MINIMAL DEFECTS

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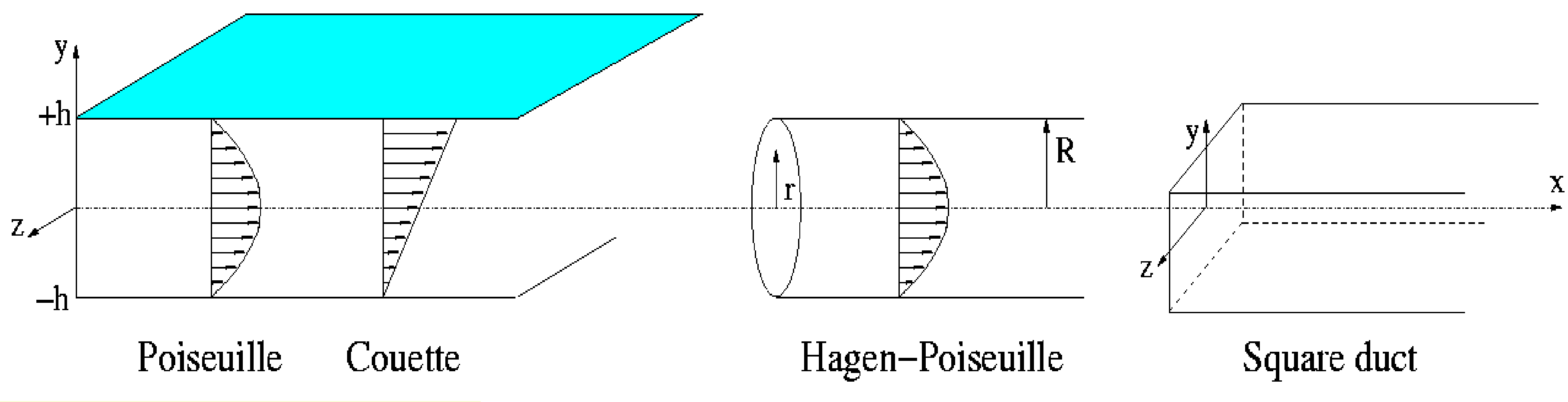
work conducted together with:

I. Gavarini and F.T.M. Nieuwstadt

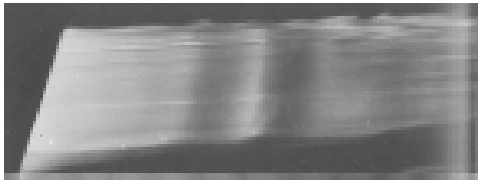


1. ***TODAY, TRANSITION IN SHEAR FLOWS IS STILL NOT FULLY UNDERSTOOD***. For the simplest parallel flows there is poor agreement between predictions from the classical linear stability theory (Re_{crit}) and experimentals results (Re_{trans})

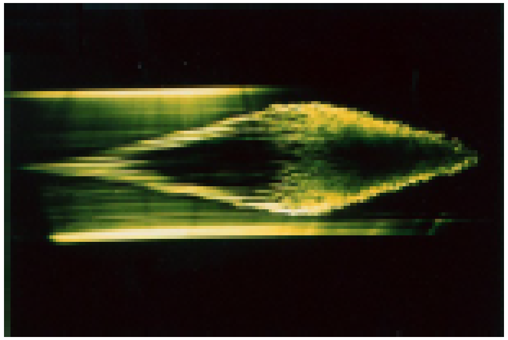
	Poiseuille	Couette	Hagen-Poiseuille	Square duct
Re_{crit}	5772	∞	∞	∞
Re_{trans}	$\sim$ 1000	$\sim$400	$\sim$2000	$\sim$2000

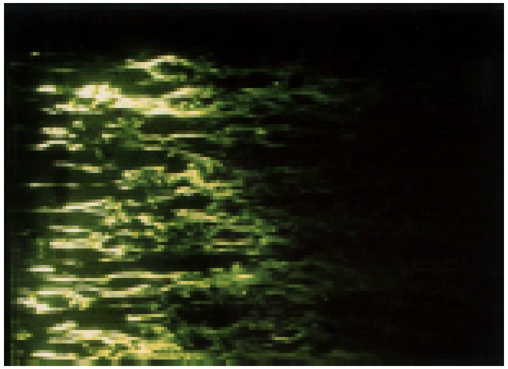
2. TODAY, TRANSITION IN SHEAR FLOWS IS STILL NOT FULLY UNDERSTOOD.

- Classical theory predicts *Tollmien-Schlichting waves* in Poiseuille and boundary layer flows:



- Except in very noise-free and controlled experiments, flow structures in transition are more like turbulent spots and streaky boundary layers:





HYDRODYNAMIC STABILITY THEORY

- Given the disagreement (*critical conditions and type of transition*) between theory and experiments is small perturbation theory at all relevant?
- Yes, it still is!
- … despite the fact that classical linear stability theory does not explicitly contain effects of free-stream turbulence, uncertain body forces, wall roughness (geometrical uncertainties), poorly modeled base flow conditions, etc.

- Issues:
  - **Initial conditions $\rightarrow$ transient growth**
  - **Dynamical uncertainties and poorly modeled terms $\rightarrow$ structured operator perturbations** $[L(U, \alpha; \omega, \beta, Re) + \Delta]v = 0$

THE TRANSITION PROCESS

- Receptivity phase: the flow filters environmental disturbances
- Initial phase
  - **ROUTE 1: TRANSIENT GROWTH**
  - **ROUTE 2: EXPONENTIAL GROWTH**

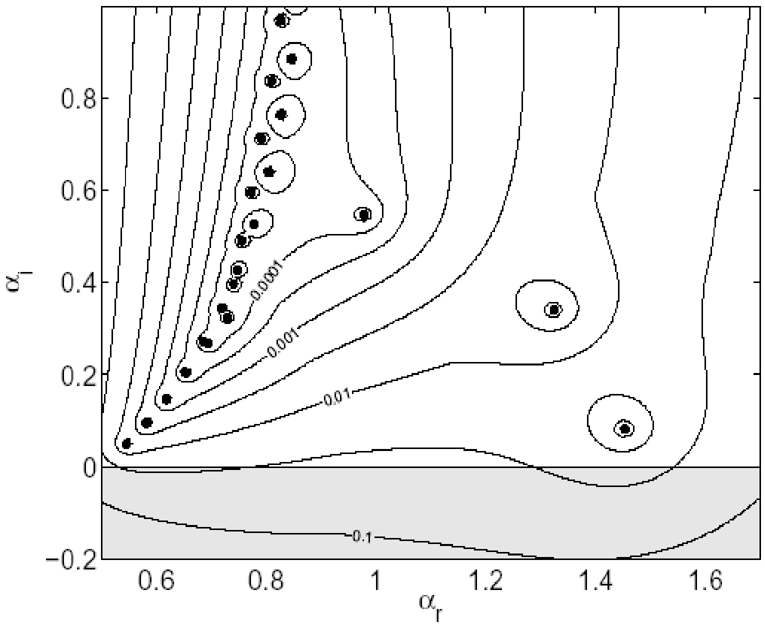
    in nominally subcritical conditions

    (related to the presence of defects in the base flow)
- Late, nonlinear stages of transition

ROUTE 2: EXPONENTIAL GROWTH

Preliminary observation: eigenvalues of the OS/Squire system are very sensitive to operator perturbations E

$$\Lambda_\varepsilon(L) = \{\alpha \in C: \ \alpha \in \Lambda(L+E), \ \text{with } E \text{ such that } \|E\| \leq \varepsilon\}$$



Consider a very particular operator perturbation, a distortion of the mean flow $U(y)$ (induced by whatever environmental forcing) $\rightarrow$

$$\Lambda_{\delta U}(L) = \left\{ \alpha \in C: \ \alpha \in \Lambda[L(U_{ref} + \delta U)], \ \text{with} \ \|\delta U\| \leq \varepsilon \right\}$$

The δU-pseudospectrum is different from the classical ε-pseudospectrum, since it considers structured dynamical uncertainties, which depend only on base flow distortions from the ideal state (Biau & Bottaro, *PoF* in press)

SENSITIVITY ANALYSIS

OS equation: $L\,(U, \alpha; \omega, \beta, Re)\, v = 0$

With a base flow variation $\delta U(y)$:

$$\delta L\, v + L\, \delta v = 0$$

$$\delta U \frac{\partial L}{\partial U} v + \delta\alpha \frac{\partial L}{\partial \alpha} v + L\, \delta v = 0$$

