J. Fluid Mech. (2003), *vol.* 476, *pp.* 293–302.
DOI: 10.1017/S002211200200318X Printed in the United Kingdom

The effect of base flow variation on flow stability

By ALESSANDRO BOTTARO[1], PETER CORBETT[1]† AND PAOLO LUCHINI[2]

[1]Institut de Mécanique des Fluides de Toulouse, Allée du Pr. Camille Soula, 31400 Toulouse, France

[2]Dipartimento di Ingegneria Meccanica, Università di Salerno, 84084 Fisciano, Italy

(Received 8 July 2002 and in revised form 29 October 2002)

The Orr–Sommerfeld operator's eigenvalues determine the stability of exponentially growing disturbances in parallel and quasi-parallel flows. This work assesses the sensitivity of these eigenvalues to modifications of the base flow, which need not be infinitesimally small. Such base flow variations may represent differences between the laboratory flow and its ideal, theoretical counterpart. The worst case, i.e. the change in base flow with the most destabilizing effect on the eigenvalues, is found using variational techniques for the plane Couette flow. Relatively small changes in the base flow are shown to be destabilizing, although the ideal flow is unconditionally stable according to linear theory. These observations inspire a velocity-based definition of pseudospectra in the hydrodynamic stability context.

1. Introduction

Classical hydrodynamic stability theory is concerned with the behaviour of normal mode solutions to the linearized equations of motion, expanded about some basic state of inherent interest. Plane Couette flow, which arises in the fluid between two parallel walls moving in opposite directions, represents one of the simplest wall-bounded shear flows which is nevertheless of great practical interest in engineering applications, geophysics and astrophysics. Linear theory predicts this flow to be unconditionally stable (Romanov 1973; Gallagher & Mercer 1962; Davey 1973; Kreiss, Lundbladh & Henningson 1994), although turbulence occurs in experiments (some recent references are Tillmark & Alfredsson 1992; Tillmark 1995; Dauchot & Daviaud 1995; Bottin *et al.* 1998). This apparent contradiction has motivated numerous analyses aimed at uncovering possible nonlinear instability mechanisms (Gill 1965; Lerner & Knobloch 1988; Grossmann 2000, and references therein).

Recent developments in hydrodynamic stability have focused on the non-normality of the underlying differential operator (Boberg & Brosa 1988; Reddy & Henningson 1993; Trefethen *et al.* 1993; Reddy, Schmid & Henningson 1993). One outcome of this work is that some eigenvalues are extremely sensitive to perturbations in the operator, and that the transient behaviour of infinitesimal flow disturbances cannot be ignored in bounded or unbounded shear flows (these effects were first explored for Couette flow by Butler & Farrell 1992). The authors cited above consider generic operator perturbations in which all vector components are weighted by a common quadratic norm. On the one hand this allows the greatest generality, on the other it gives equal weight to physically and dimensionally distinct quantities and may introduce coupling terms that are absent in the original equations.

† Present address: Airbus Deutschland, GmBH, Loads & Aeroelastics, Kreetslag 10, D-211 29 Hamburg, Germany.

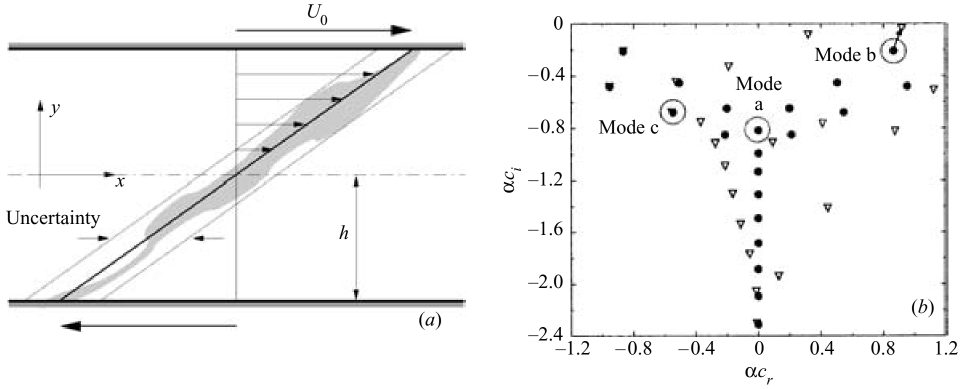

FIGURE 1. (*a*) Sketch of the canonical Couette flow, with the reference frame and the characteristic problem parameters. The shaded area indicates the range of velocity profiles an hypothetical laboratory apparatus might produce. (*b*) ●, Eigenvalue spectra for the plane Couette flow at $R = 500$, $\alpha = 1.5$; ∇, a Couette-like flow at the same conditions. Three modes are circled and labelled for later reference. Note the movement of mode b towards the real line.

