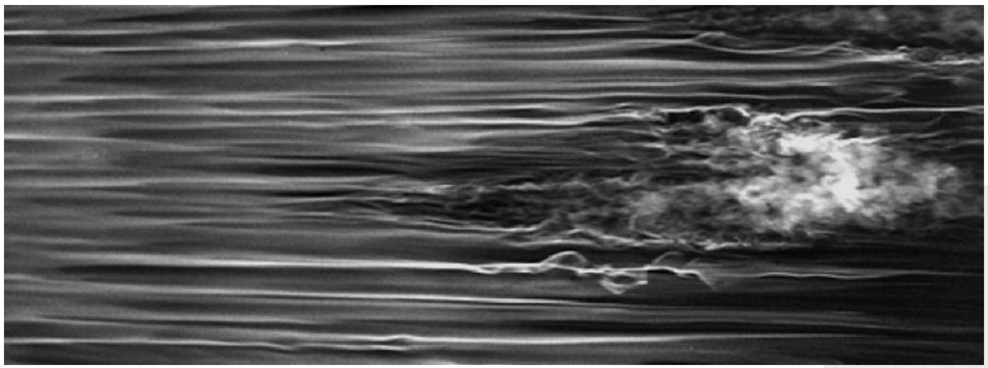




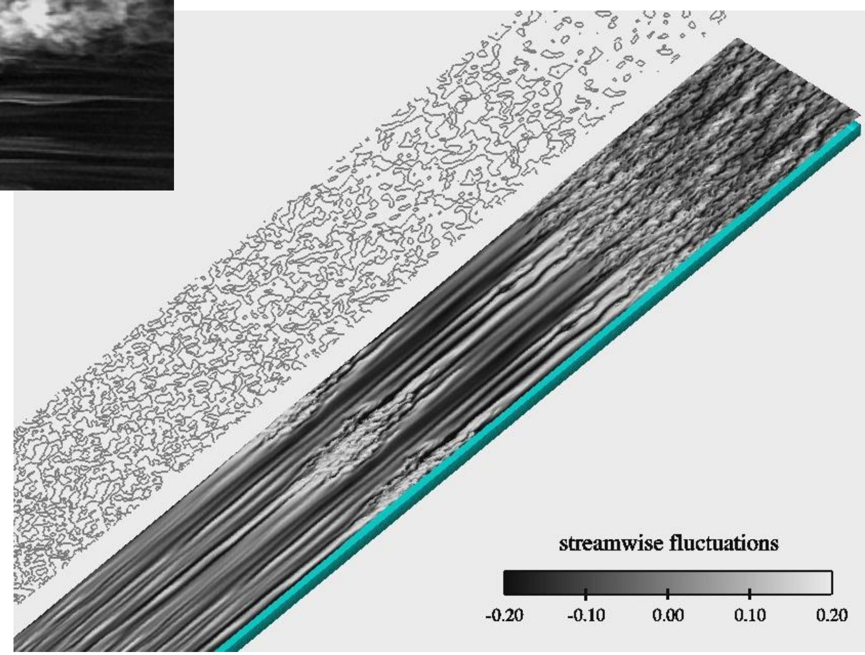
Philipp Schlatter, 2009

What we know already

- Emmons (1951) spots, induced by free-stream turbulence



Matsubara & Alfredsson, 2005



Zaki & Durbin, 2005

SUBCRITICAL (BYPASS) TRANSITION
(for 'large' Tu disturbance levels)

What we know already

'Optimal perturbations ' to explain bypass transition??

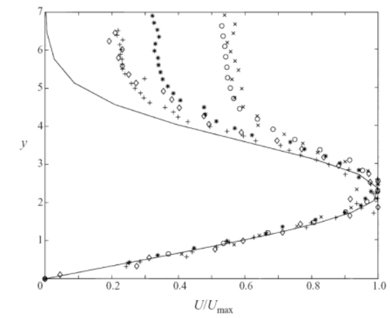
- Linear (based on B/L scalings):

  Andersson, Berggren & Henningson, 1999

  Luchini, 2000

- Nonlinear (based on B/L scalings):

  Zuccher, Luchini & Bottaro, 2004



... but α = 0 streaks are not good at kicking transition

Waleffe, 1995

Andersson et al., 2001

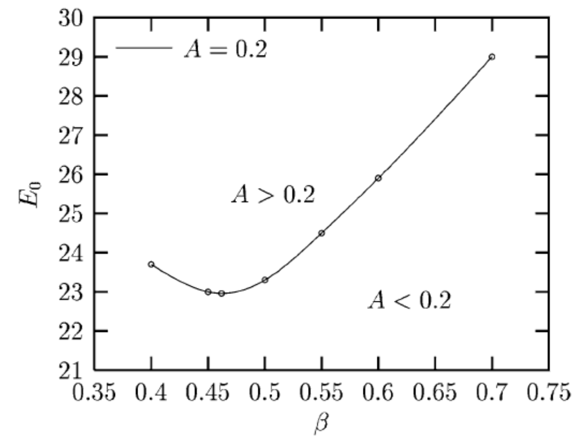
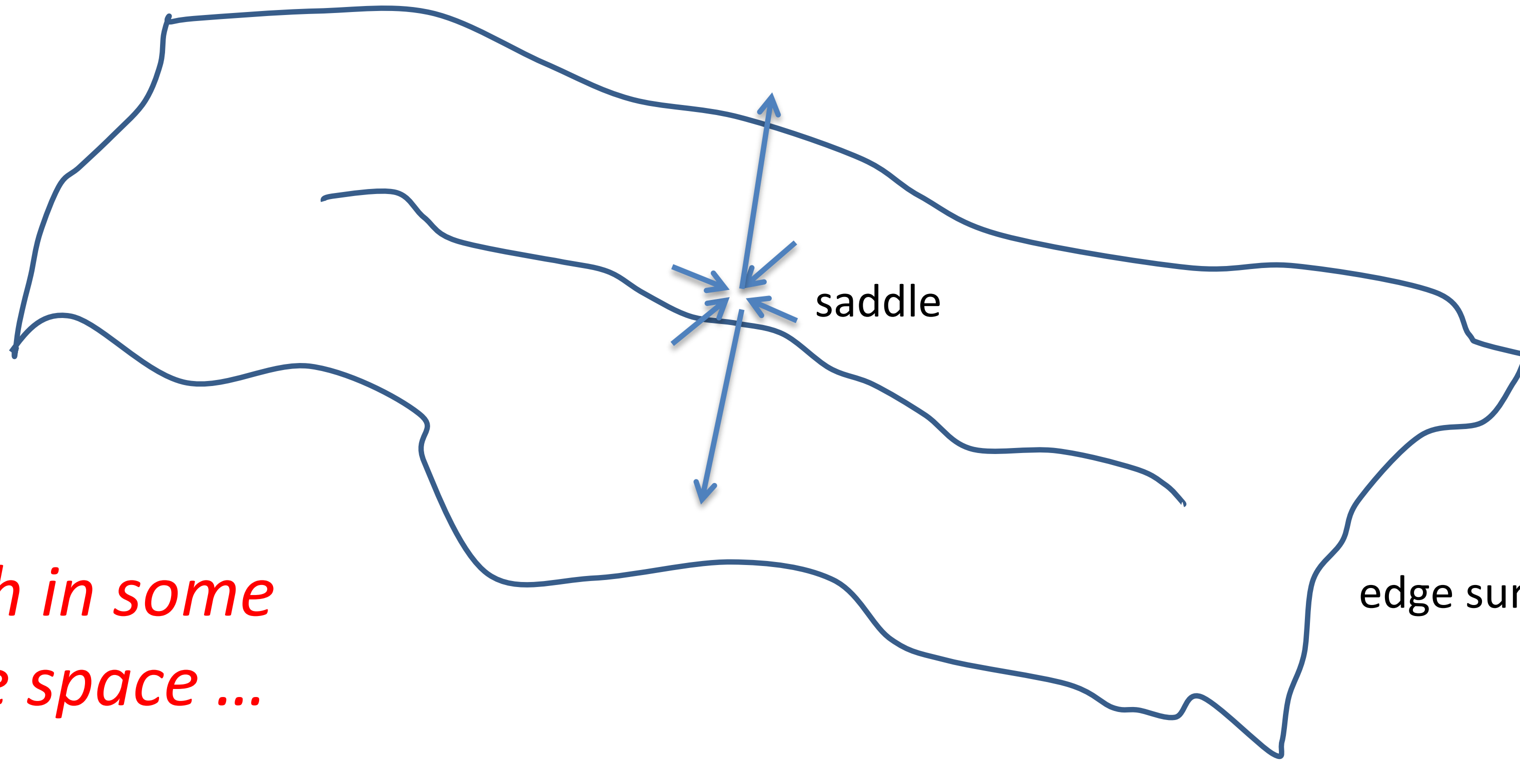


Fig. 12. Curve of initial perturbation energy E_0 as a function of β for which $A = 0.2$ somewhere in the domain.

What we know already

What about ECS, saddles, edge states, etc.?