Projecting on a, eigenfunction of the adjoint system ($L^*a=0$) we find

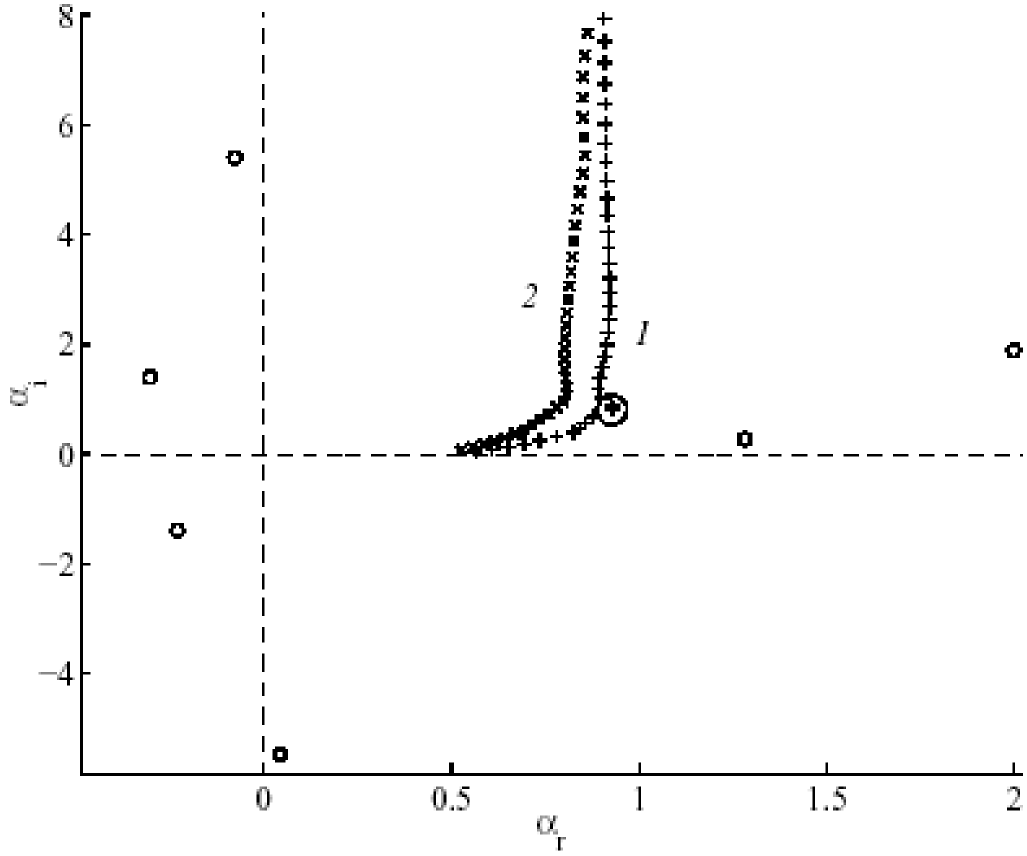
$$a \cdot \delta U \frac{\partial L}{\partial U} v + \delta\alpha \, a \cdot \frac{\partial L}{\partial \alpha} v + a \cdot L \, \delta v = 0$$

and hence,

$$\delta\alpha = - \frac{a \cdot \delta U \frac{\partial L}{\partial U} v}{a \cdot \frac{\partial L}{\partial \alpha} v} = \ldots = \int_{-1}^{1} G_U \, \delta U \, dy$$

In practice, for each eigenvalue α_n we can tie the base flow variation δU to the ensuing variation $\delta\alpha$ via a <u>sensitivity function</u> G_U

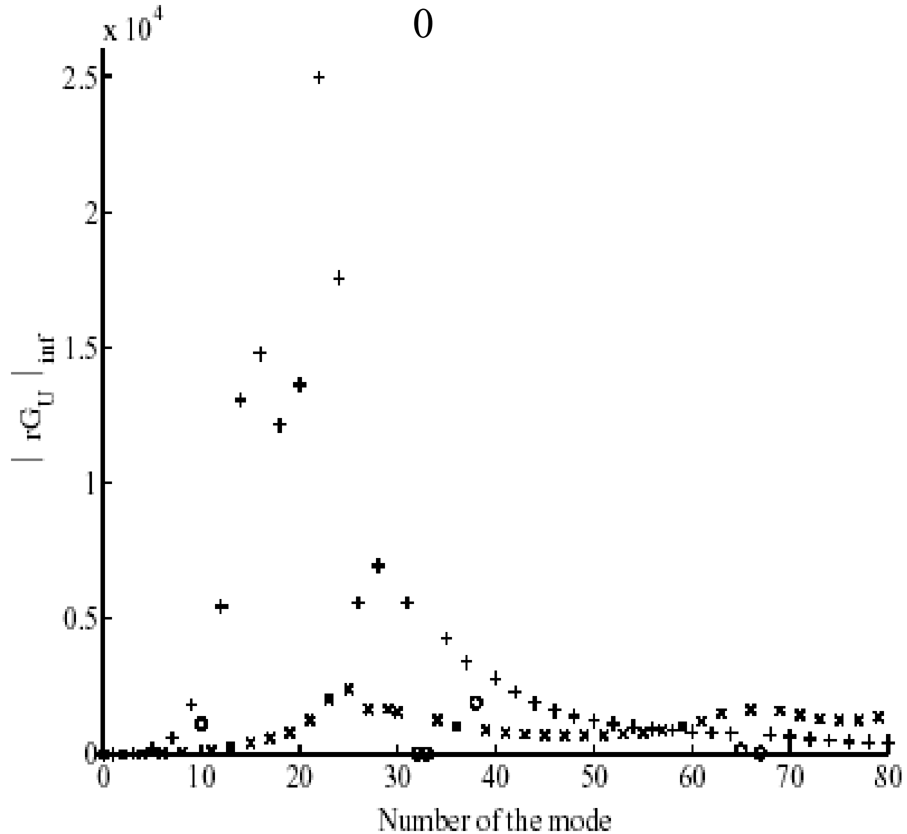
HAGEN-POISEUILLE FLOW



Spectrum of eigenvalues at $Re = 3000$, $m = 1$, $\omega = 0.5$.
The circle includes the two most receptive eigenvalues

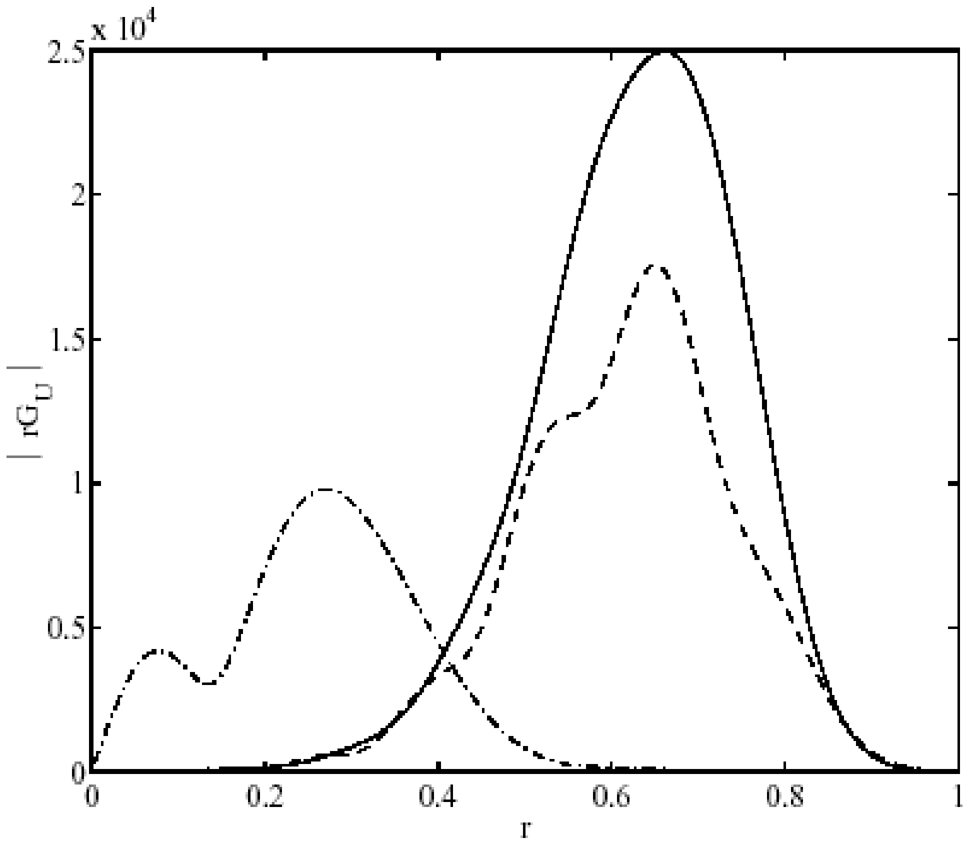
ROUTE 2: EXPONENTIAL GROWTH

$$\delta\alpha = \int_0^1 rG_U \delta U \; dr$$



Corresponding ∞ norm of rG_u. Modes arranged in order of increasing $|\alpha_i|$

SENSITIVITY FUNCTIONS



Mode 22 (solid); mode 24 (dashed); 10^3*mode 1 (dash-dotted)

"Optimization"

look for optimal base flow distortion (**minimal defect**) of given norm ε, so that the growth rate of the instability ($-\alpha_i$) is maximized:

Find $\min(\alpha_i)$ with $U - U_{ref}$ of norm ε

$$\mathrm{Min}(\alpha_i) = \mathrm{Min}\left\{\alpha_i + \gamma\left[\int_{-1}^{1}(U - U_{ref})^2\,dy - \varepsilon\right]\right\}$$

Necessary condition is that:

$$\delta\alpha_i + \gamma\left[\int_{-1}^{1} 2(U - U_{ref})\ \delta U\ dy\right] = 0$$

Employing the previous result:

$$\int_{-1}^{1} \left[\mathrm{Im}(G_U) + 2\gamma(U - U_{ref})\right] \delta U \; dy = 0$$