Both classical linear stability theory and the more recent work on algebraic growth mechanisms consider the behaviour of small disturbances in a basic flow state of inherent interest. The base flow can usually be described by an analytical expression. In practice, however, one often wishes to investigate the stability of velocity profiles measured in experiments, which can only be poorly approximated by analytical means, or which result from third-party computations of unknown quality. Measurement error, unknown computational precision and the use of an approximation outside its range of validity all imply a degree of uncertainty associated with the base flow. Figure 1(*a*) illustrates the situation: the canonical Couette flow profile is shown as a diagonal line; the shaded area shows the range of velocity profiles one might actually measure in an experiment.

This article focuses on the underlying assumption in stability theory that the base flow is fixed and known. Assuming that deviations of a given amplitude from the ideal flow state may occur, we seek to determine the resultant effects on the flow's linear stability. How are the growth rate and frequency of the corresponding disturbance modes affected by changes in the shape of the base flow velocity profile? Can the results be related to physical mechanisms? Answers to these questions may prove significant for the transition problem, and provide some insight into how reasonable comparisons between more practical and purely theoretical results may be made.

As illustrated in figure 1(*b*), there may be considerable differences between the stability characteristics of the ideal flow (the Couette spectrum is shown by solid dots) and a flow whose velocity profile differs only very slightly from the ideal (whose spectrum is indicated by inverted triangles). Below, a method is described for determining the sensitivity of individual eigenvalues to changes in the base flow using techniques from classical eigenanalysis. Then a standard variational procedure is employed to find the worst-case base flow modifications, i.e. those velocity deviations of given absolute value r that maximize the growth rate of one eigenmode. Concepts are illustrated in the context of the plane Couette flow, and some parallels are drawn with previous work. Sufficiently large modifications to the base flow are found to induce linear instability. Critical Reynolds numbers are found to vary as r^{-1} when r

is small. The paper concludes with a proposal for a velocity-based definition of the pseudospectrum for hydrodynamic stability.

2. Sensitivities

Although only plane Couette flow is discussed below, the following arguments apply to exponentially growing disturbances of arbitrary three-dimensional form occurring in parallel shear flows with one or two horizontal velocity components.

The behaviour of infinitesimal three-dimensional disturbances in an arbitrary, parallel base flow $(U(y), 0, W(y))$ in the domain $y \in [-h, +h]$ is described by the linearized Navier–Stokes equations. These can be recast as a system of ordinary differential equations: the Orr–Sommerfeld equation for the vertical velocity disturbance $v(y)$, and the Squire equation for the vertical vorticity disturbance $\eta(y)$. The Reynolds number $R = U_0 h/\nu$ appears as the only parameter when these are non-dimensionalized using U_0 as a velocity scale and h as a length scale, where ν is the kinematic viscosity. This work adopts a temporal setting, i.e. disturbances are bounded in space and decay or grow in time as $\exp(-\mathrm{i}\omega t)$, with ω the complex frequency, $\omega = \omega_r + i\omega_i$. The dimensionless system can be compactly written as,

$$\begin{bmatrix} \mathscr{C}\nabla^2 - \mathrm{i}\alpha U'' - \mathrm{i}\beta W'' & 0 \\ \mathrm{i}\beta U' - \mathrm{i}\alpha W' & \mathscr{C} \end{bmatrix} \begin{bmatrix} v \\ \eta \end{bmatrix} = 0, \tag{2.1}$$

with boundary conditions,

$$v = \mathrm{d}v/\mathrm{d}y = 0, \quad \eta = 0 \quad \text{at} \quad y = \pm 1.$$

Here $\mathscr{C} = -\mathrm{i}\omega + \mathrm{i}\alpha U + \mathrm{i}\beta W - R^{-1}\nabla^2$, the Laplacian operator is $\nabla^2 = \mathrm{d}^2/\mathrm{d}y^2 - \alpha^2 - \beta^2$, α and β are real wavenumbers in the streamwise (x) and spanwise (z) directions, respectively, and primes denote differentiation with respect to y.