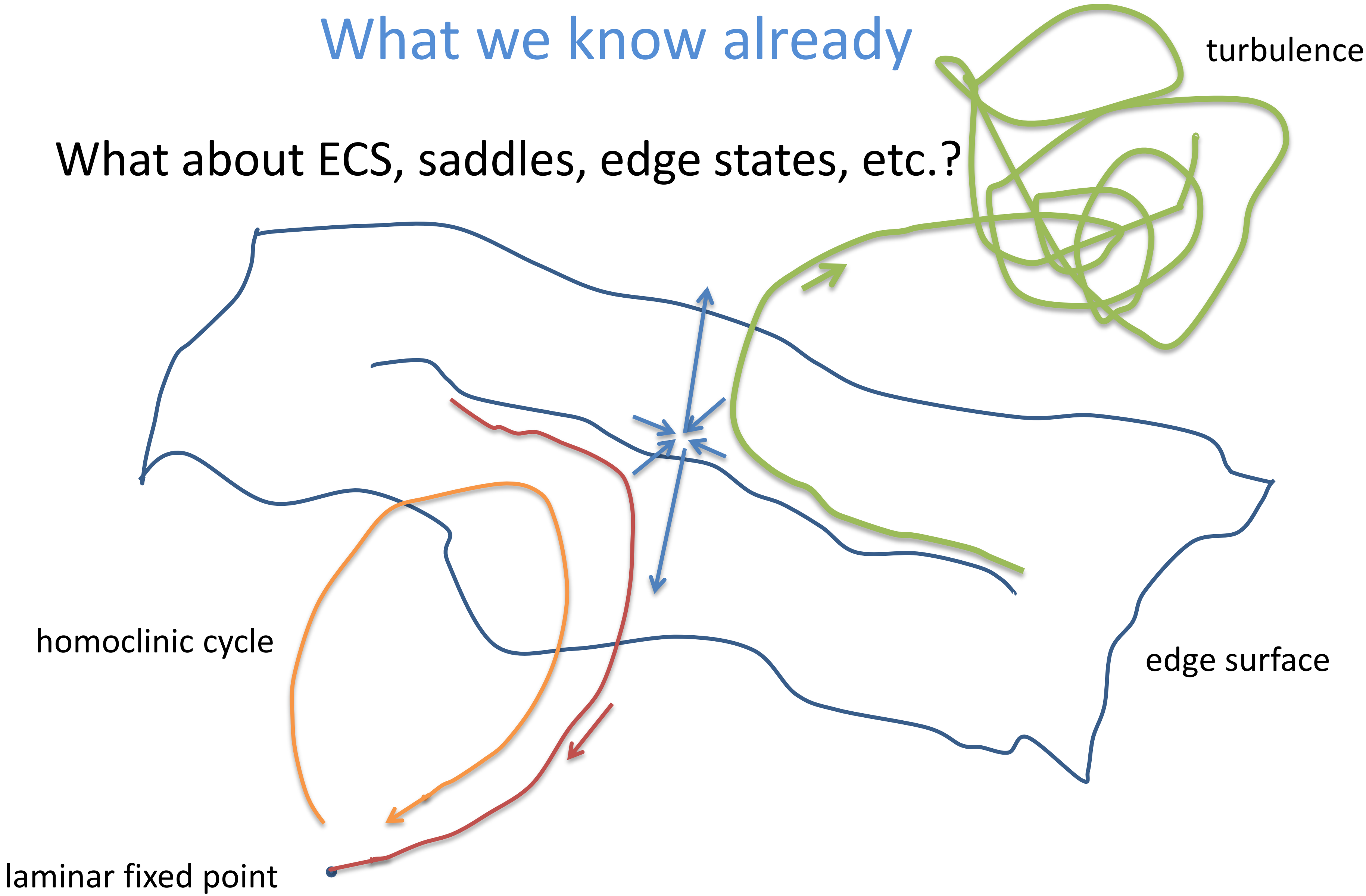
edge surface

Sketch in some phase space …

laminar fixed point

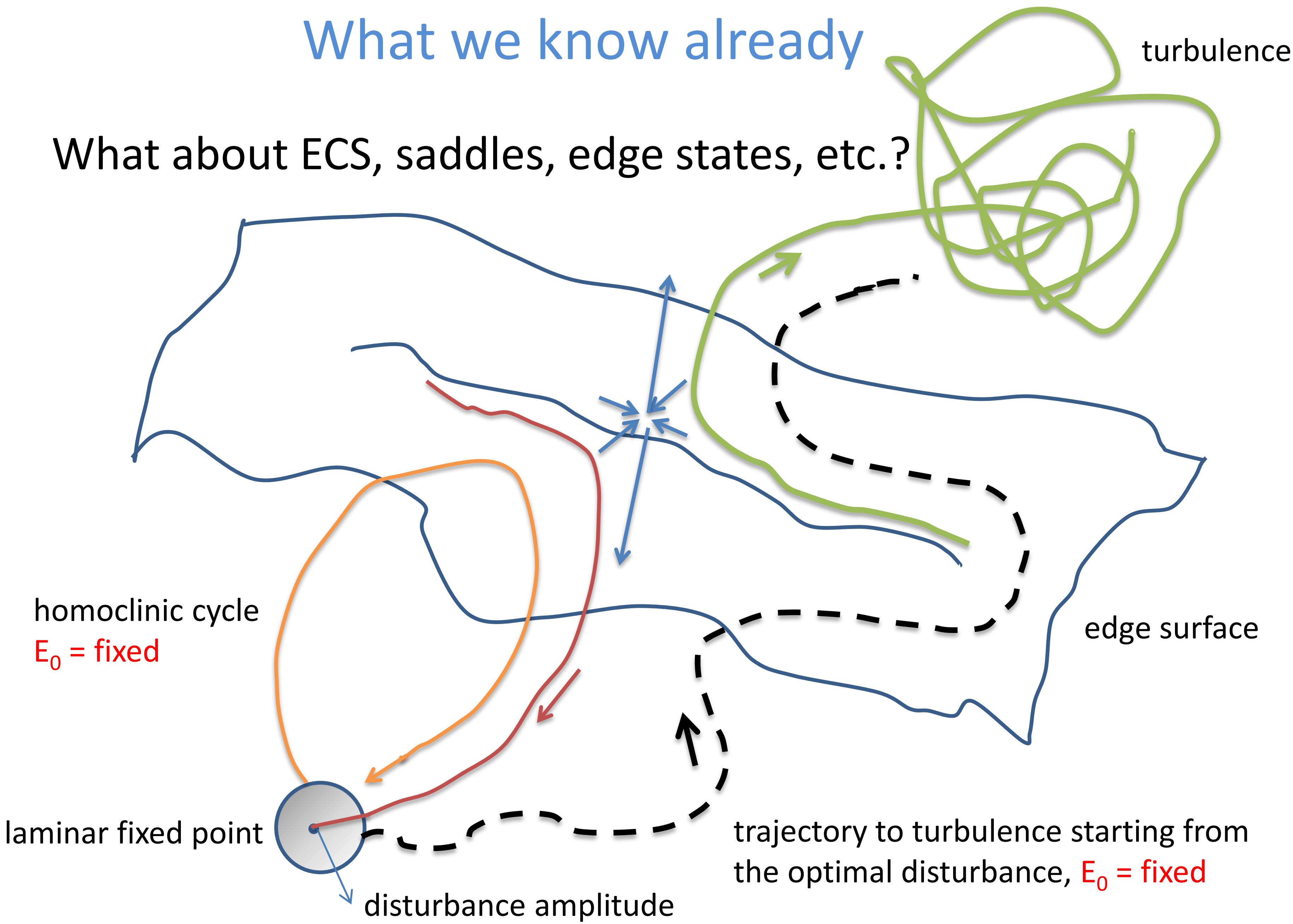
What we know already

What about ECS, saddles, edge states, etc.?



What we know already

What about ECS, saddles, edge states, etc.?



What we know already

What about ECS, saddles, edge states, etc.?

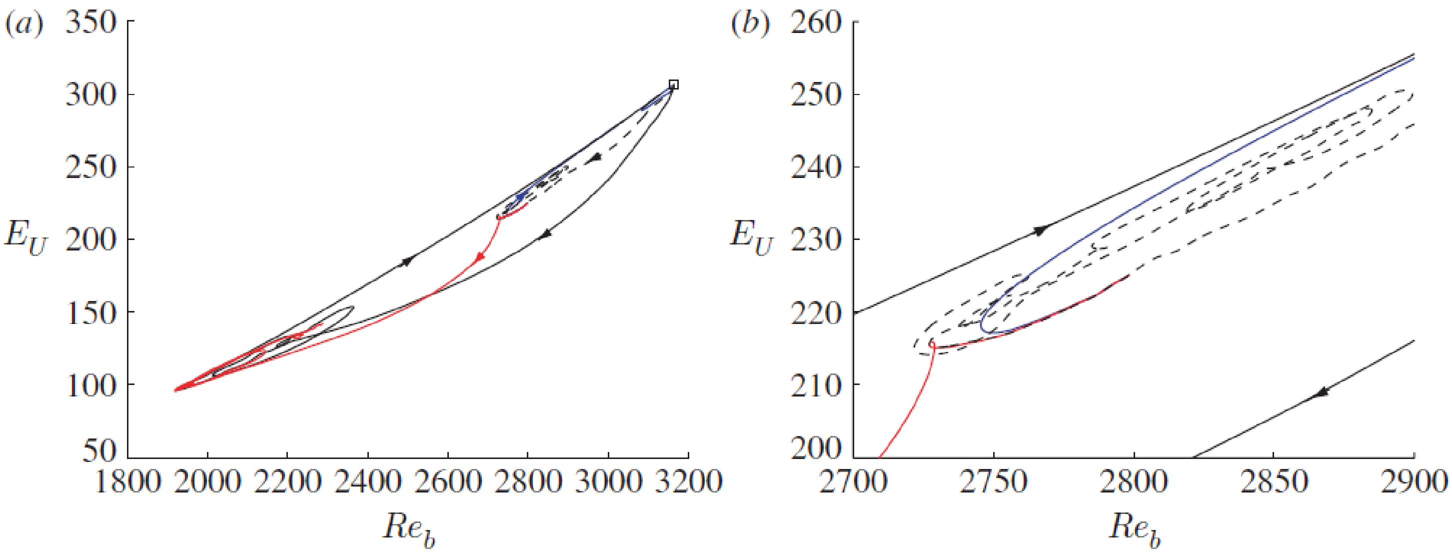


Figure 7. (*a*) E_U ‘energy’ versus Re_b. The laminar fixed point with Re_b=3163 and E_U=306.45 is denoted by a square. (*b*) Better details of the flow trajectory on the edge (dashed curve).

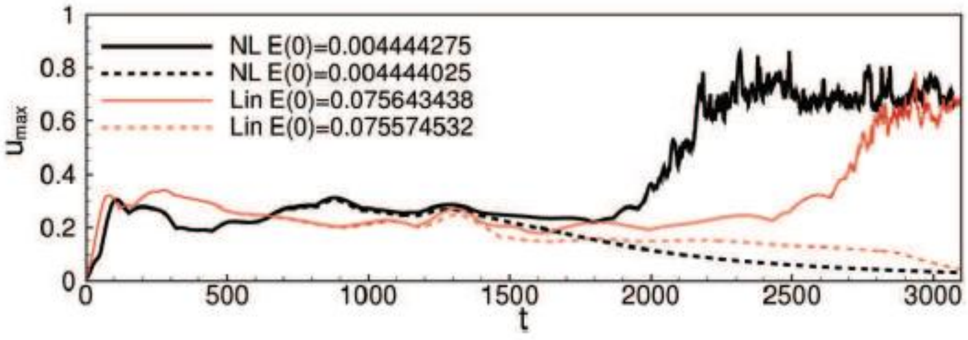
Biau & Bottaro, 2009 (square duct)

What we know already

What about ECS, saddles, edge states, etc.?

Cherubini et al., 2011

Duguet et al., 2011



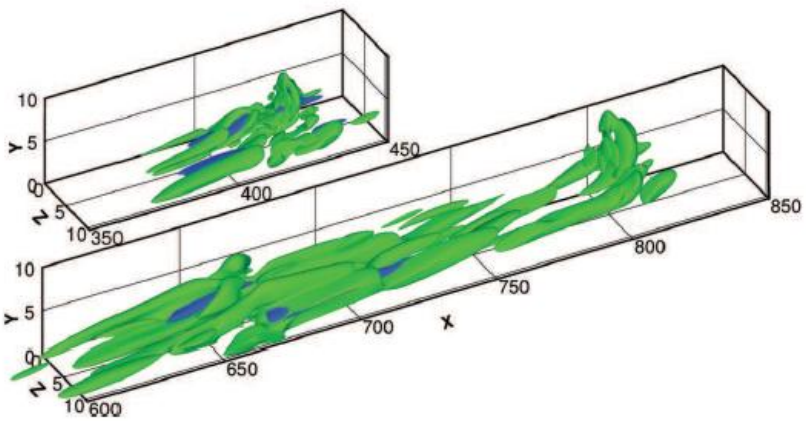
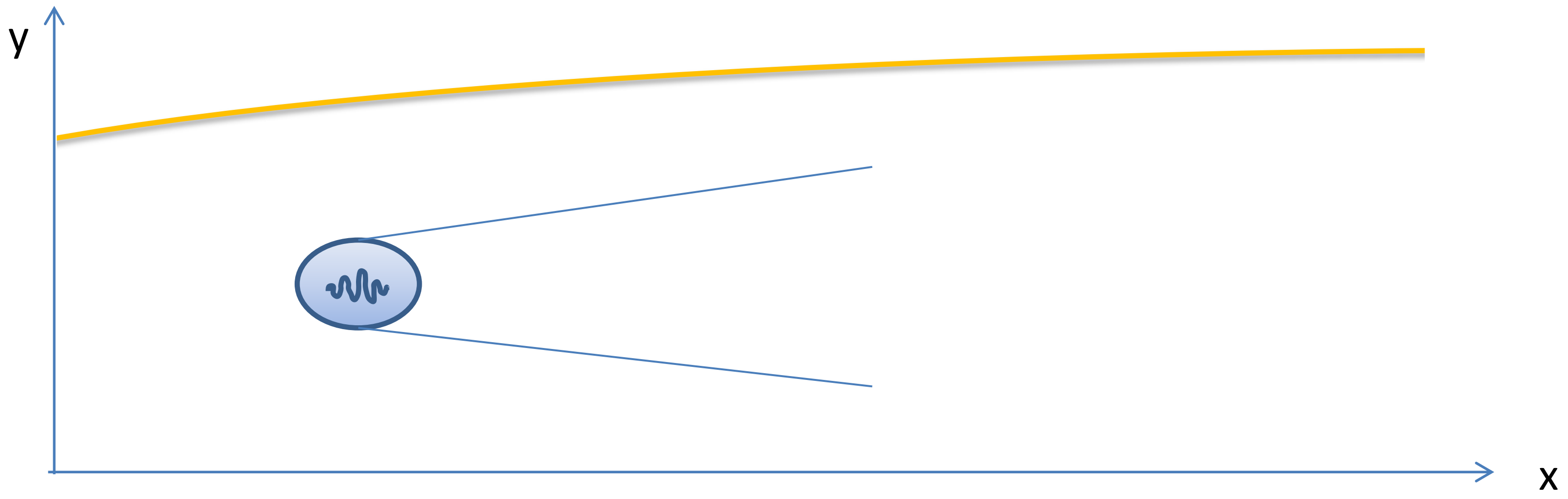


FIG. 2. (Color online) Snapshots of the streamwise component of the perturbation (darker surfaces, blue online, for $u = -0.13$) and of the Q-criterion (lighter surfaces, green online) at $t = 300$ and $t = 700$ (top and bottom, respectively) obtained by the DNS initialized with the nonlinear optimal perturbation with $E_0 = 0.004444275$.