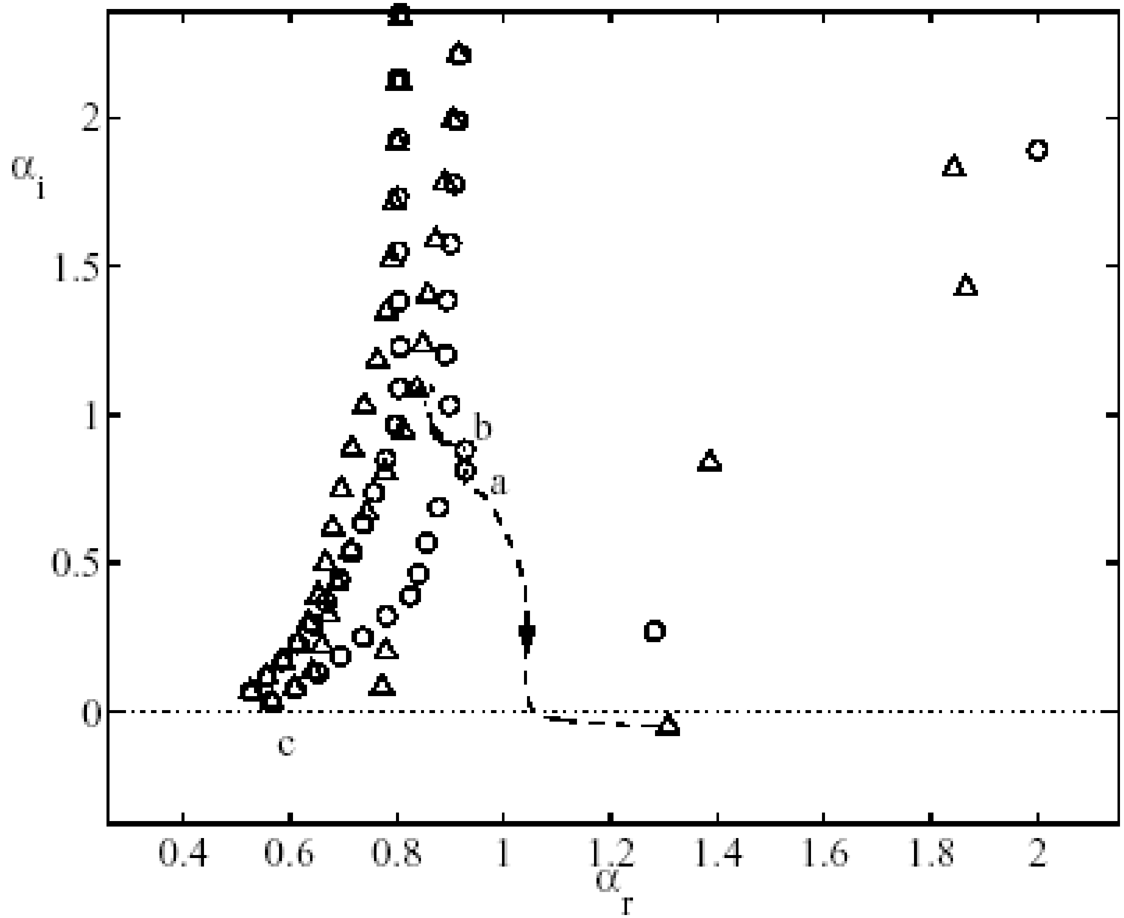
A simple gradient algorithm can be used to find the new base flow that maximizes the growth rate, for any α_n and for any given base flow distortion norm ε:

$$U^{(i+1)} = U^{(i)} + \vartheta \left[\mathrm{Im}(G_U{}^{(i)}) + 2\gamma^{(i)}(U^{(i)} - U_{ref})\right]$$

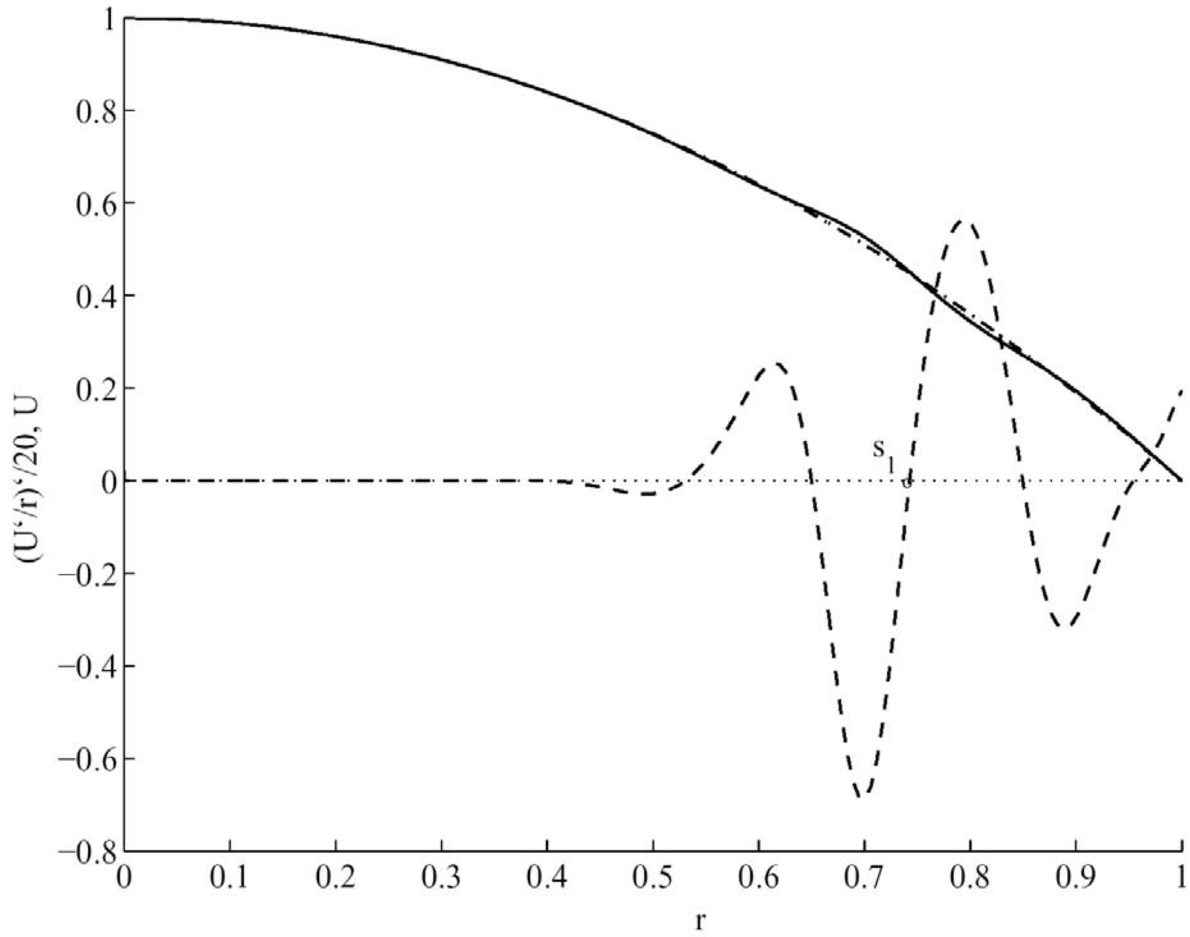
with

$$\gamma^{(i)} = \mu \left\{ \frac{\int_{-1}^{1} \left[\mathrm{Im}(G_U{}^{(i)})^2\right]}{4\varepsilon} \right\}^{\frac{1}{2}}$$

ROUTE 2: EXPONENTIAL GROWTH

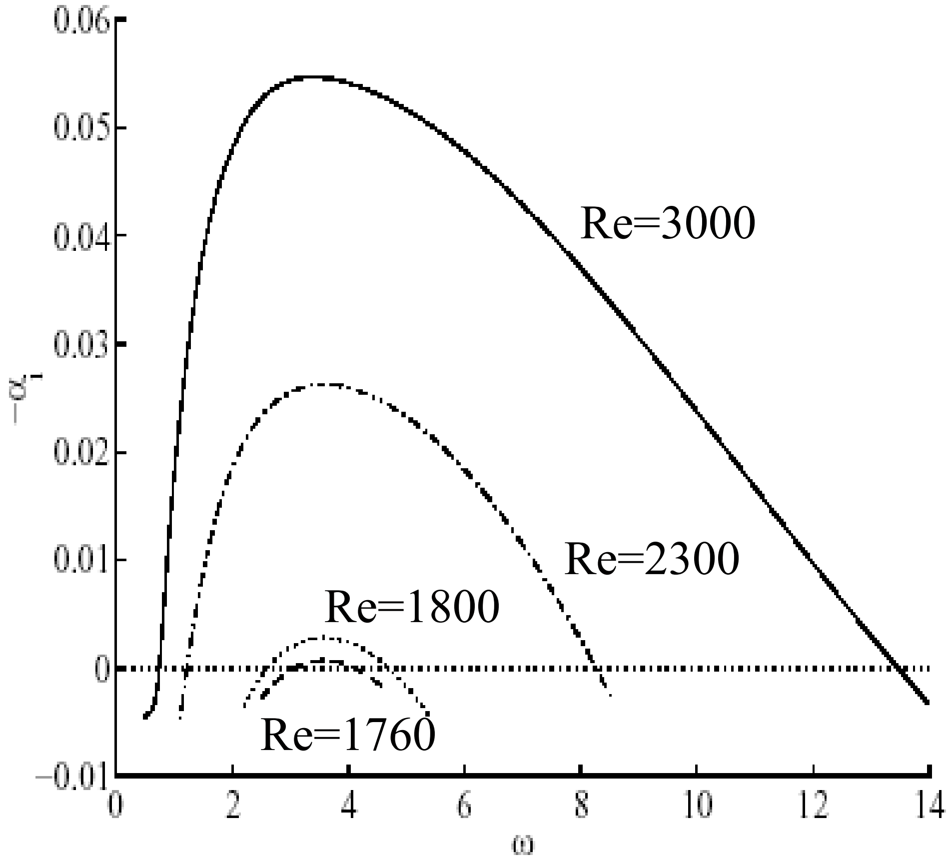


Re = 3000, m = 1, ω = 0.5. HP flow (circles), OD flow (triangles) with $\varepsilon = 2.5*10^{-5}$ which minimizes α_i of mode 22.