The spectrum of the system consists of the eigenvalues of the two diagonal terms of the square matrix in (2.1), called Orr–Sommerfeld and Squire modes respectively. In both entries the base flow appears in the form $\alpha U + \beta W$ or its second derivative, thereby permitting application of Squire's transformation

$$\tilde{\alpha}\tilde{U} = \alpha U + \beta W, \quad \tilde{\alpha}^2 = \alpha^2 + \beta^2, \tag{2.2}$$

and reducing the system to

$$\begin{bmatrix} \tilde{\mathscr{C}}\nabla^2 - \mathrm{i}\tilde{\alpha}\tilde{U}'' & 0 \\ \mathrm{i}\beta U' - \mathrm{i}\alpha W' & \tilde{\mathscr{C}} \end{bmatrix} \begin{bmatrix} v \\ \eta \end{bmatrix} = 0, \tag{2.3}$$

with $\tilde{\mathscr{C}} = -\mathrm{i}\omega + \mathrm{i}\tilde{\alpha}\tilde{U} - R^{-1}\nabla^2$ and $\nabla^2 = \mathrm{d}^2/\mathrm{d}y^2 - \tilde{\alpha}^2$. Since Squire modes are always damped (cf. Schmid & Henningson 2001), it suffices to consider modes of the form $\exp[\mathrm{i}(\tilde{\alpha}x - \omega t)]$ developing in an unidirectional base flow $\tilde{U}$ when seeking the critical Reynolds number, below which no exponentially growing instability will appear. Note that the use of Squire's transform by no means limits consideration to two-dimensional disturbances; any unstable three-dimensional mode can be recovered by applying (2.2).

After dropping the tildes, the eigenrelation for v reads simply

$$\left[(U - c)\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \alpha^2\right) - U'' + \frac{\mathrm{i}}{\alpha R}\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \alpha^2\right)^2\right] v = 0 = \mathscr{L}_{\mathrm{OS}}\, v, \tag{2.4}$$

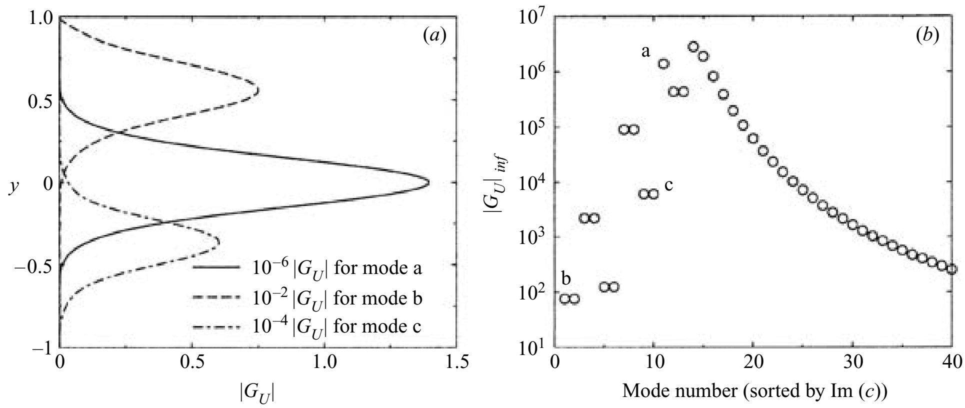

FIGURE 2. (*a*) Sensitivities for modes labelled a, b and c in figure 1(*b*). (*b*) Infinity-norm of the first 40 sensitivity functions for this flow, sorted by imaginary part.

where $c = \omega/\alpha$ is the wave speed. Operator perturbation theory is a well-developed mathematical field, and the behaviour of an operator's eigenvalues subject to changes in the operator is of interest in functional and numerical analysis as well as physics (Kato 1995; Reed & Simon 1978). A small variation in the base flow profile will cause corresponding variations in the eigenvalues and eigenfunctions of the operator $\mathscr{L}_{\mathrm{OS}}$,

$$U \rightarrow U + \delta U \Rightarrow \begin{cases} c \rightarrow c + \delta c \\ v \rightarrow v + \delta v. \end{cases} \tag{2.5}$$

Introducing these variations in (2.4), it is possible to eliminate factors of δv by taking an inner product with the adjoint eigenfunction a. This quantity is defined by the relation $(a, \mathscr{L}_{\mathrm{OS}}\, v) = (\mathscr{L}^{\dagger}_{\mathrm{OS}}\, a, v)$, where the inner product is given by $(p, q) \equiv \int_y \bar{p}\, q\, \mathrm{d}y$. The overbar indicates complex conjugate and the † denotes the adjoint operator. From the above an expression results for the adjoint Orr–Sommerfeld equation,

$$\left[(U - \bar{c})\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \alpha^2\right) + 2U'\frac{\mathrm{d}}{\mathrm{d}y} - \frac{\mathrm{i}}{\alpha R}\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \alpha^2\right)^2\right] a = 0 = \mathscr{L}^{\dagger}_{\mathrm{OS}}\, a, \tag{2.6}$$

with homogeneous boundary conditions, plus the following relation defining the sensitivity to variations in the base flow:

$$\delta c \int_y \bar{a}\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \alpha^2\right) v\, \mathrm{d}y = \int_y \delta U \left[\bar{a}\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - \alpha^2\right) v - (\bar{a}v)''\right] \mathrm{d}y = \int_y \delta U\, G_U\, \mathrm{d}y. \tag{2.7}$$

Provided the leftmost integral is unity, and in this work it is always normalized in such a manner, G_U is a linear response function tying changes in the eigenvalues to changes in the mean flow.