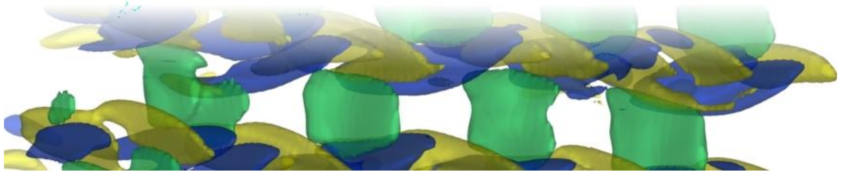
What else?

- Localized initial disturbance (in x, y and z)
- Efficient (small input → *catastrophic* output)
- Nonlinear interactions
- Maintain ’obliquity’ (i.e. no B/L scales)

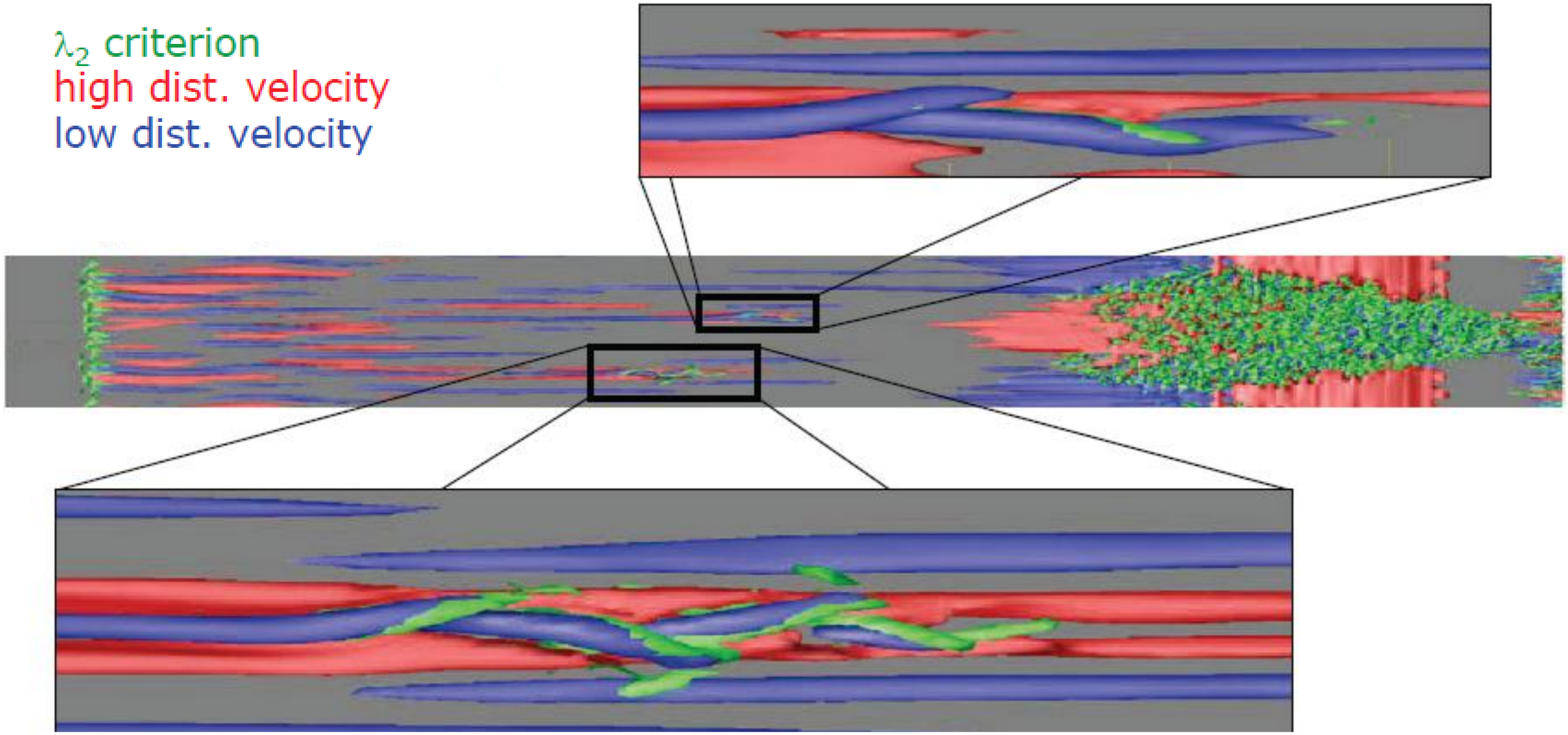


Questions we will try to answer …

- Linear/nonlinear mechanisms
- What is the most dangerous, localized initial flow state (which we will call the *minimal seed*)?
- How *robust* is it with respect to flow domain constraints, Re, initial energy level … ?
- Path to transition? Going near some saddle point in phase space (the *edge state*)?
- Can we imagine something like a *cycle* for the regeneration of flow structures?



Effect of 3D inlet noise: **suboptimal**



Philipp Schlatter et al., 2009

Optimizing the initial disturbance field

- Direct-adjoint procedure to maximize the disturbance energy at given target time T

Polack & Ribière, 1969, conjugate gradient approach needed to converge also for ‘large’ initial disturbance energies

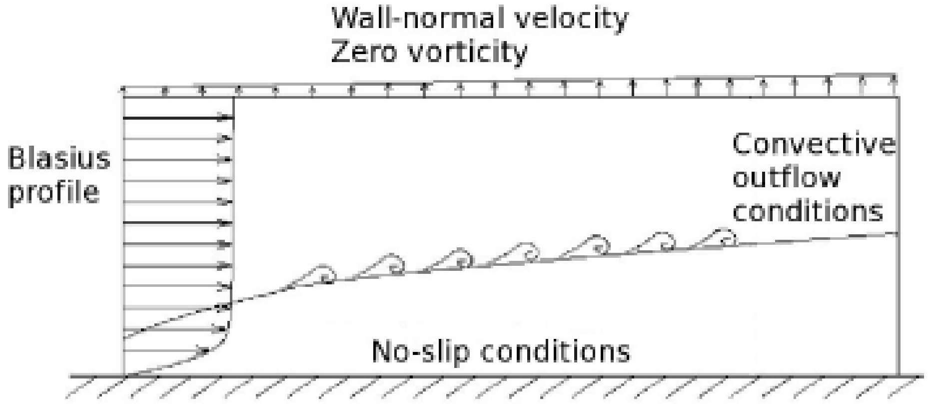
The DNS code

Non-dimensional incompressible Navier–Stokes equations:

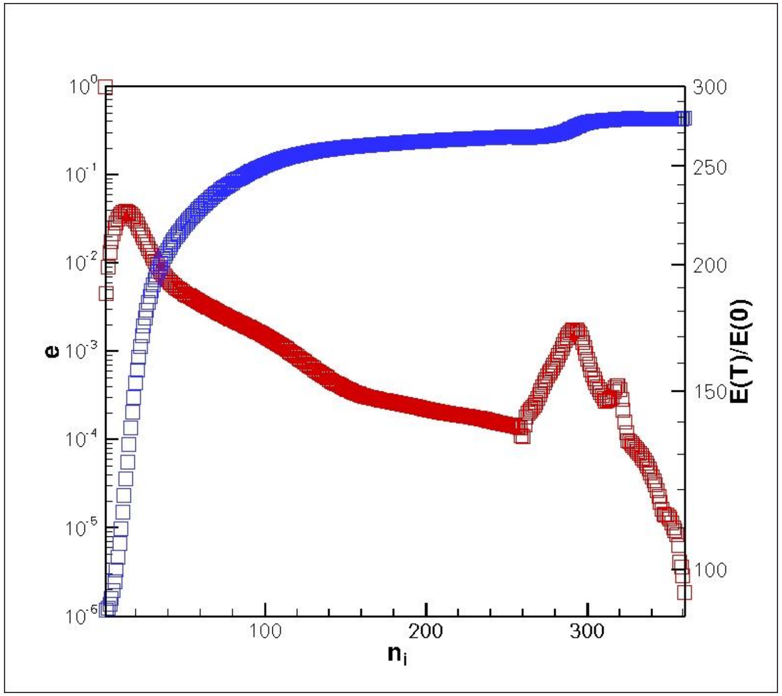
$$\begin{aligned} \mathbf{u}_t + (\mathbf{u} \cdot \nabla)\mathbf{u} &= -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}, \\ \nabla \cdot \mathbf{u} &= 0, \end{aligned}$$

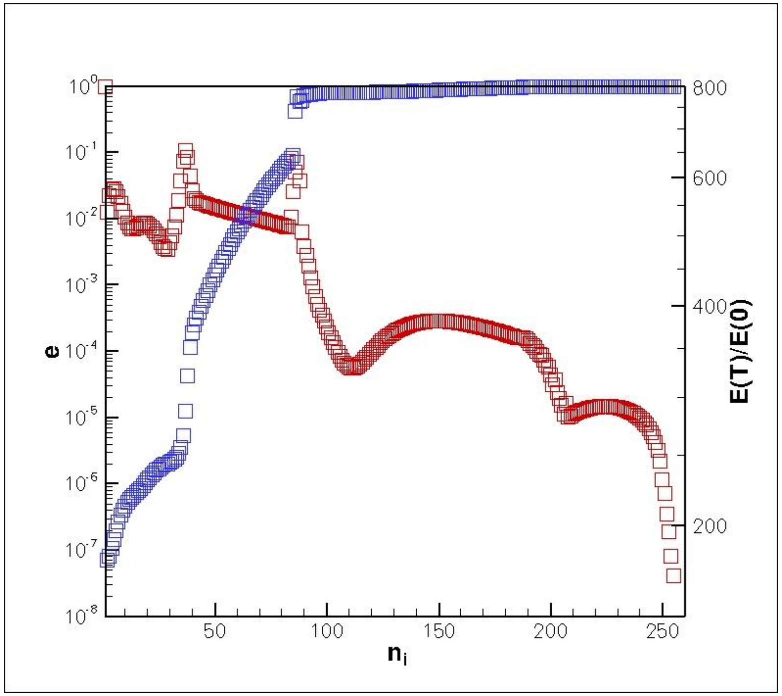
with $\mathbf{u} = (u, v, w)^T$ the velocity vector, p the pressure and $Re = \frac{U_\infty \delta^*}{\nu}$

- '*Fractional step*' method on a '*staggered*' grid.
- Centered second-order spatial discretization
- Temporal discretization: Crank–Nicolson for the viscous terms, third-order Runge–Kutta for non-linear ones.
- Domain: $200 \times 20 \times 10.5$ in terms of δ_1, discretized on a $901 \times 150 \times 61$ grid