Optimally distorted base flow *vs* Hagen-Poiseuille flow. The curve of (U'/r)'/20 indicates an inflectional instability

ROUTE 2: EXPONENTIAL GROWTH



Growth rate as function of ω for $m = 1$ and $\varepsilon = 10^{-5}$

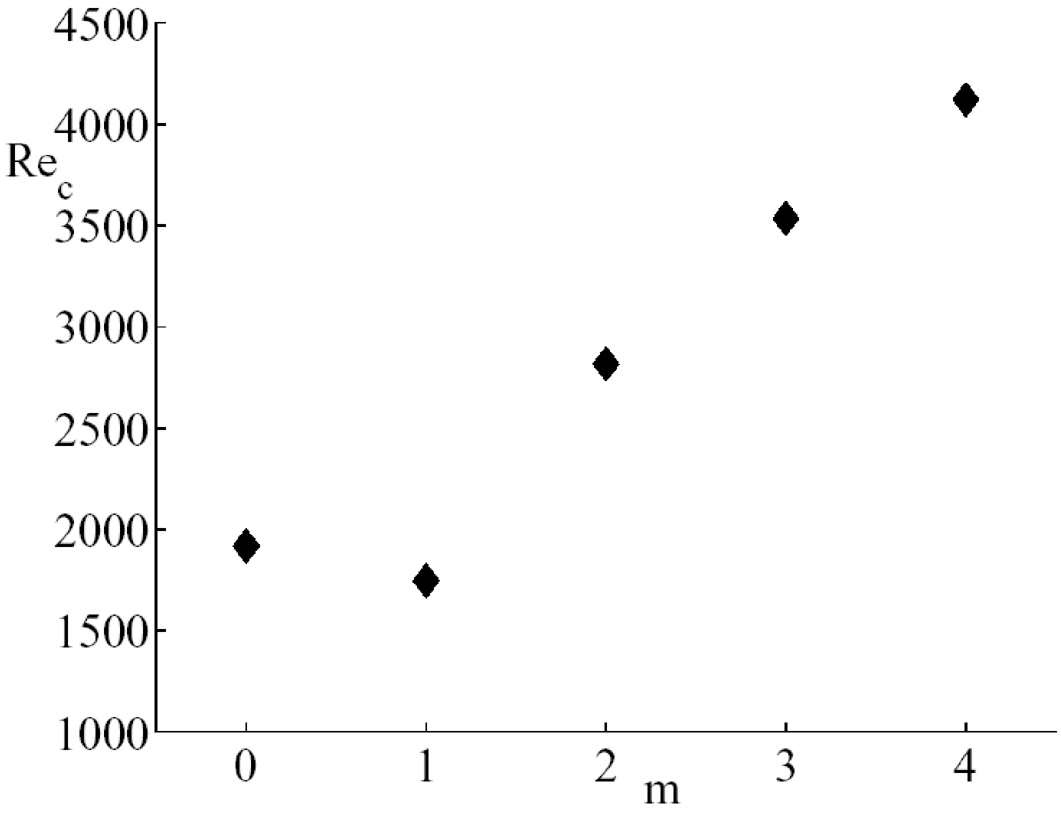
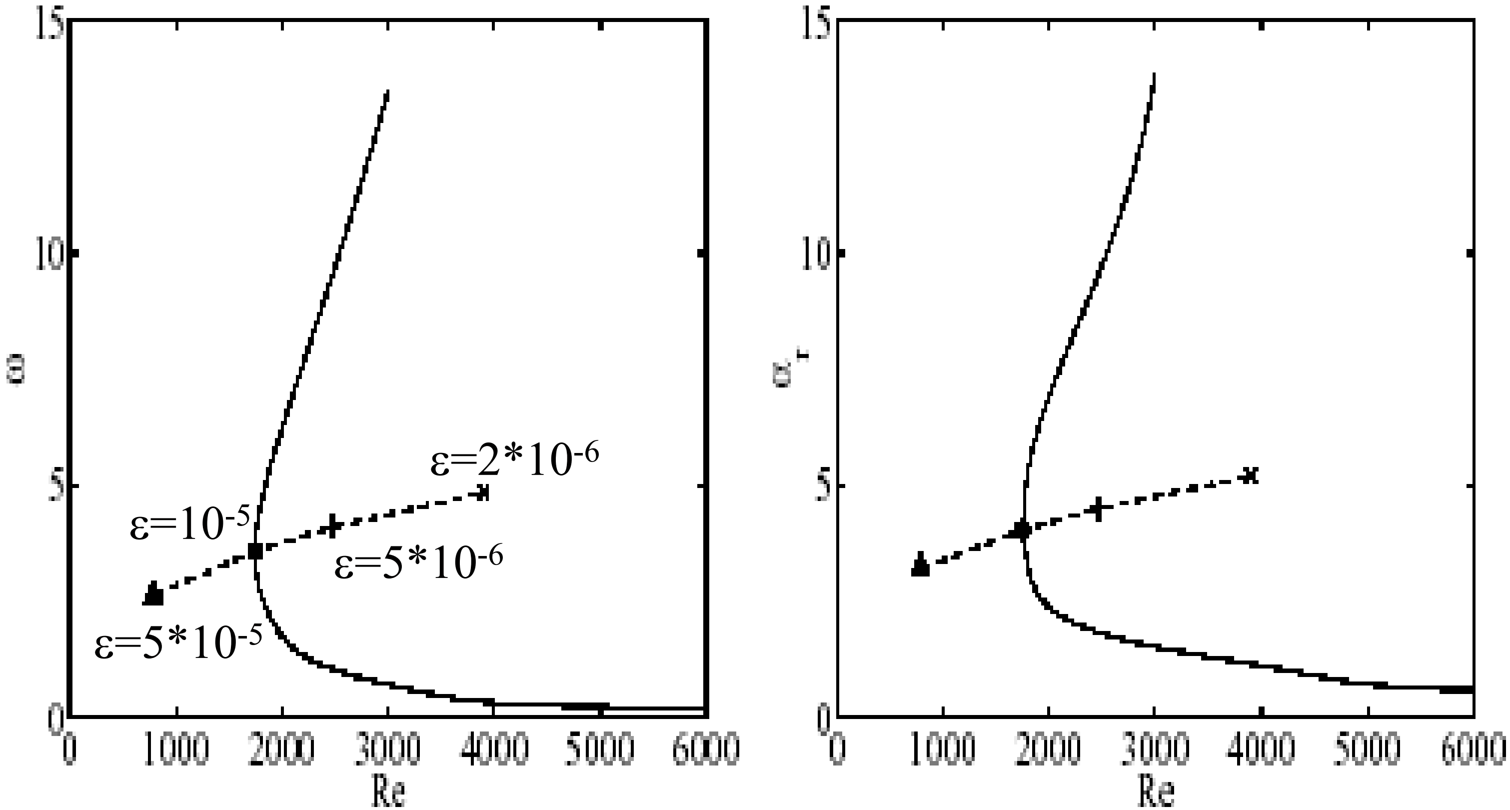


FIGURE 18. Critical Reynolds number for optimally perturbed base flows with $\epsilon = 10^{-5}$, as a function of the azimuthal wavenumber m.