The above relationships are best explored by way of example. The solid circles in figure 1(*b*) show the eigenvalues of (2.4) for plane Couette flow, $U = y$, at $R = 500$ and $\alpha = 1.5$. These have been obtained using the QZ algorithm (which also provides the corresponding eigenvectors) in conjunction with a Chebyshev collocation technique with sufficient points to ensure convergence (Weideman & Reddy 2000). Computing the adjoint eigenvectors in like fashion, the sensitivities can be obtained: figure 2(*a*) shows the amplitudes of G_U for the three modes labelled a, b and c in figure 1(*b*).

These results illustrate that mean flow modifications above the centreline and relatively close to the upper wall will affect primarily mode b, whereas mode c will feel changes in the base flow below the centreline but further from the lower wall, and that deviations from the ideal flow in the vicinity of the centreline will have a large effect on mode a, with comparatively negligible effects on modes b and c. The latter two modes belong to conjugate pairs; it so happens that the sensitivity functions for their conjugate counterparts are mirrored about $y = 0$, with maxima near the opposite walls.

Of interest is the two orders of magnitude separation between the amplitudes of the different sensitivity functions, as well as the fact that there appears to be no correlation between sensitivity amplitude and proximity of the corresponding eigenvalue to the real line. This is shown quantitatively in figure 2(*b*), which displays the infinity-norm of the sensitivities sorted by imaginary part. Here it can be seen that the mode immediately underneath a in figure 1(*b*) has the largest sensitivity. These results are in qualitative agreement with studies on stability operator pseudospectra for plane Couette and Poiseuille flows, which show that eigenvalues near the confluence of eigenvalue branches are extremely sensitive to perturbation (Reddy & Henningson 1993; Trefethen *et al.* 1993; Schmid & Henningson 2001).

3. Worst case

The results of §2 can be used to determine the change, of specified magnitude, in base flow velocity profile which has the largest effect on the eigenvalues of the system, and hence on its linear stability in the classical asymptotic sense. Naturally, interest will focus upon changes which maximize growth rate, i.e. are destabilizing. More specifically, one wishes to maximize $\mathrm{Im}(c)$ via a modification to the ideal base flow, denoted U_{ref}. Let this deviation, which need not be infinitesimal, be quantified with an energy-like norm,

$$r^2 = \int_y (U - U_{\mathrm{ref}})^2 \,\mathrm{d}y. \tag{3.1}$$

Introducing the Lagrange multiplier λ to enforce the constraint (3.1), a variational approach to maximizing $\mathrm{Im}(c)$ leads to

$$\mathrm{Im}(\delta c) = \lambda \int_y 2(U - U_{\mathrm{ref}})\delta U \,\mathrm{d}y.$$

Employing the sensitivity (2.7) in this relation one obtains

$$\mathrm{Im}(G_U) = 2\lambda(U - U_{\mathrm{ref}}), \tag{3.2}$$

whence

$$U = U_{\mathrm{ref}} + \frac{1}{2\lambda}\mathrm{Im}(G_U) = U_{\mathrm{ref}} + \Delta U. \tag{3.3}$$

It is possible to determine λ by making recourse to (3.1) and (3.2),

$$\lambda = \pm\sqrt{\frac{1}{4r^2}\int_y \mathrm{Im}(G_U)^2 \,\mathrm{d}y}. \tag{3.4}$$

Although the positive root of (3.4) is of interest here, it is equally possible to minimize $\mathrm{Im}(c)$ by working with the negative root. This has interesting repercussions for the active control of hydrodynamic instabilities. Some results on damping unstable modes in Poiseuille flow via slight modifications to the base flow were reported by Bottaro, Corbett & Luchini (2001).