Convergence

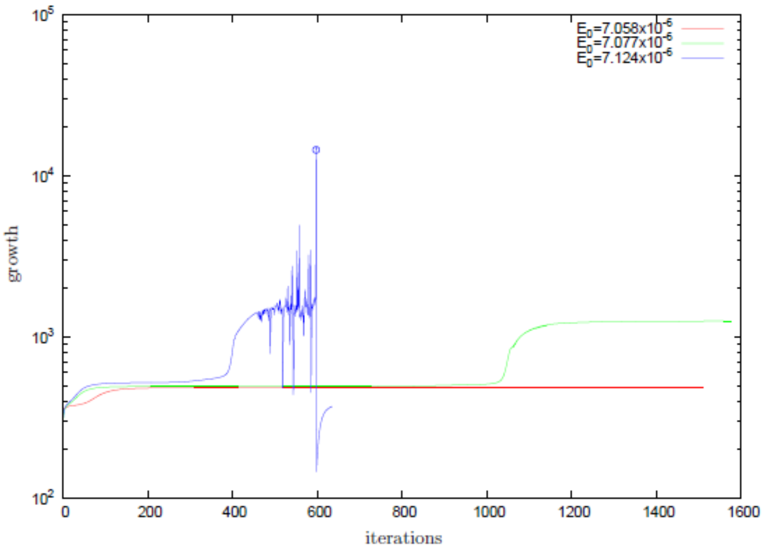


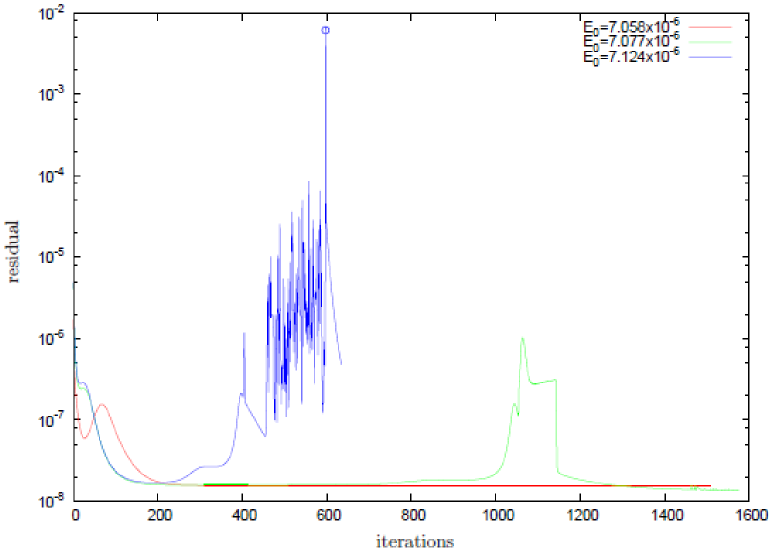


T = 75, E_0 = 0.001, Re = 610

T = 75, E_0 = 0.01, Re = 610

Convergence

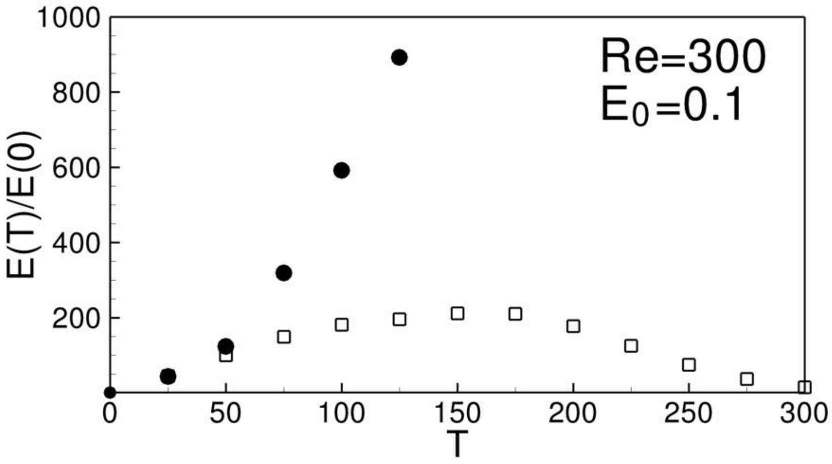


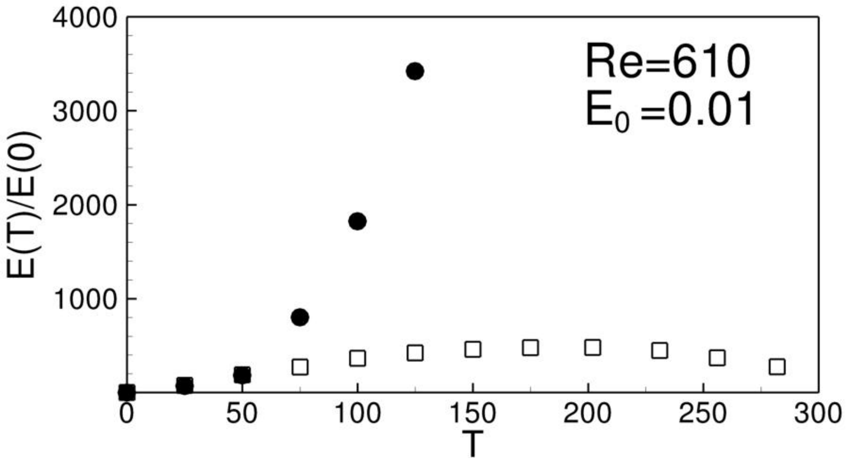


Pringle, Willis & Kerswell, 2011 (periodic pipe flow)

'… however the domain is by no means long enough for us to observe truly localisd optimals as opposed to periodic disturbances.'

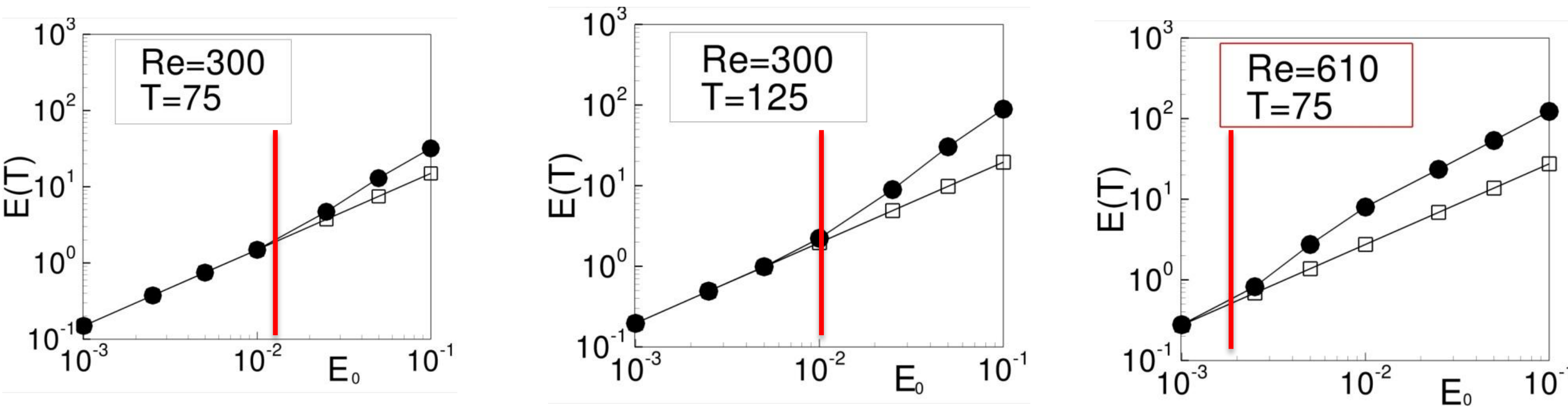
Linear versus nonlinear





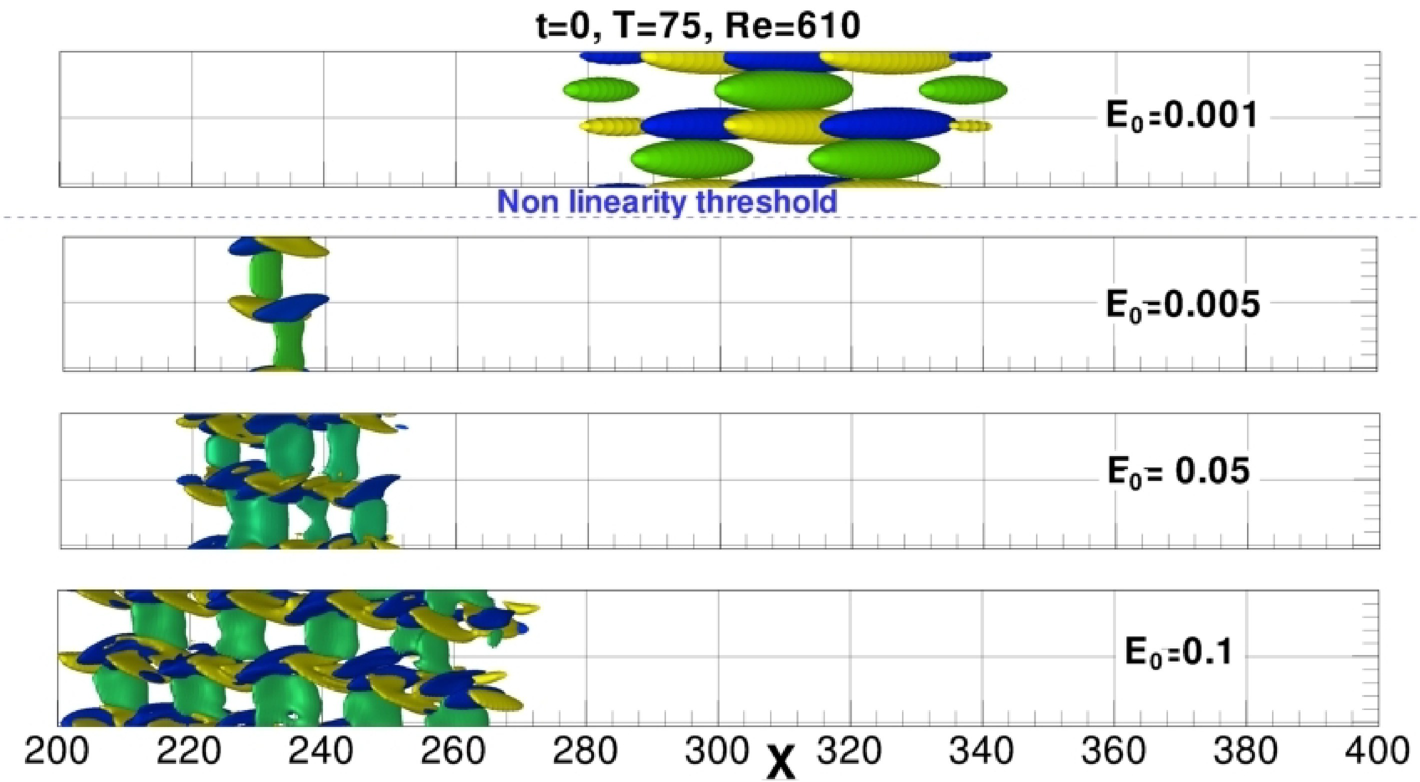
For target time T sufficiently large nonlinear optimals produce much larger gains

Linear versus nonlinear



For given Re and T, a *threshold* on E_0 exists above which nonlinear effects become important