ROUTE 2: EXPONENTIAL GROWTH



Neutral curves for $m = 1$ and $\varepsilon = 10^{-5}$. Symbols give Re_{crit}

ROUTE 2: EXPONENTIAL GROWTH

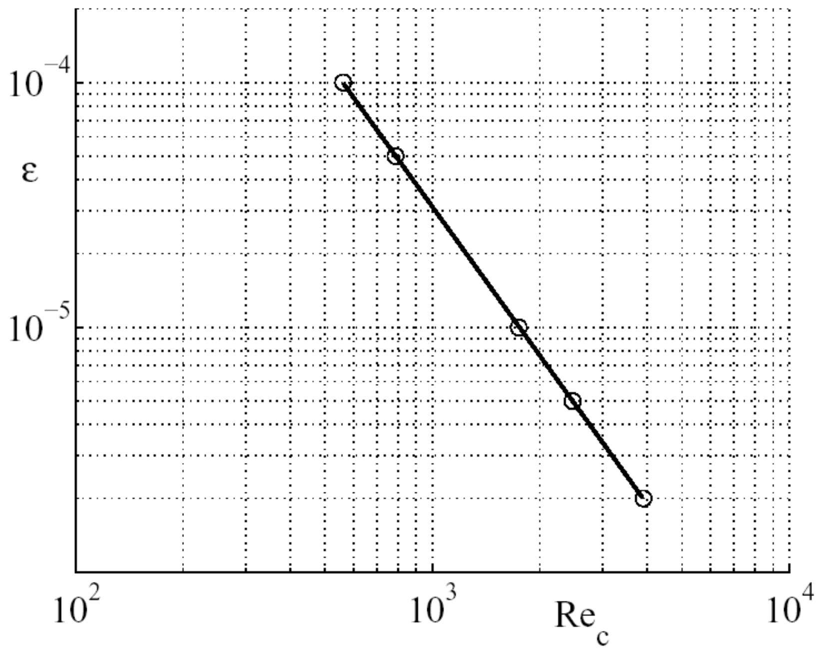


FIGURE 16. Norm of the base-flow deviation as a function of the critical Reynolds number for $m = 1$.

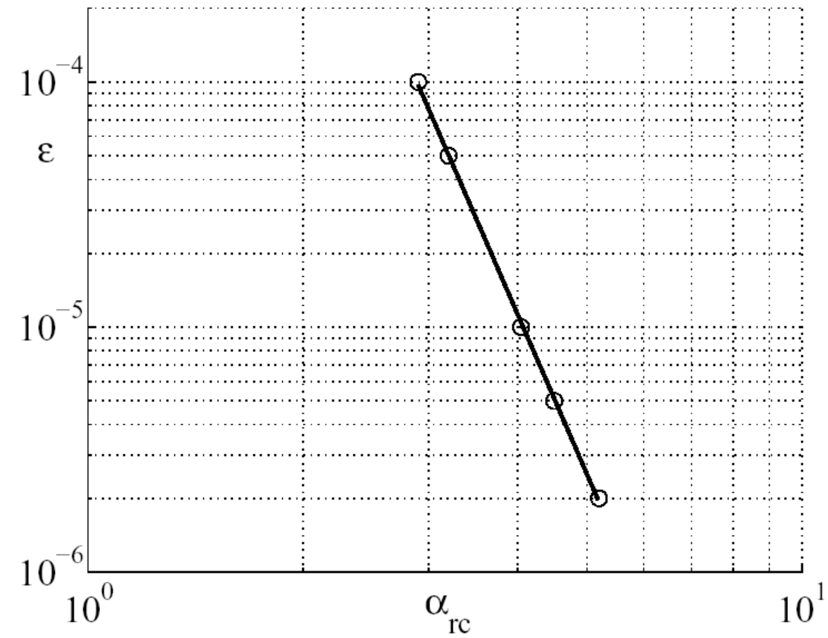


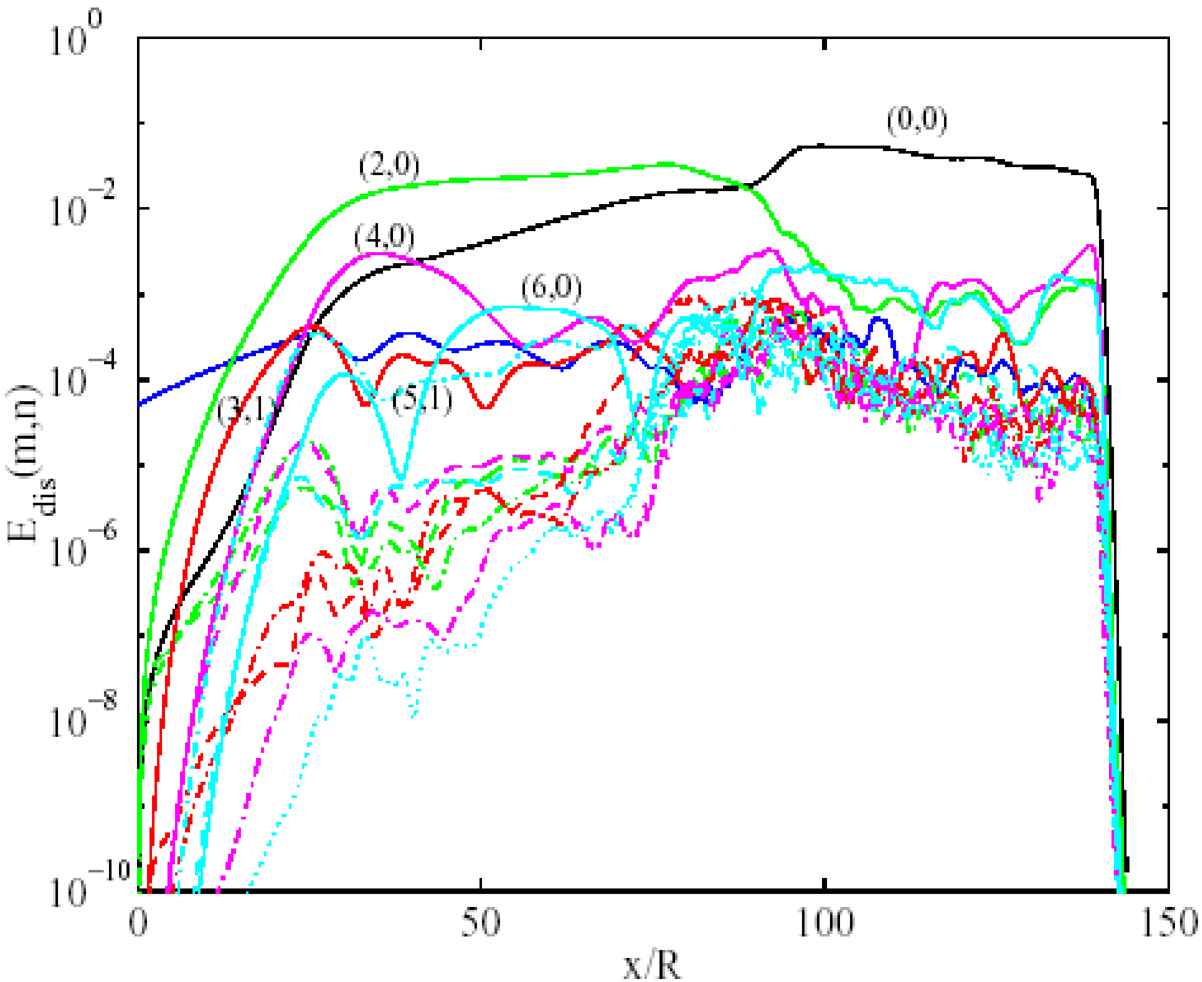
FIGURE 17. Norm of the base-flow deviation as a function of the critical streamwise wavenumber for $m = 1$.

ε scales as Re^{-2} $\rightarrow$ the critical amplitude scales as Re^{-1}

FULL NONLINEAR SIMULATIONS

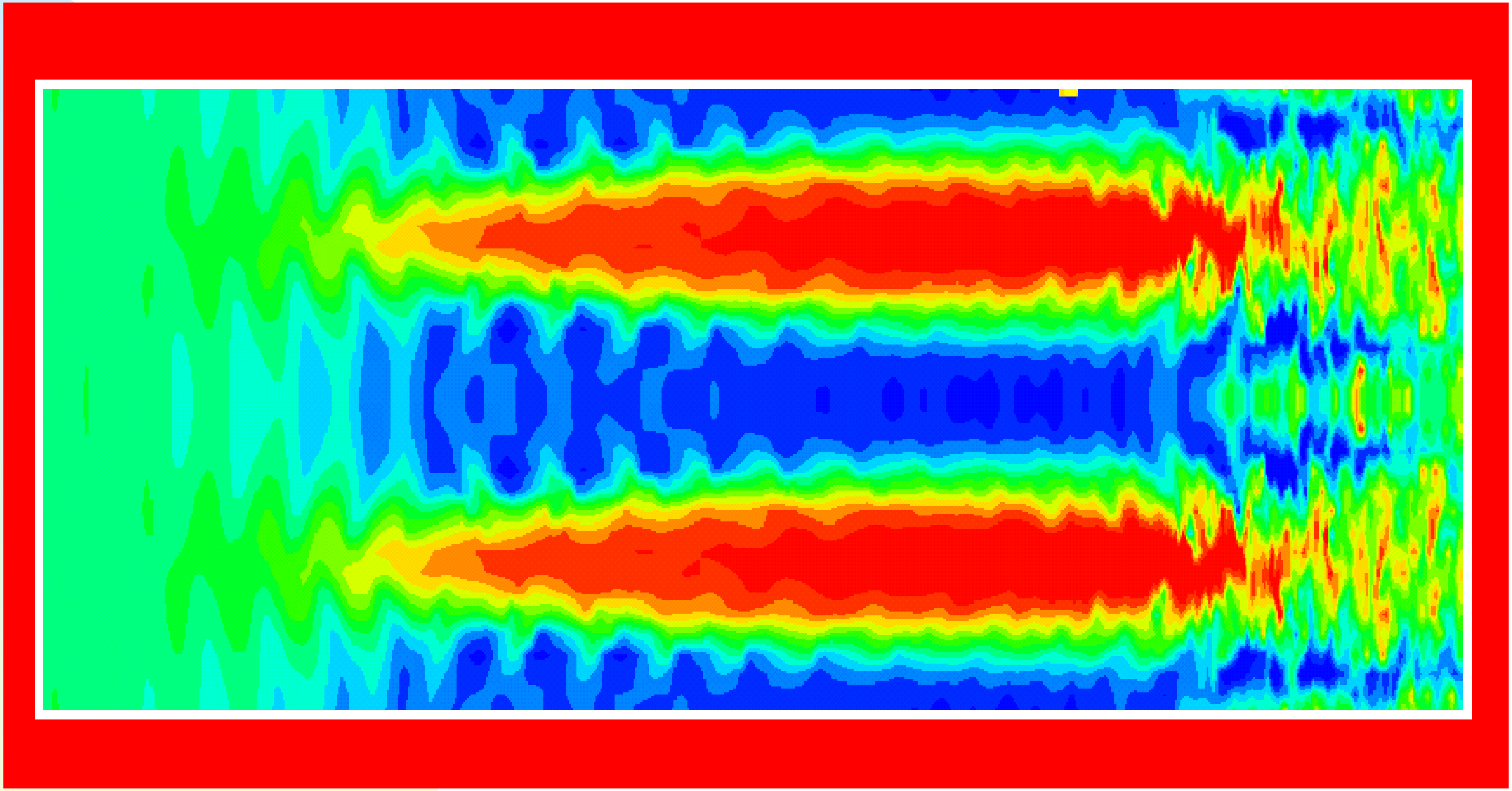
1. *Oblique* transition
2. TS-like transition