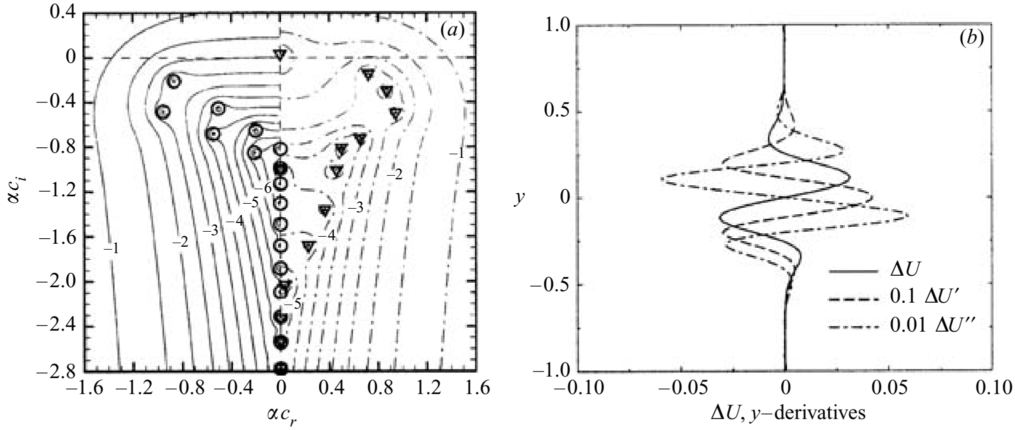

FIGURE 3. (a) Spectra and ϵ-pseudospectra (depicted by contour lines of $\log \epsilon$) of Couette (○, solid lines) and Couette-like (∇, dash-dot lines) flows at $R = 500$, $\alpha = 1.5$, the latter resulting from maximization of Im(c) using mode a. (b) Mean flow modification, ΔU and normal derivatives.

When r is infinitesimal, the most destabilizing deviation from $U_{\rm ref}$ is simply the appropriately scaled imaginary part of the sensitivity. When r is small but not infinitesimal, an iterative procedure is necessary to determine the 'optimally destabilizing' base flow. In this case a mode is selected, the direct and adjoint eigenproblems are solved and (3.3) is applied to find U. The mode is tracked until subsequent changes in the inner product of ΔU with itself drop below a threshold value (results were computed using 10^{-16}), meaning that changes in the eigenvalue have also become negligible.

Illustrating this procedure via application to the plane Couette flow discussed previously ($R = 500$, $\alpha = 1.5$) with the arbitrary, and relatively large, choice of $r = 5 \times 10^{-2}$ produces the results in figure 3. The left-hand side of figure 3(a) shows the spectrum and ϵ-pseudospectrum of the ideal Couette flow (ϵ-pseudospectra, defined in § 4, were computed using the Matlab function `psa.m` provided by Wright & Trefethen 2001). The right-hand side reports those of the modified flow at the end of the extremization procedure, with one mode in the upper half-plane (conjugate pairs have been omitted for legibility). Figure 3(b) shows the flow modification responsible for destabilizing the flow, and its normal derivatives. As can be seen from the latter, a vorticity peak near $y = 0$ has generated inflection points, with consequences in accordance with the inviscid instability criteria of Rayleigh and Fjørtoft (Schmid & Henningson 2001). The resultant flow in fact differs only very slightly from canonical Couette flow. Perhaps significantly, it bears a resemblance to the Couette flow profile modified by the presence of a fixed ribbon, which becomes linearly unstable for Reynolds numbers on the order of a few hundred, as computed by Barkley & Tuckerman (1999) who sought to duplicate the experiments of Bottin *et al.* (1998). The behaviour of the system under parametric variation is as expected. At fixed wavenumber, holding r constant and increasing R has a destabilizing effect, which is analogous to fixing R and increasing r. In either case, a clear maximum in Im(c) is associated with a wavenumber on the order of unity.

Figure 4(a) shows the lowest R for which the indicated growth rate occurs over a range of r representing a reasonably small departure from the ideal, here between 1% and 10%. The neutral curve ($\omega_i = 0$) defines the critical Reynolds number, which

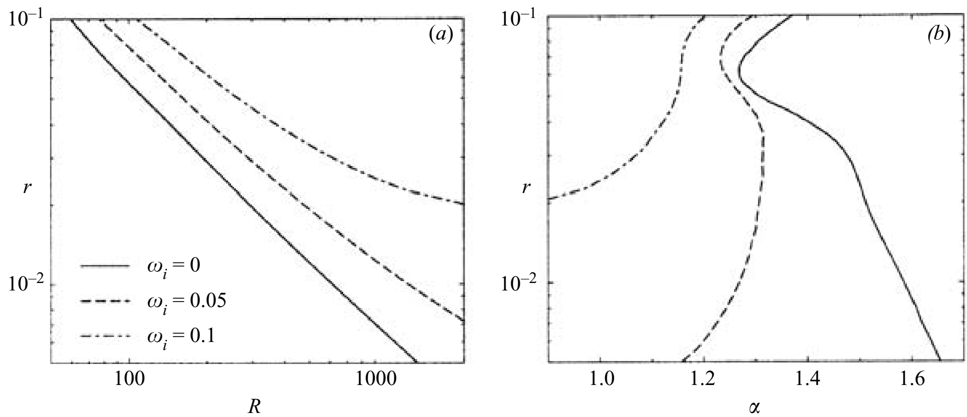

FIGURE 4. (*a*) Lowest R at which a given, constant value of ω_i occurs, as a function of r. (*b*) α for which the relation in (*a*) holds.