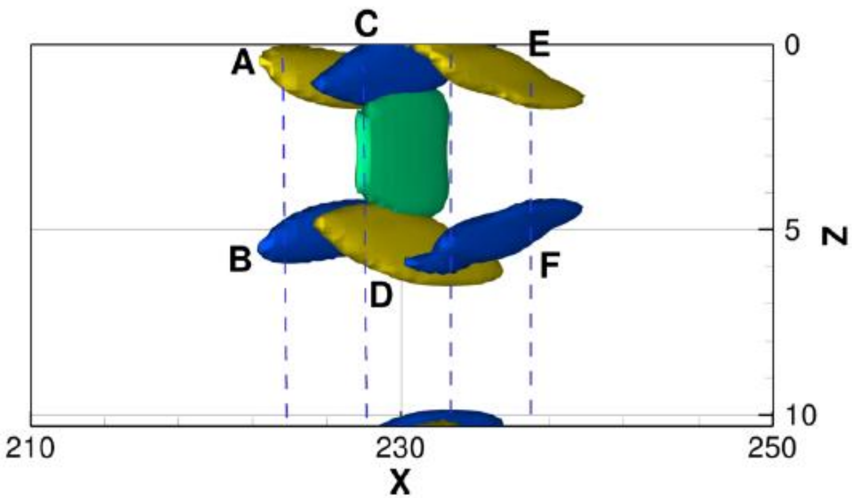
Dependence of nonlinear optimal on E_0



Above the *threshold* the same basic building block reappears …

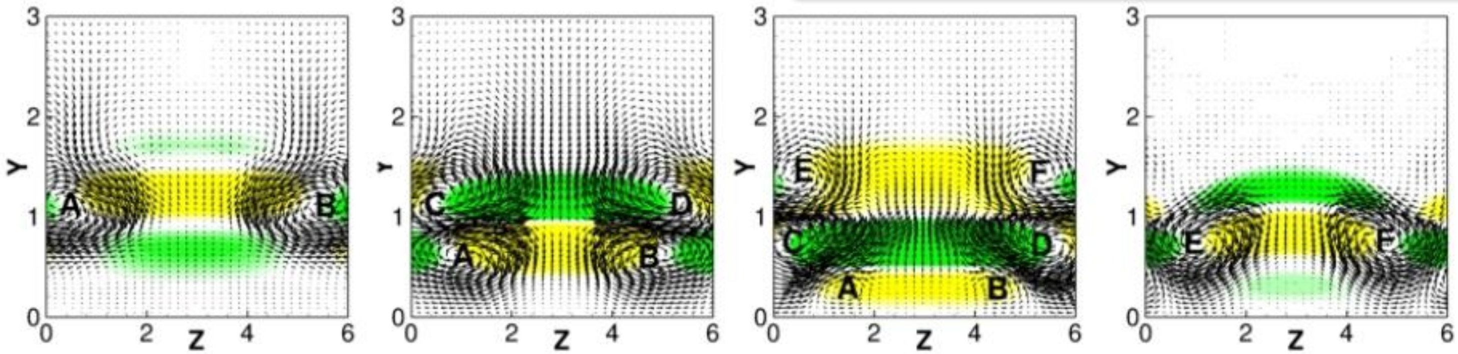
THE MINIMAL SEED

Optimal initial perturbation at $T = 75$, $E_0 = 0.01$ and $Re = 610 \rightarrow$ *alternated vortices inclined in x and tilted upstream (yellow and blue), which lay on the flanks of a region of high negative streamwise disturbance (green).*

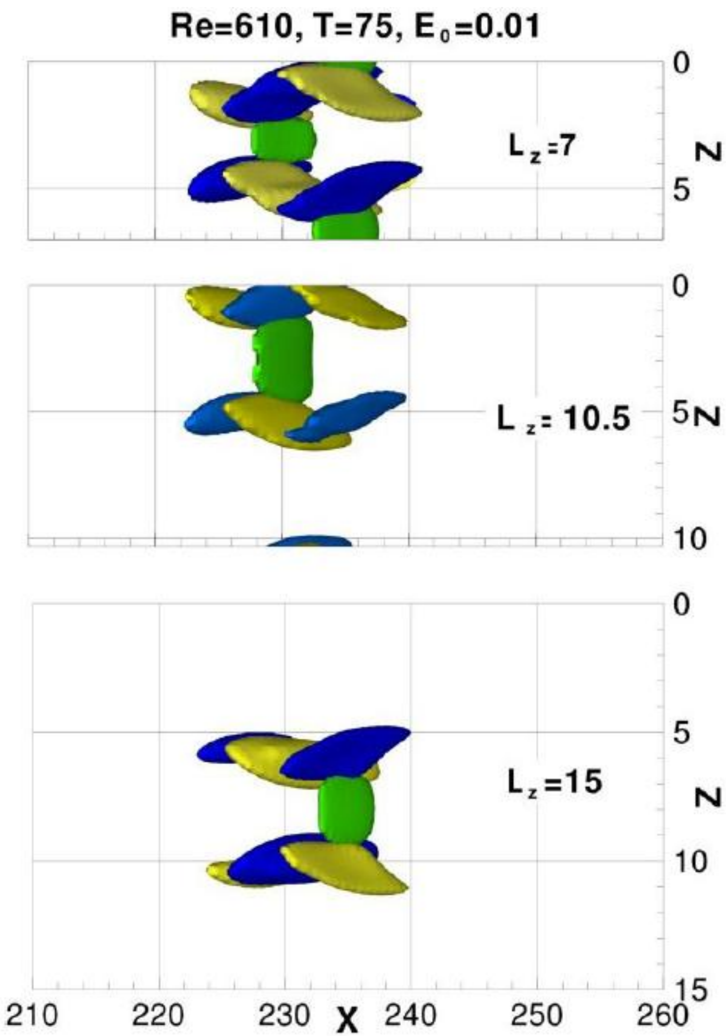


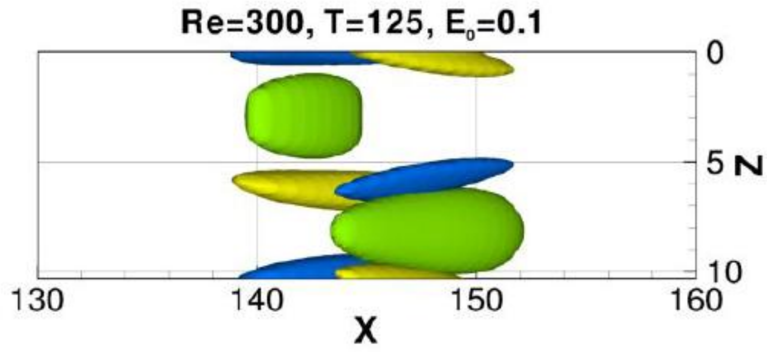
Large differences w.r.t. the linear optimal:

- it is localized in x and z
- vortices are streamwise-inclined
- u' is the largest component ($|u'_{max}| = 0.018$)
- regions with high negative u' are associated with high positive v'



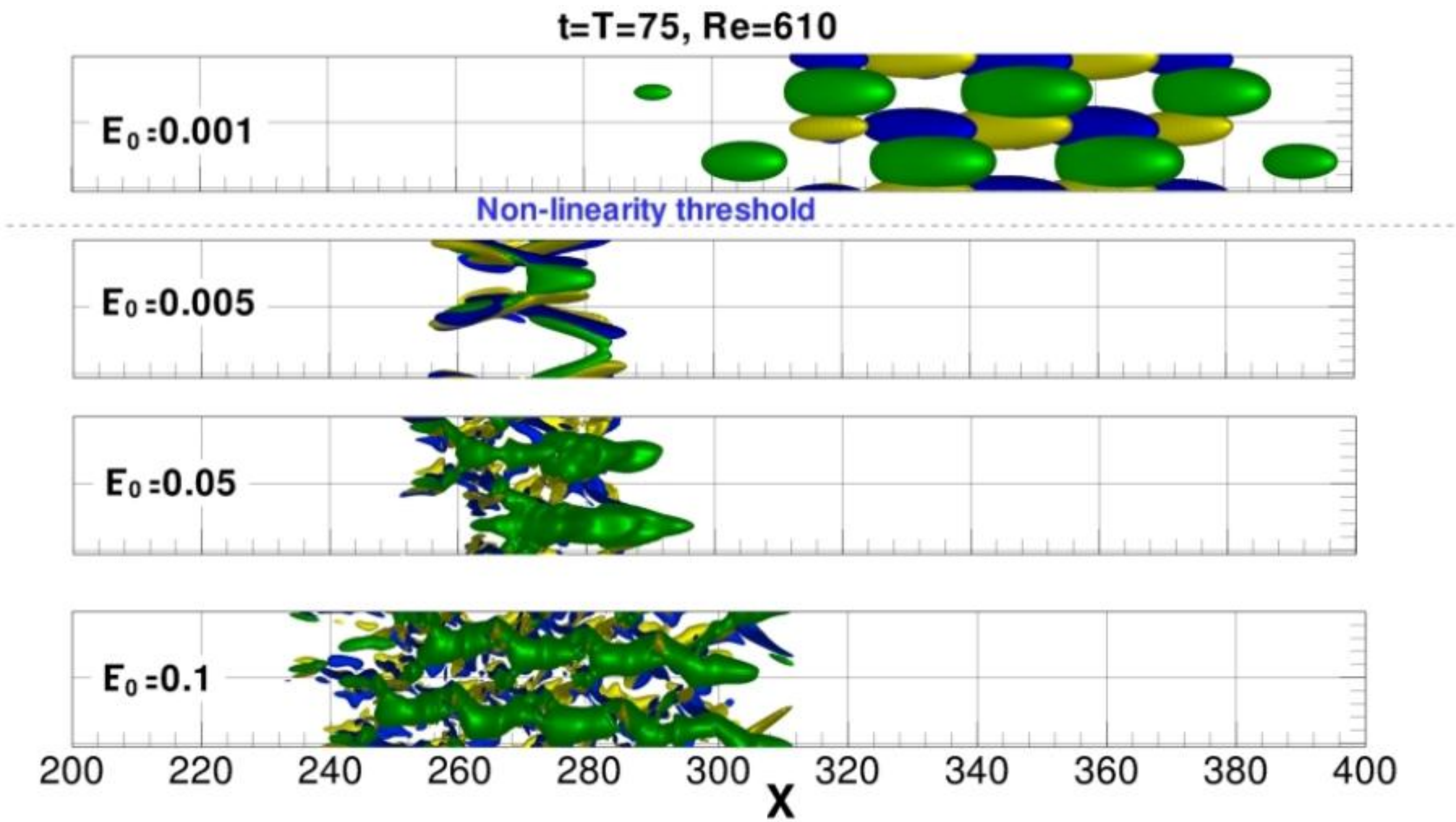
≈ universality of the minimal seed





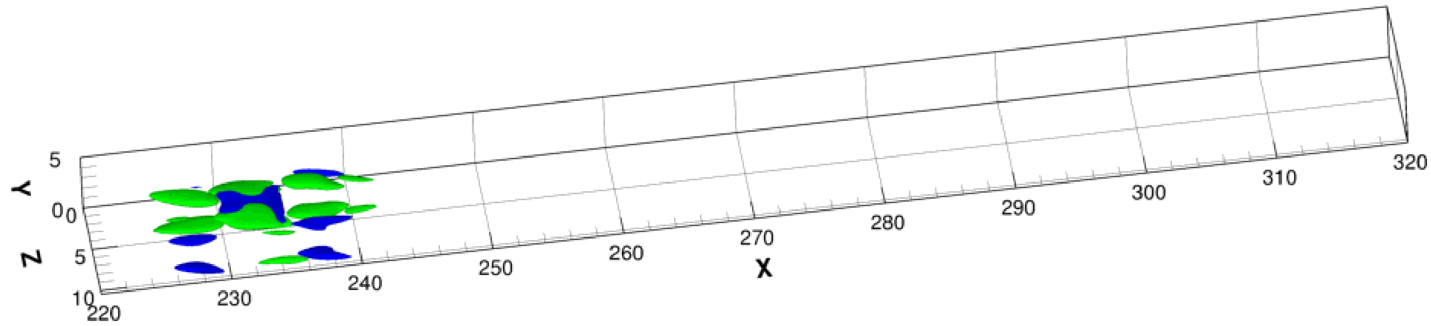
- The minimal seed is observed at different Re.
- It slightly depends on the domain length
- It is has a characteristic spanwise and streamwise size

What happens at the target time T?