$$E_{dis}^{(m,n)}(x) = \sum_{j=\pm m} \sum_{k=\pm n} \frac{1}{2T} \int_{\tau}^{\tau+T} \int_{0}^{2\pi} \int_{0}^{1} \left| \hat{\mathbf{u}}\, e^{i(j\vartheta - k\omega t)} \right|^2 r\,dr\,d\vartheta\,dt$$



Spatial evolution of the disturbance energy for the Fourier modes (m, n), with $\omega = 0.5$. Initial amplitude of the $(\pm1, \pm1)$ mode (shown with thick blue line) is 0.002. $Re = 3000$, $\varepsilon = 2.5*10^{-5}$.

Instantaneous streamwise disturbance velocity at r = 0.7



Red: high velocity streaks

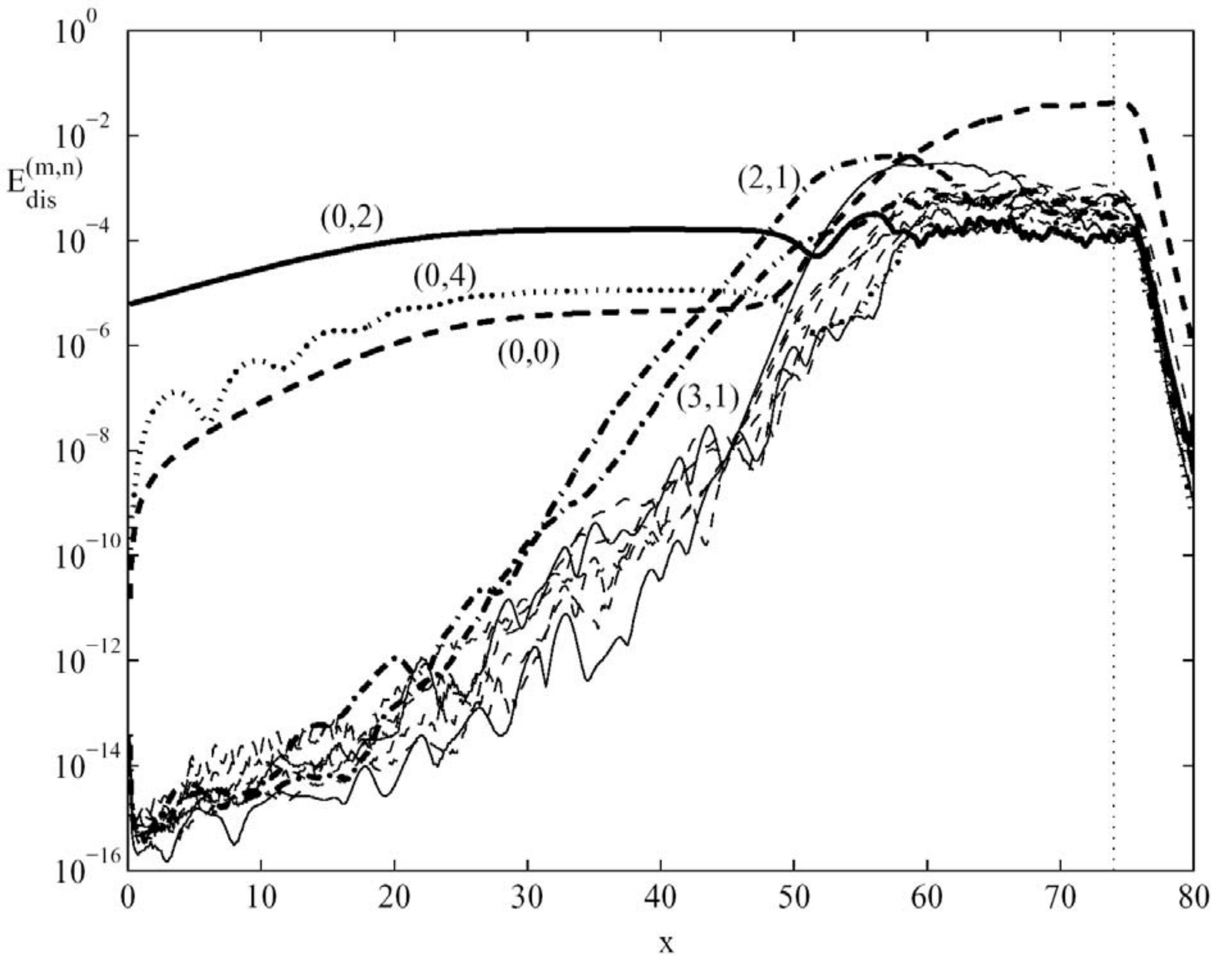
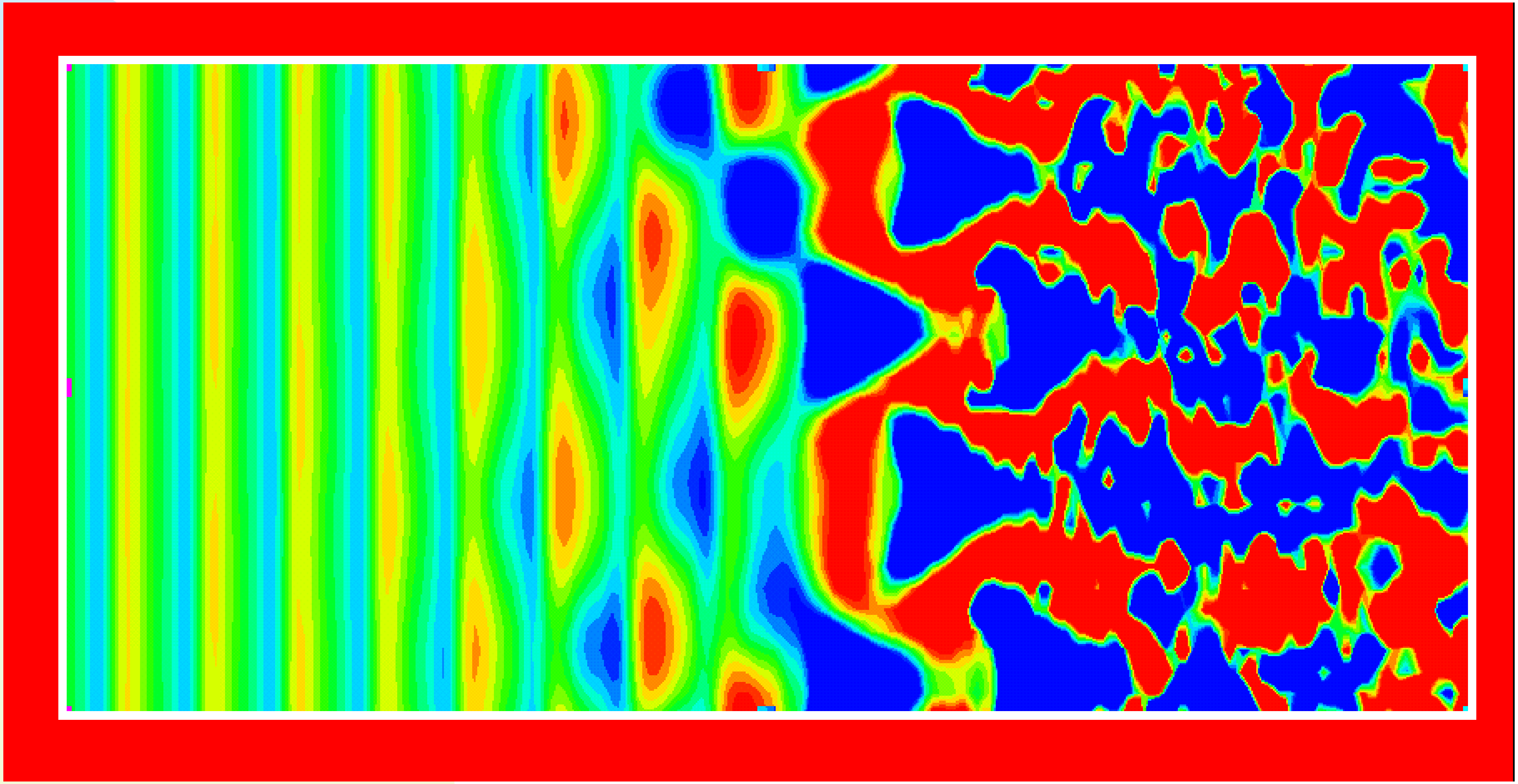