varies in inverse proportion to the disturbance amplitude. The wavenumber for which the relationship holds is reported in figure 4(*b*). As r increases, the flow modification ΔU increases in both amplitude and breadth, both the lobes in each half-plane of figure 3(*b*) becoming larger and moving towards the walls.

It might be argued that the modified base flow no longer satisfies the Navier–Stokes equations. However, any unidirectional base flow $U(y)$ is an exact solution to the Euler equations. Therefore an arbitrary profile, which may have been produced by initial or boundary perturbations at a certain instant of time, is in convective equilibrium and will only slowly diffuse at later times, at a lower and lower rate as the Reynolds number increases. Provided the growth rate of the instability *a posteriori* turns out to be faster than this diffusion, any velocity profile can legitimately be frozen for the purpose of stability analysis. In fact, the vertical diffusion of U scales as R^{-1}. Since the critical amplitude is found to scale as R^{-1} as well, it is reasonable to expect that larger perturbation amplitudes on the same order of magnitude will induce $O(R^{-1})$ growth rates, as confirmed by figure 4(*a*). So, if the modified base flow comes about and persists over a sufficiently long spatial extent, transition could be triggered by an exponential instability. The present analysis thus provides a lower bound on the norm of ΔU below which exponential amplification cannot occur.

4. Pseudospectra for hydrodynamic stability

Pseudospectra have proven extremely useful tools for numerical analysis (Trefethen *et al.* 1993; Wright & Trefethen 2001, and references therein); moreover they can be linked to physical processes in transitional flows (Reddy & Henningson 1993; Reddy *et al.* 1993). The ϵ-pseudospectrum of $\mathscr{L}_{OS} = \mathscr{L}_{OS}(U_{ref})$ is defined to be the union of spectra of all perturbed operators $\mathscr{L}_{OS} + \mathscr{P}$, with $\|\mathscr{P}\| \leqslant \epsilon$. Denoting spectra by Λ, one of several equivalent definitions for the ϵ-pseudospectrum Λ_ϵ is

$$\Lambda_\epsilon(\mathscr{L}_{OS}) = \{c \in \mathbb{C} \; : \; c \in \Lambda[\mathscr{L}_{OS}(U_{ref}) + \mathscr{P}] \text{ for some } \mathscr{P} \text{ with } \|\mathscr{P}\| \leqslant \epsilon\}. \quad (4.1)$$

The more non-normal the dynamical operator $\mathscr{L}_{OS}$, the greater the potential a disturbance operator $\mathscr{P}$ has for affecting its eigenvalues. Such a disturbance operator is meant to characterize very general perturbations to the system, whose physical origin is unspecified. A computationally efficient manner of determining the boundaries of the ϵ-pseudospectrum, defining the maximum effect of a disturbance operator of given norm, is to plot the contours of the resolvent operator (Wright & Trefethen 2001), cf. figure 3(*a*). One of Trefethen *et al.*'s (1993) key findings was that the minimal norm of a destabilizing perturbation scales as R^{-2} when general operator perturbations are allowed.

It is possible to introduce a class of ΔU-pseudospectra based on perturbations of the base flow velocity only:

$$\Lambda_{\Delta U}(\mathscr{L}_{\rm OS}) = \{c \in \mathbb{C} \ : \ c \in \Lambda[\mathscr{L}_{\rm OS}(U_{\rm ref} + \Delta U)] \text{ for some } \Delta U \text{ with } \|\Delta U\| \leqslant r\}. \quad (4.2)$$

This definition imposes restrictions on admissible disturbance operators $\mathscr{P}$, limiting these to variations in the mean flow and consequently increasing the bound on the minimal norm of destabilizing (operator) perturbations to a more conservative R^{-1}.

Restricting consideration to base flow velocity perturbations seems justified in hydrodynamic stability, where slight and practically unavoidable modifications in the base flow constitute the primary source of differences between theory and experiment. It is argued that the ΔU-pseudospectrum represents an alternative to the conventional ϵ-pseudospectrum which is based on a practically relevant norm.