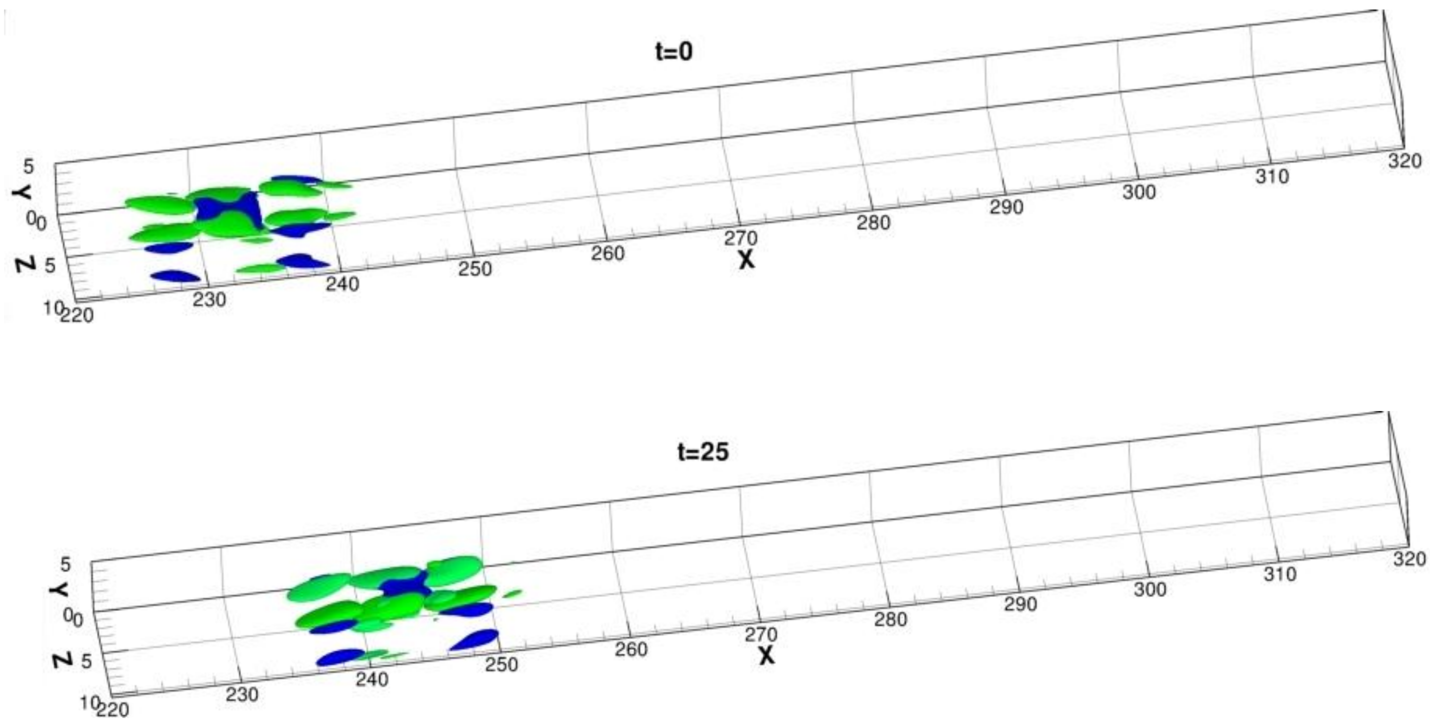


Beyond the non-linearity *threshold* Λ-vortices appear; their interactions lead the flow to turbulence when several *minimal seeds* are present in the initial field

Path to turbulence of the minimal seed

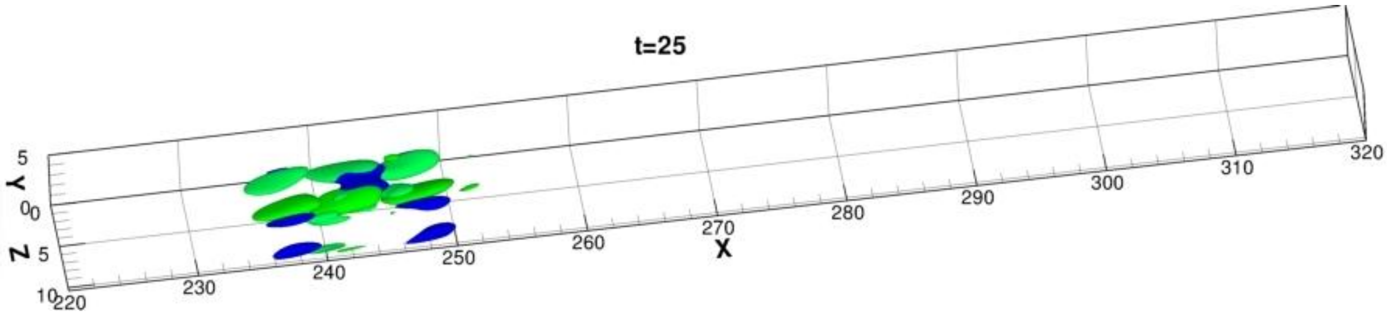


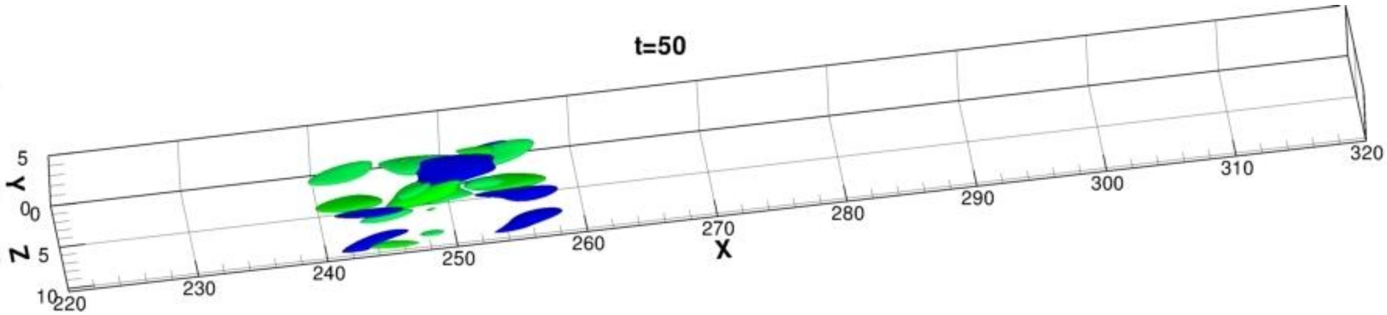
Path to turbulence of the minimal seed



Orr mechanism tilts the vortices (drawn in green via the Q criterion) downstream

Path to turbulence of the minimal seed

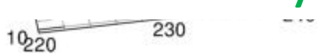




Lift up – related to $v'U_y$ – to amplify the streamwise disturbance field (drawn in blue)

Path to turbulence of the minimal seed

"Landahl (1975, 1980) studied the linear evolution of localized disturbances and formalized a physical explanation for the streak growth mechanism, which we denote the lift-up effect. Since a fluid particle in a streamwise vortex will initially retain Its horizontal momentum if displaced in the wall-normal direction, such a disturbance in the wall-normal velocity will cause in a shear layer a perturbation in the streamwise velocity. "

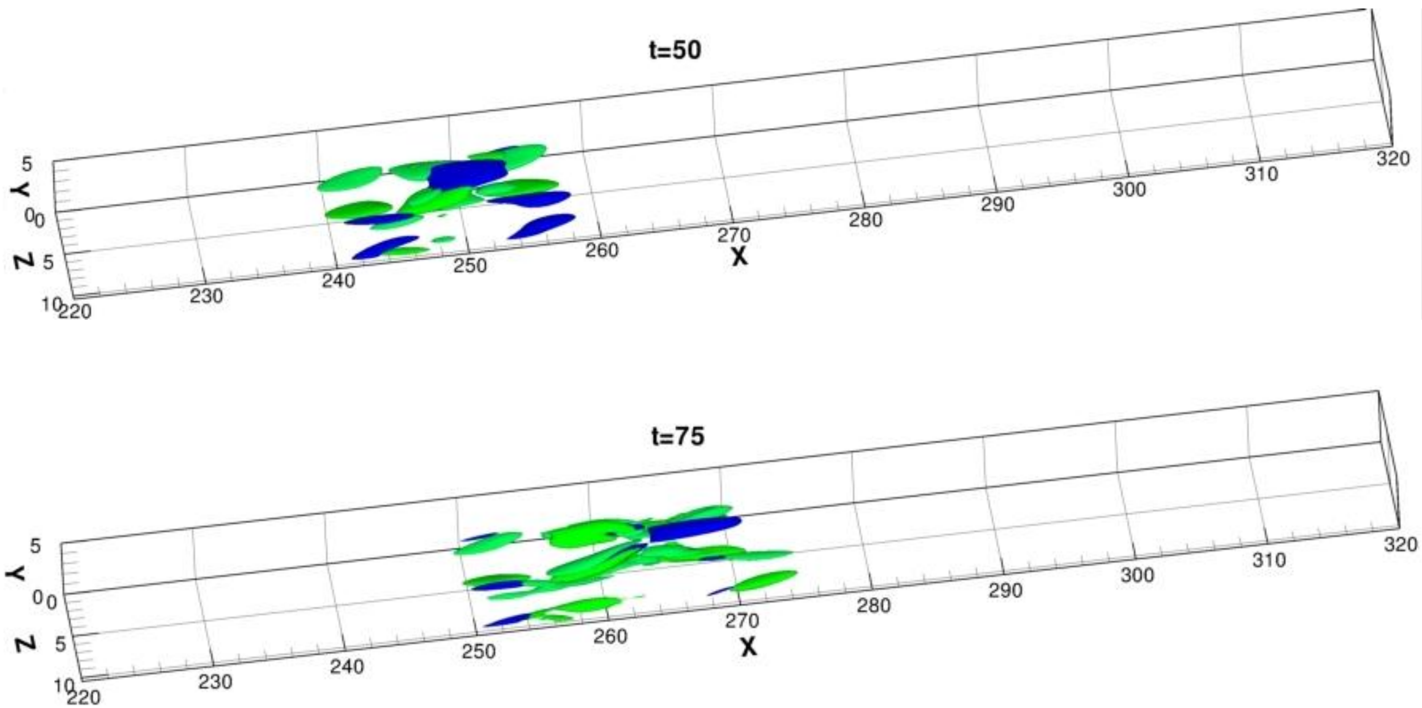


L. BRANDT, P. SCHLATTER AND D.S. HENNINGSON, JFM 2004

Lift up to amplify the streamwise disturbance field

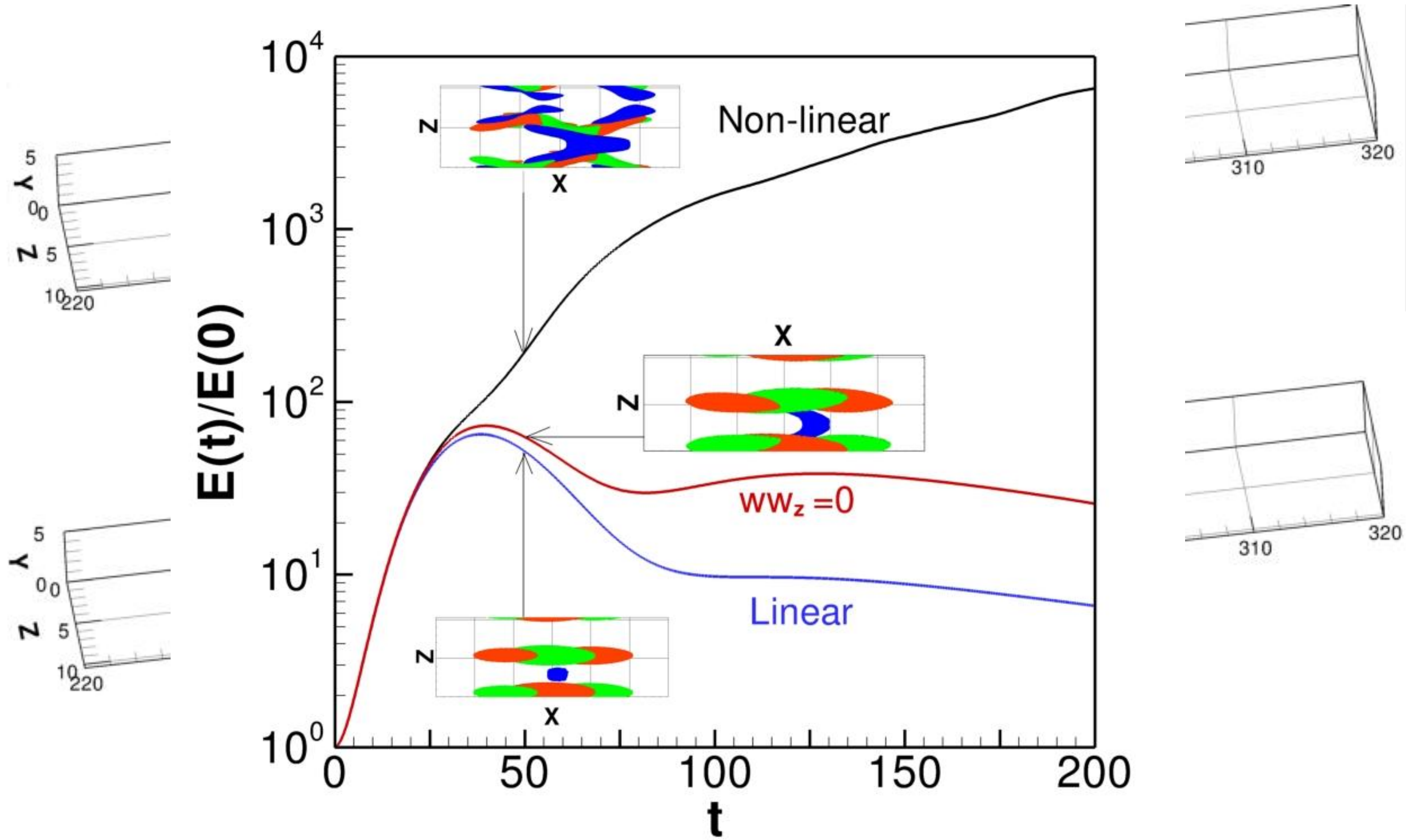
(drawn in blue)