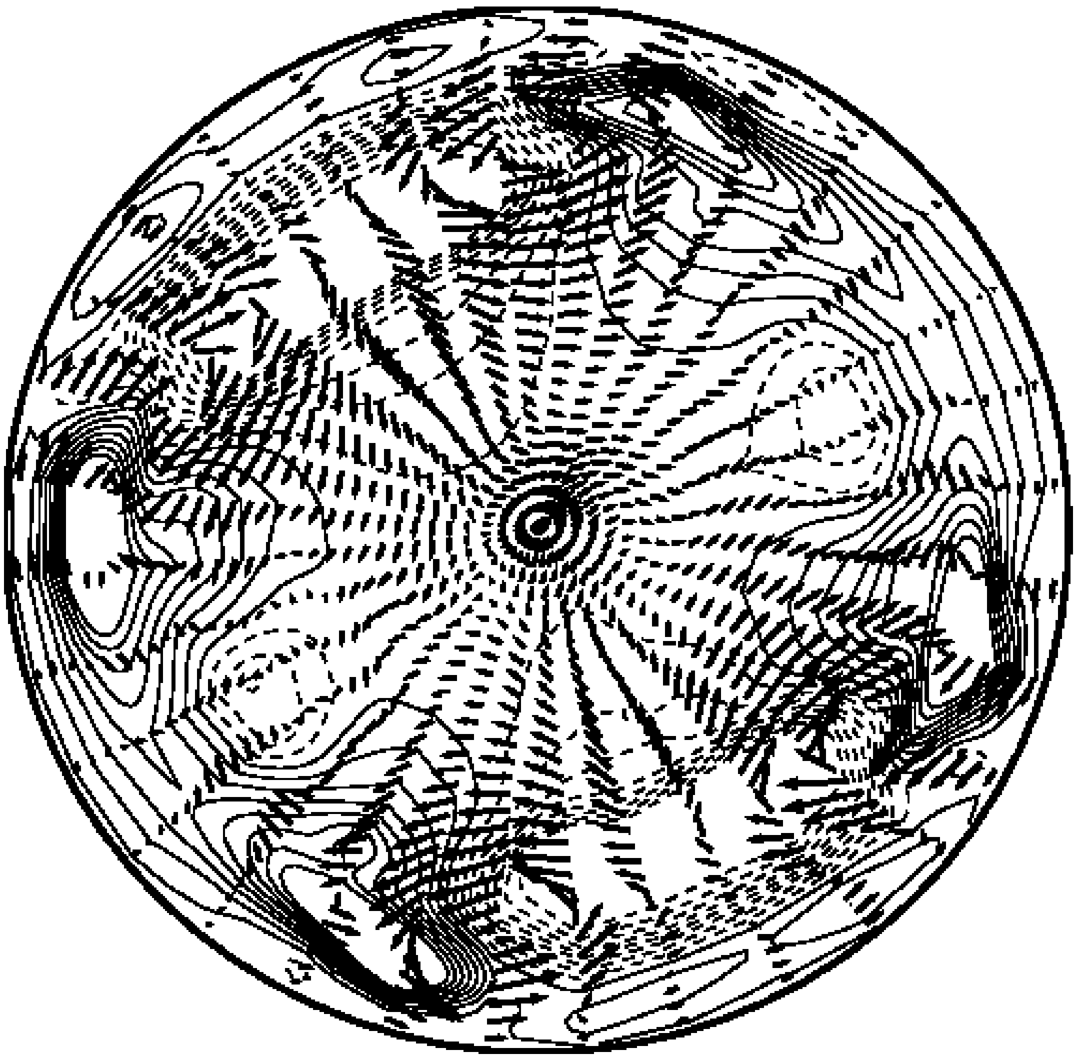
FIGURE 24. Spatial evolution of the disturbance kinetic energy for various Fourier modes (m, n), with $\omega = 0.5$ fundamental frequency. The inflow condition consists of a combination of the unstable eigenmodes at $\mathrm{Re} = 3000$, $m = 0$, $n = \pm 2$, $\epsilon = 2.5 \times 10^{-5}$, each one with $A_v = 0.001$, plus small-amplitude random perturbations. The dotted vertical line indicates the start of the fringe region.