5. Two-dimensional versus three-dimensional phenomena

The difference between infinitesimal disturbances occurring in a given base flow, and changes in that base flow (either of very small or of finite magnitude) cannot be overemphasized. The present work is concerned with the effect variations in the horizontal base flow velocity components U and W can have on the spectrum of linear stability eigenvalues. As pointed out in § 2, no restriction is imposed on the form of the eigenfunctions; they are merely considered from an apposite frame of reference rendering the analysis far less complex: the disturbances may be two- or three-dimensional in the original frame. Exponentially growing disturbances are found in base flows which experience $O(R^{-1})$ modifications to their canonical velocity profiles, measured in a velocity norm.

Previous studies on pseudospectra and algebraic growth have considered only generic perturbations to the full Orr–Sommerfeld/Squire system, measuring them with a matrix norm. In contrast, the ΔU-pseudospectrum does not affect the upper off-diagonal element of (2.3), with the following consequences:

(i) Since the Orr–Sommerfeld equation remains uncoupled from the Squire equation, Squire's transform may be used.

(ii) The asymptotic trend for destabilizing base flow modifications as $R \to \infty$ is proportional to $O(R^{-1})$, compared to generic disturbances to the full operator, found by Trefethen *et al.* (1993) to be proportional to $O(R^{-2})$. Note that this difference would not arise for a hypothetical system which only permitted two-dimensional modes, in which case either method would give the same estimate.

Algebraically growing disturbances may be important when the base flow deviates from its canonical form, whether or not this deviation is sufficient to drive an eigenmode into the unstable half-plane. Butler & Farrell (1992) showed that non-modal growth mechanisms favour amplification of three-dimensional disturbances, and it is essential to consider the full original system (2.3) when seeking those perturbations undergoing the most transient growth.

The purpose of this work is not to refute the importance of either non-modal growth or exponentially growing waves, but rather to study another aspect of the linear stability problem. It is quite likely that transition depends on more than one of these mechanisms acting in concert, in addition to the receptivity conditions of the configuration at hand.

6. Summary

The impact on classical linear stability of small departures from ideally prescribed velocity profiles is studied. In practice, these imperfections may stem from measurement errors or environmental factors, or they may simply be signature characteristics unique to the experimental facility. A key to understanding the relation between mode and mean flow is the derivation of sensitivity functions, which are appropriate combinations of the direct and adjoint Orr–Sommerfeld eigenfunctions. Sensitivities indicate the regions in which base flow modification has the most effect on linear stability. Conversely, these sites will most reward efforts aimed at ensuring high flow quality. For the plane Couette flow used as an example here, modes relatively far from the real line (which determines their decay or growth in time) are more sensitive to changes in the mean flow than those close to it.

It is possible to construct a constrained extremization problem to determine the mean flow modification of given magnitude having the most effect on an eigenmode. Using an energy-like quadratic norm to quantify changes in the base flow velocity profile alone, rather than allowing more general operator perturbations having different physical dimensions and possibly different physical origins, the choice here has been to concentrate on finding modifications leading to larger growth rates. For relatively minor deviations from the ideal linear velocity profile, Couette-like flows become unstable. Only the two-dimensional case need be treated since a simple projection onto the wavenumber vector allows the general problem to be simplified. The results maintain full generality and a three-dimensional disturbance can be recovered by projecting back onto a different system of horizontal coordinates. Some examples of three-dimensional base flow modifications have been reported by Bottaro *et al.* (2001) and Gavarini, Bottaro & Nieuwstadt (2002).

On the basis of the above results a transition scenario involving the exponential growth of small disturbances can be envisioned for the flow between imperfect parallel plates in relative motion. This does not rule out transient growth as a relevant mechanism at play in this flow (figure 3(a) shows that the linear stability operator of the perturbed Couette flow is still highly non-normal); conversely it does not rely exclusively on transient effects to trigger the sequence of events leading to the onset of shear flow turbulence (Grossmann 2000). Rather, it is suggested that both mechanisms (transient and exponential amplification) can occur concurrently during transition; some of the effects noted in experiments on transition in Couette flows, namely the oblique wave fronts appearing on the flanks of turbulent spots, appear to be of inflectional origin (Tillmark & Alfredsson 1992).

Finally, the results obtained here inspire the definition of an alternative pseudospectrum particularly suited for stability studies, in which perturbations to the Orr–Sommerfeld operator are restricted to the mean flow, giving it added physical relevance. A major difference between the ϵ- and ΔU-pseudospectra is that the former permits a two-way coupling between the Orr–Sommerfeld and Squire equations, whereas the latter excludes it. Despite this significant eigenvalue sensitivity to base flow variations is found.