Path to turbulence of the minimal seed



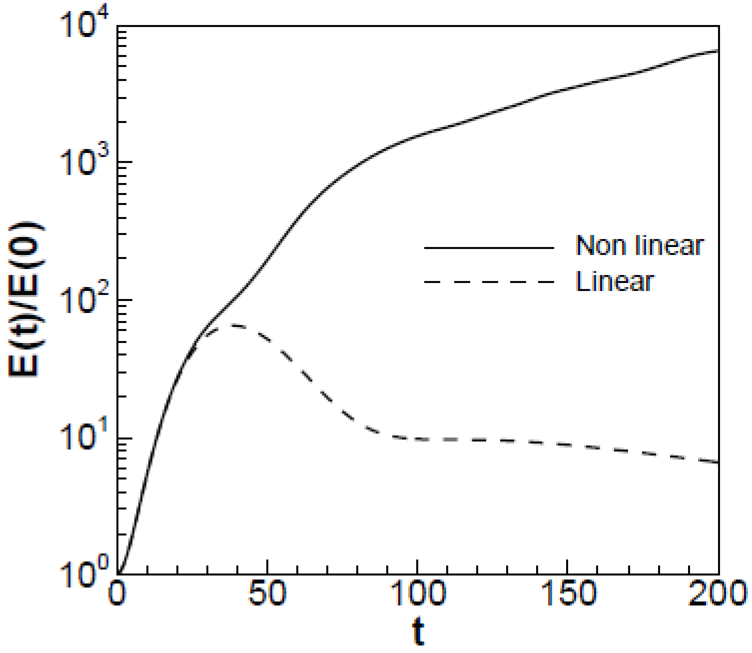
Structures remain *oblique* thanks to the term ww_z

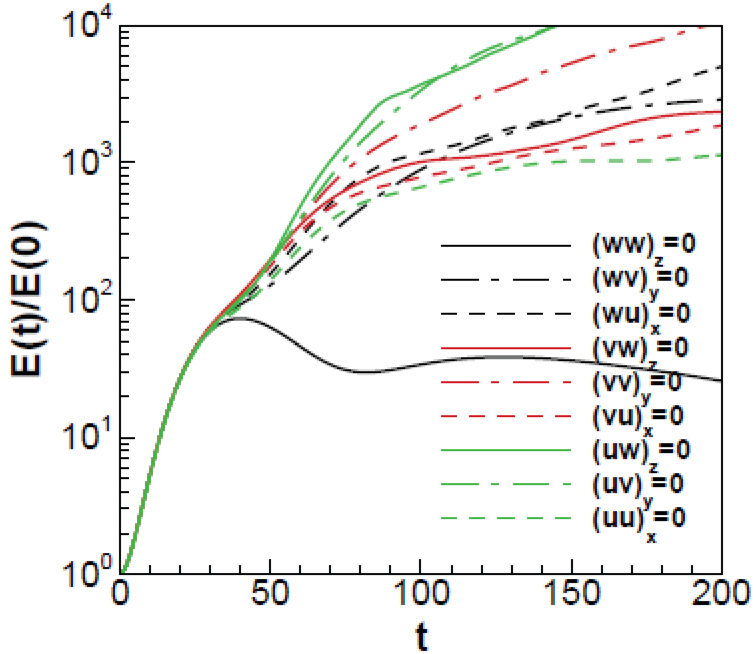
Path to turbulence of the minimal seed



Structures remain *oblique* thanks to the term ww_z

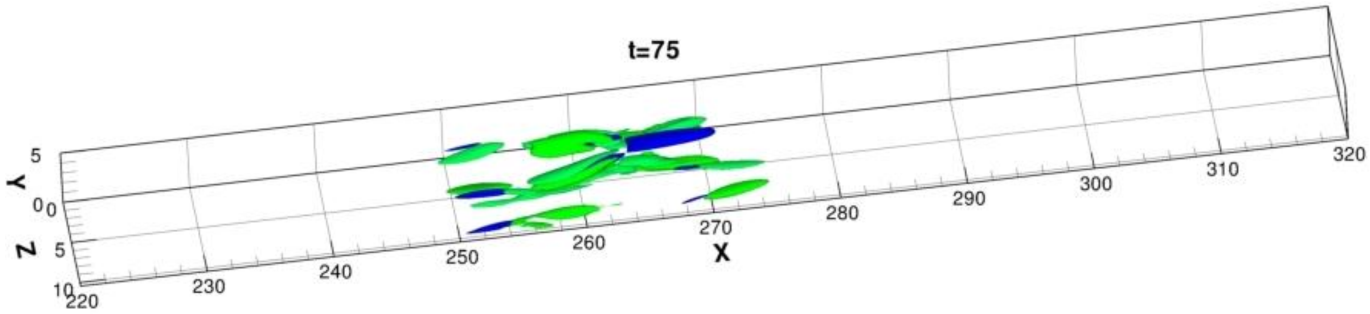
Path to turbulence of the minimal seed

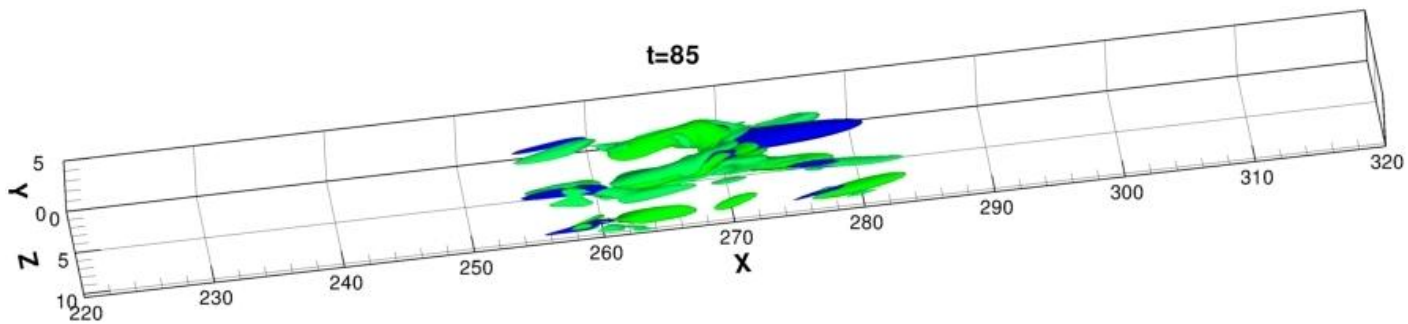




Structures remain *oblique* thanks to the term ww_z

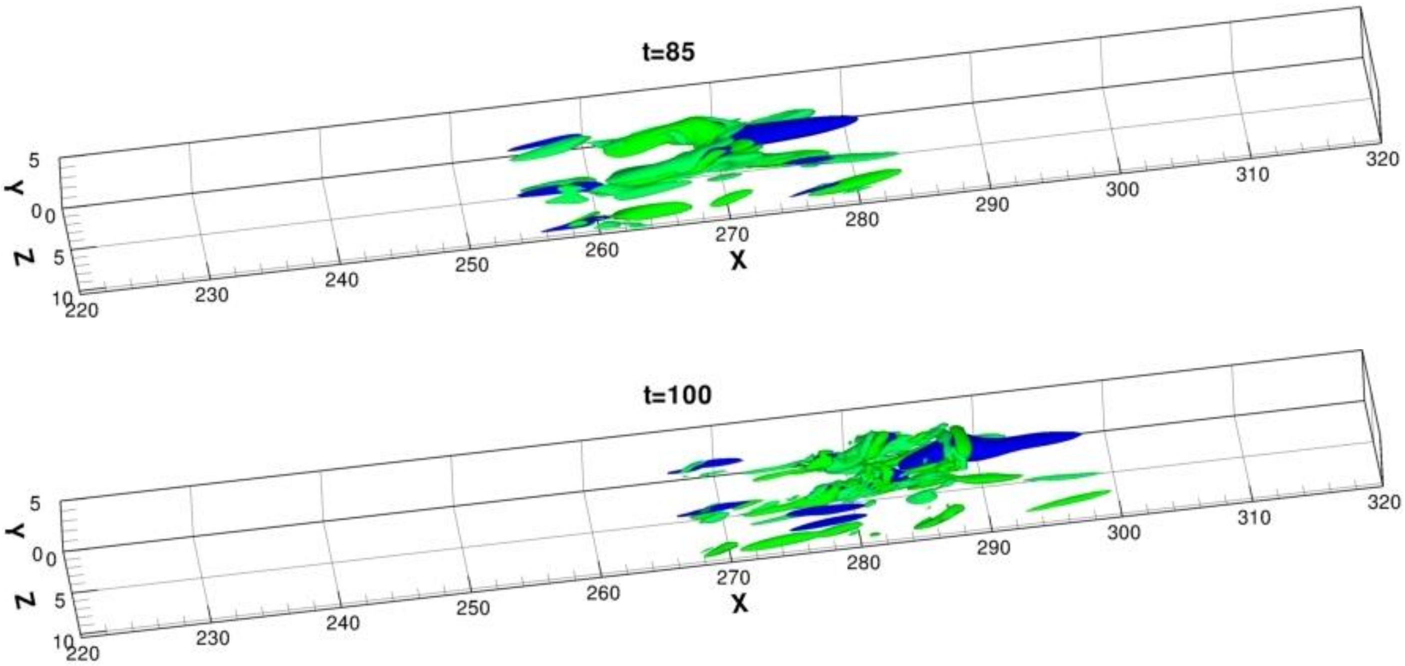
Path to turbulence of the minimal seed





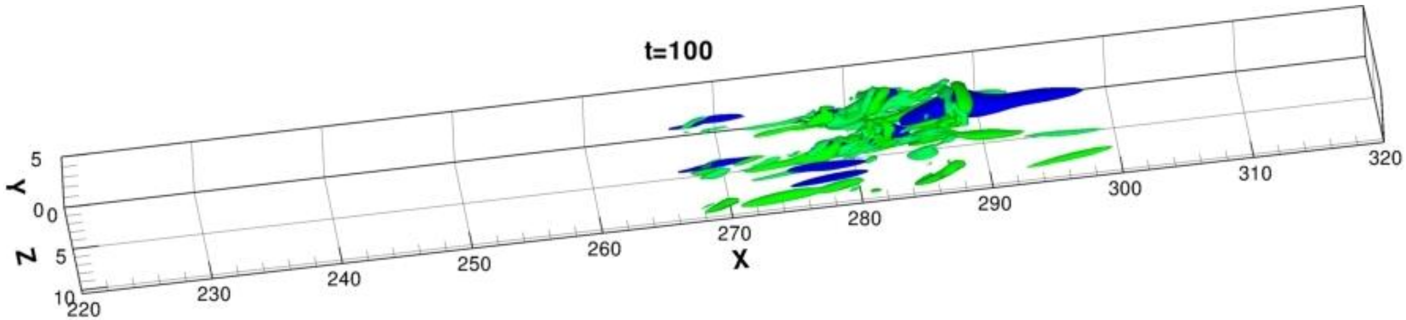
Creation of a Λ-vortex because of stretching of the vortical disturbances by the mean flow, via the term Uu'_x