The authors would like to acknowledge discussions with J. M. Floryan, M. I. Gavarini and F. T. M. Nieuwstadt whose comments helped to improve the manuscript.

REFERENCES

Barkley, D. & Tuckerman, L. S. 1999 Stability analysis of perturbed plane Couette flow. *Phys. Fluids* **11**, 1187–1195.

Boberg, L. & Brosa, U. 1988 Onset of turbulence in a pipe. *Z. Naturforsch.* **43a**, 697–726.

Bottaro, A., Corbett, P. & Luchini, P. 2001 Hydrodynamic stability: Eigenvalue sensitivity to base flow variations. ERCOFTAC SIG 33 *Workshop on Flow Control, April 25–27, 2001, Abisko, Sweden.*

Bottin, S., Dauchot, O., Daviaud, F. & Manneville, P. 1998 Experimental evidence of streamwise vortices as finite amplitude solutions in transitional plane Couette flow. *Phys. Fluids* **10**, 2597–2607.

Butler, K. M. & Farrell, B. F. 1992 Three-dimensional optimal perturbations in viscous shear flow. *Phys. Fluids* A **4**, 1637–1650.

Dauchot, O. & Daviaud, F. 1995 Finite amplitude perturbation and spots growth mechanism in plane Couette flow. *Phys. Fluids* **7**, 335–343.

Davey, A. 1973 On the stability of plane Couette flow to infinitesimal disturbances. *J. Fluid Mech.* **57**, 369–380.

Gallagher, A. P. & Mercer, A. M. 1962 On the behaviour of small disturbances in plane Couette flow. *J. Fluid Mech.* **13**, 91–100.

Gavarini, M. I., Bottaro, A. & Nieuwstadt, F. T. M. 2002 Eigenvalue sensitivity to base flow variations in Hagen–Poiseuille flow. In *Proc. ASME FEDSM'02, July 14–18, 2002, Montreal, Quebec, Canada*, Paper FEDSM2002–31050.

Gill, A. E. 1965 A mechanism for instability of plane Couette flow and of Poiseuille flow in a pipe. *J. Fluid Mech.* **21**, 503–511.

Grossmann, S. 2000 The onset of shear flow turbulence. *Rev. Mod. Phys.* **72**, 603–618.

Kato, T. 1995 *Perturbation Theory for Linear Operators*. Springer.

Kreiss, G., Lundbladh, A. & Henningson, D. S. 1994 Bounds for threshold amplitudes in subcritical shear flows. *J. Fluid Mech.* **270**, 175–198.

Lerner, J. & Knobloch, E. 1988 The long-wave instability of a defect in a uniform parallel shear. *J. Fluid Mech.* **189**, 117–134.

Reddy, S. C. & Henningson, D. S. 1993 Energy growth in viscous channel flows. *J. Fluid Mech.* **252**, 209–238.

Reddy, S. C., Schmid, P. J. & Henningson, D. S. 1993 Pseudospectra of the Orr–Sommerfeld operator. *SIAM J. Appl. Maths* **53**, 15–47.

Reed, M. & Simon, B. 1978 *Methods of Modern Mathematical Physics*. Academic.

Romanov, V. A. 1973 Stability of plane-parallel Couette flow. *Funct. Anal. Applics.* **7**, 137–146.

Schmid, P. J. & Henningson, D. S. 2001 *Stability and Transition in Shear Flows*. Springer.

Tillmark, N. 1995 Experiments on transition and turbulence in plane Couette flow. PhD thesis, Royal Institute of Technology, Dept of Mechanics, Stockholm, Sweden.

Tillmark, N. & Alfredsson, P. H. 1992 Experiments on transition in plane Couette flow. *J. Fluid Mech.* **235**, 89–102.

Trefethen, L. N., Trefethen, A. E., Reddy, S. C. & Driscoll, T. A. 1993 Hydrodynamic stability without eigenvalues. *Science* **261**, 578–584.

Weideman, J. A. C. & Reddy, S. C. 2000 A Matlab differentiation matrix suite. *ACM Trans. Math. Software* **26**, 465–519; Matlab functions available at http://www.onid.orst.edu/~weidemaj/differ.html.

Wright, T. G. & Trefethen, L. N. 2001 Large-scale computation of pseudospectra using Arpack and eigs. *SIAM J. Sci. Comput.* **23**, 591–605; Matlab functions available at http://web.comlab.ox.ac.uk/projects/pseudospectra/psagui.