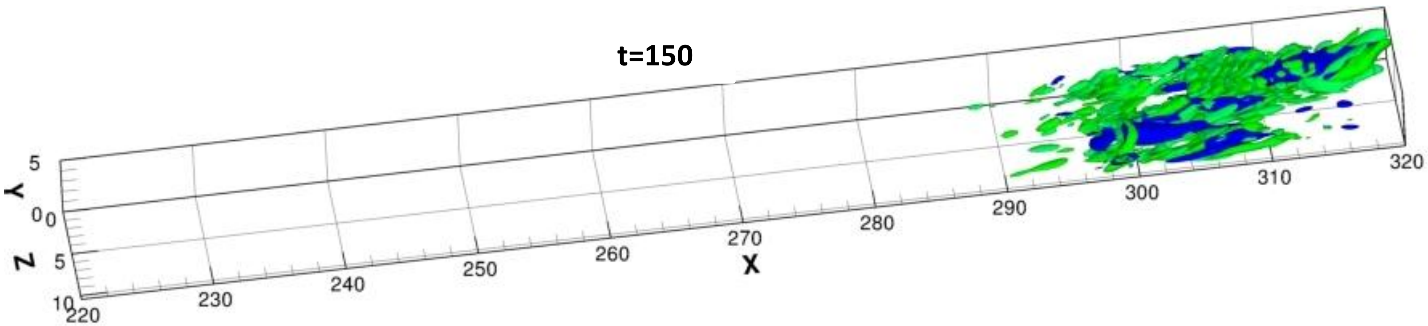
Path to turbulence of the minimal seed



Formation of an arch-vortex → hairpin, (switching off the term $(u'v')_x$ inhibits the development of the hairpin head)

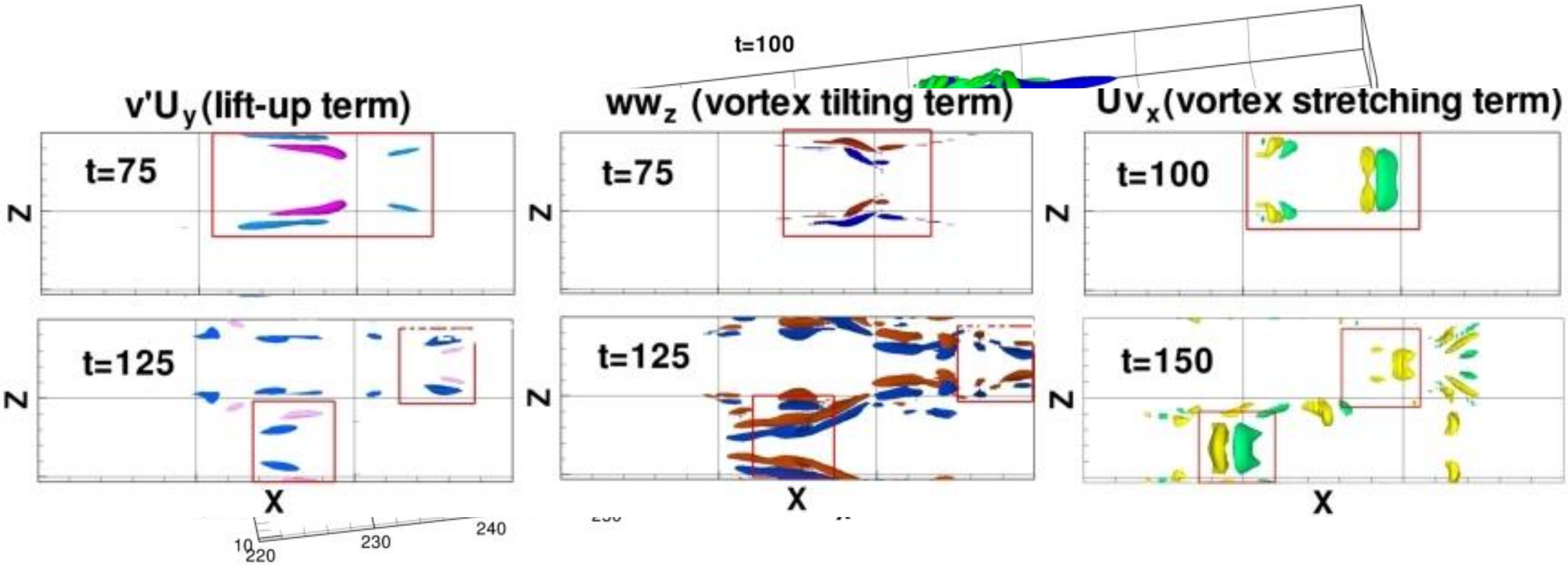
Path to turbulence of the minimal seed



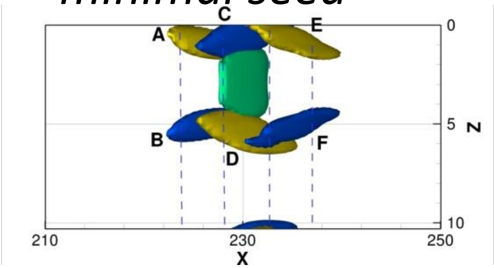


Smaller scale vortices and subsidiary hairpins

Path to turbulence of the minimal seed



Smaller scale vortices and subsidiary hairpins *minimal seed*



snapshot from a simulation of turbulence

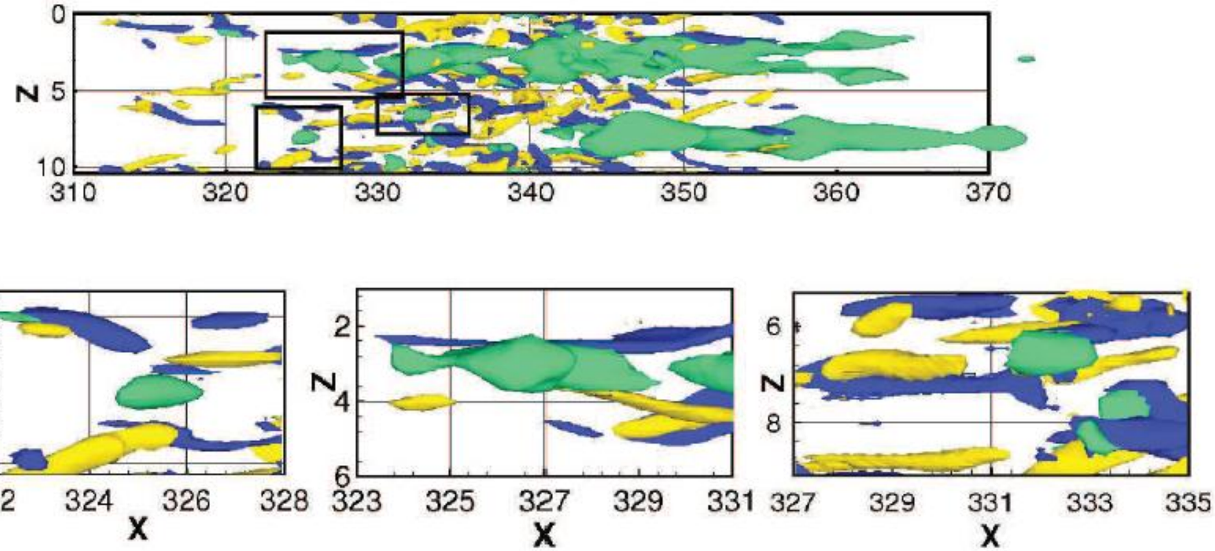
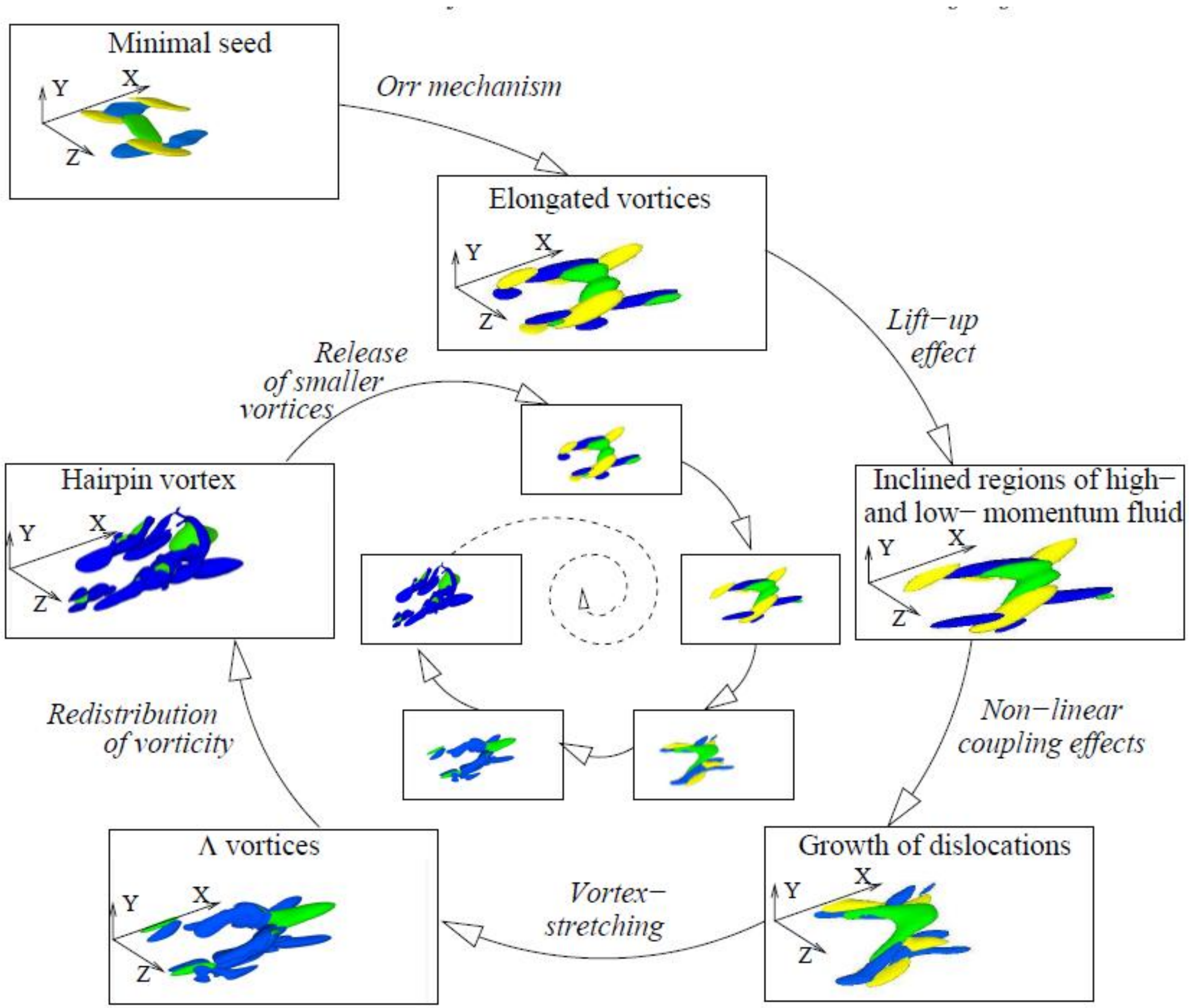


FIGURE 23. Iso-surfaces of the streamwise component of the velocity (green, $u' = -0.25$) and streamwise vorticity perturbations (yellow and blue for $\omega'_x = 0.6$ and $\omega'_x = -0.6$), respectively. The top frame shows the entire view of the wave packet, whereas the bottom ones provide the local view of the three regions of the flow marked by black rectangles on the top.

This suggests a cycle for the regeneration of flow structures at smaller/faster space/time scales …

The disturbance regeneration cycle



Summing up

- *Minimal seed*, localized spatial structure invariant w.r.t. Re, E_0, domain size, target time
- Minimal seed differs in shape and amplitude from both classical OP (Andersson et al. 1999, Luchini 2000) and from *linear*, localized OP
- It triggers transition faster than any other IC (better than oblique transition, for details consult the paper by Cherubini et al., in press). **NONLINEARITY IS CRUCIAL!**
- Steps: Orr mechanism, lift-up ($v'U_y$), maintain obliquity $(ww)_z$, Λ-vortices ($U\mathbf{u}'_x$), hairpins → **THEN REPEAT**!
- *Disturbance regeneration cycle* could start from other disturbances, such as free-stream turbulence …