Instantaneous streamwise disturbance velocity at r = 0.7

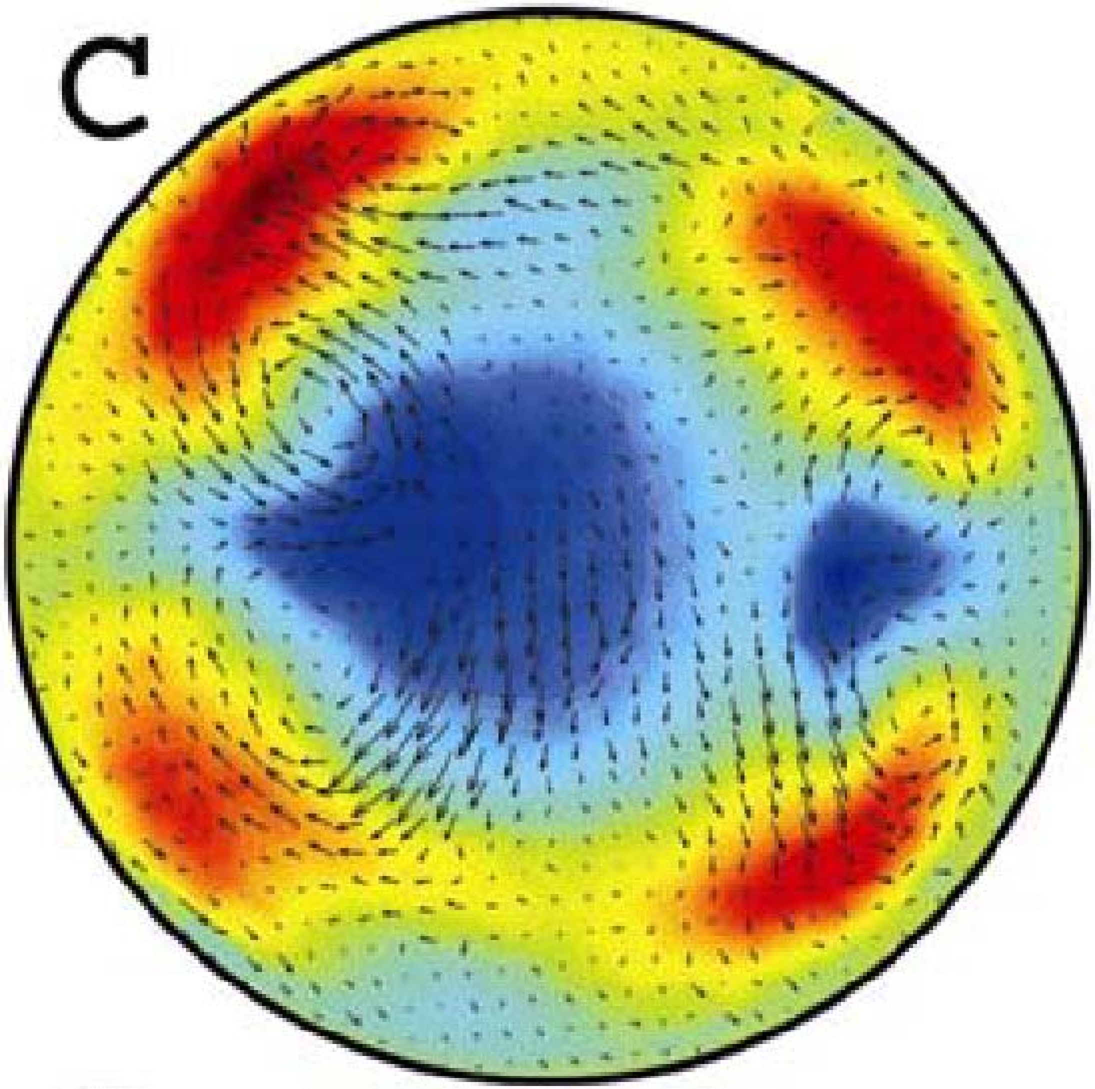


x = 56

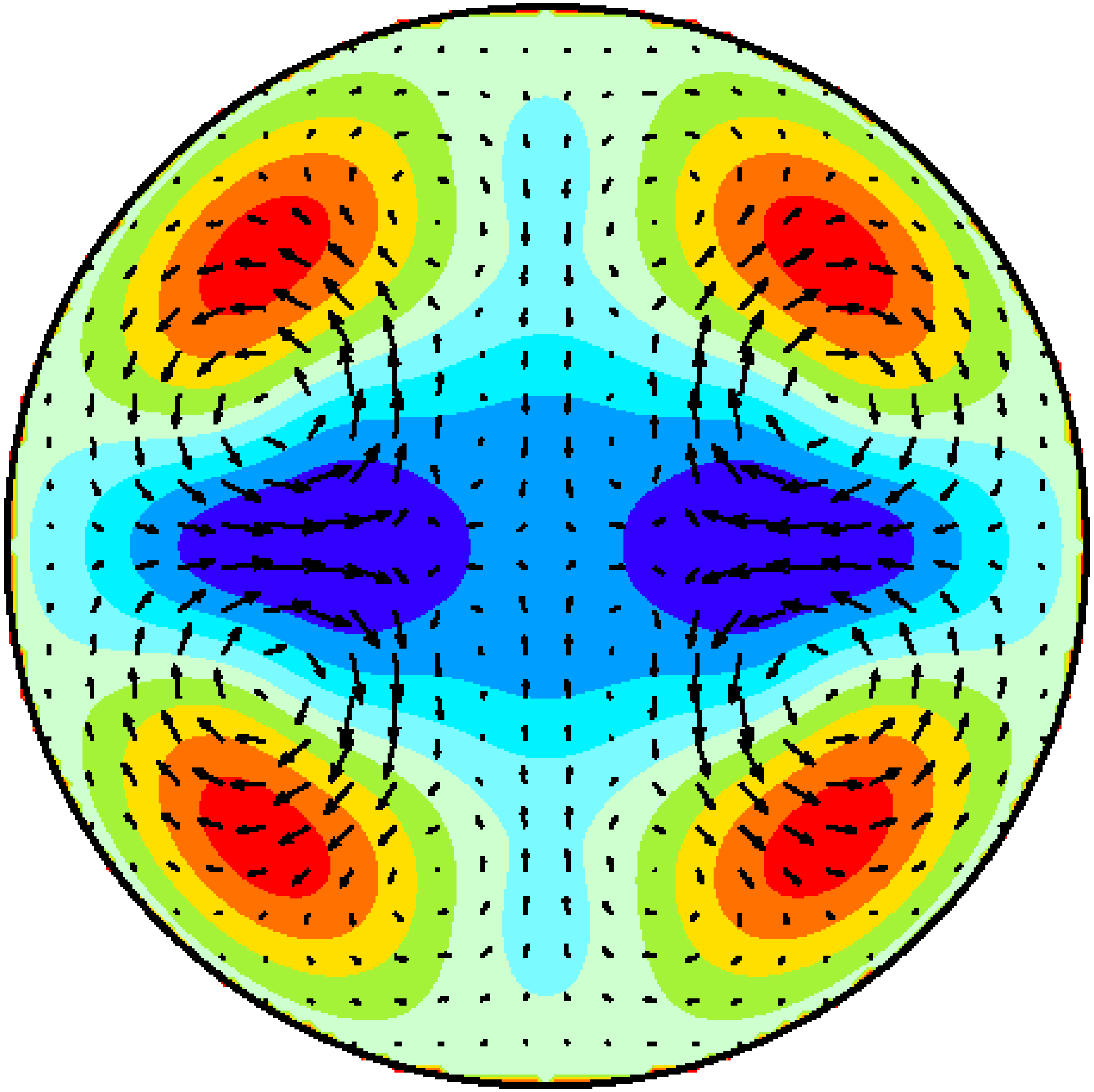
Instantaneous velocity field, x = 56



C_2 symmetric state in a turbulent puff at Re=2500
B. Hof *et al.*, this meeting, 8 Aug. 2004



H. Faisst & B. Eckhardt, *PRL* 2003
streamwise-averaged C_2 state



Oblique transition:
non-linearities are destabilizing
subcritical bifurcation

TS-like transition:
non-linearities saturate
supercritical bifurcation

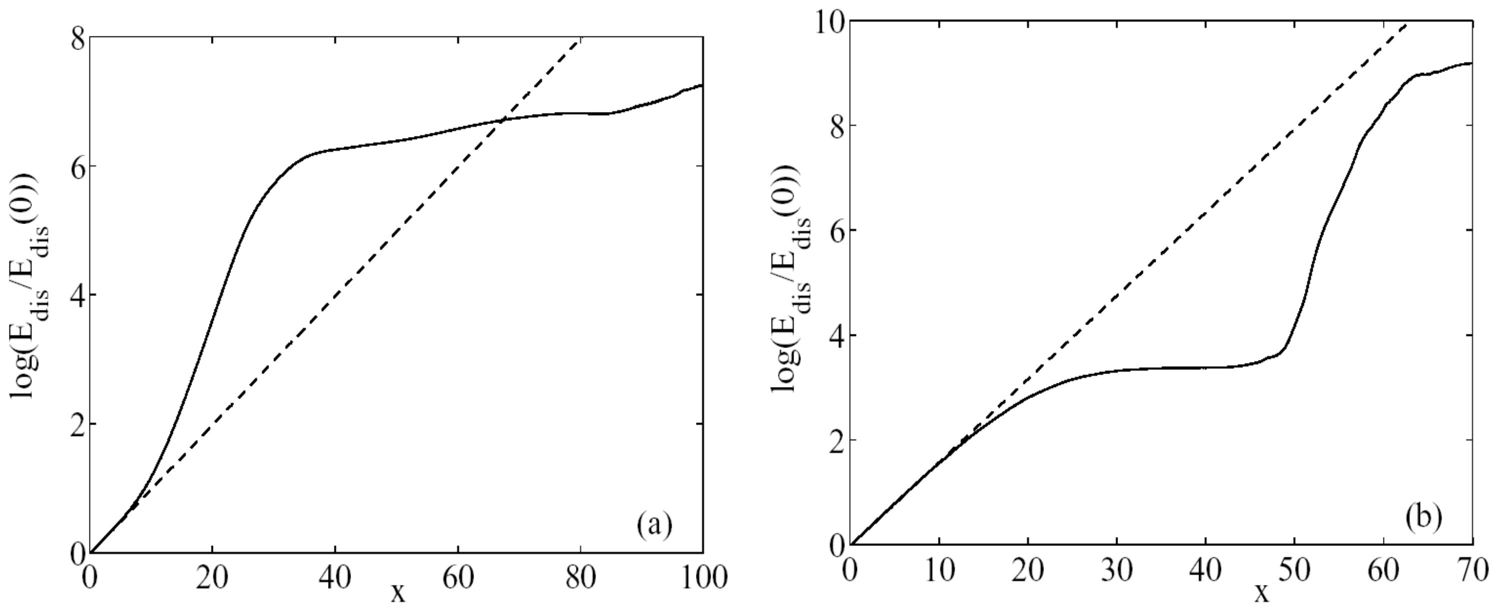


FIGURE 28. Spatial evolution of the total disturbance energy, for the case in which the optimal base-flow distortion is maintained over the whole length of the pipe. Left: the inflow condition is $(\pm 1, \pm 1)$. Right: the inflow condition is $(0, \pm 2)$. The dashed lines indicate the exponential growth predicted by linear theory for the two cases.

In both cases transition can take place also when the minimal defect is imposed only at the pipe entrance, provided its amplitude is sufficiently large for the instability to grow faster than the viscous diffusion of the defect (in the TS-like case we also need a sufficiently noisy background for the (2,1) mode to survive long enough).

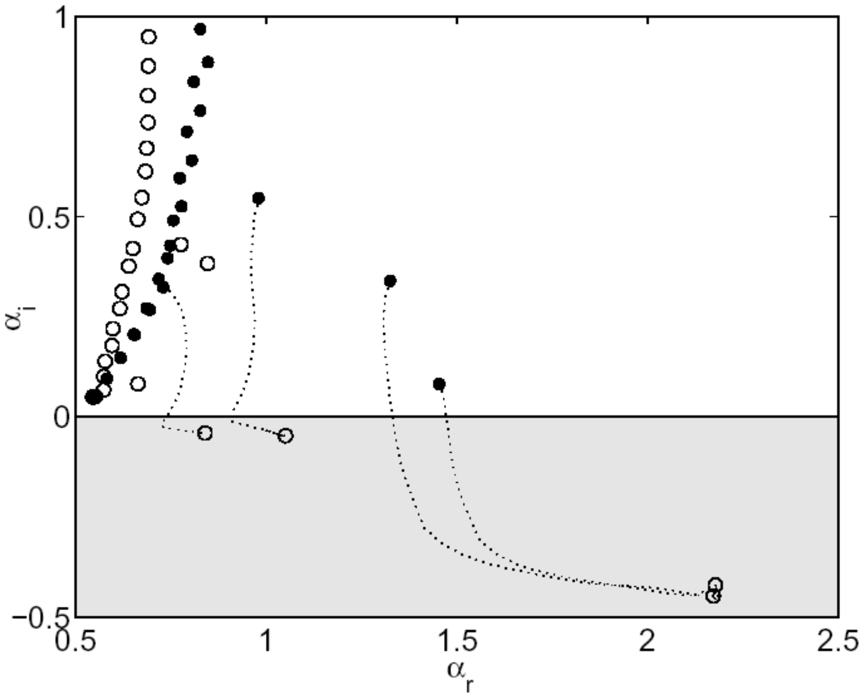
More details in Gavarini *et al.*, *JFM* in press

CONCLUSIONS

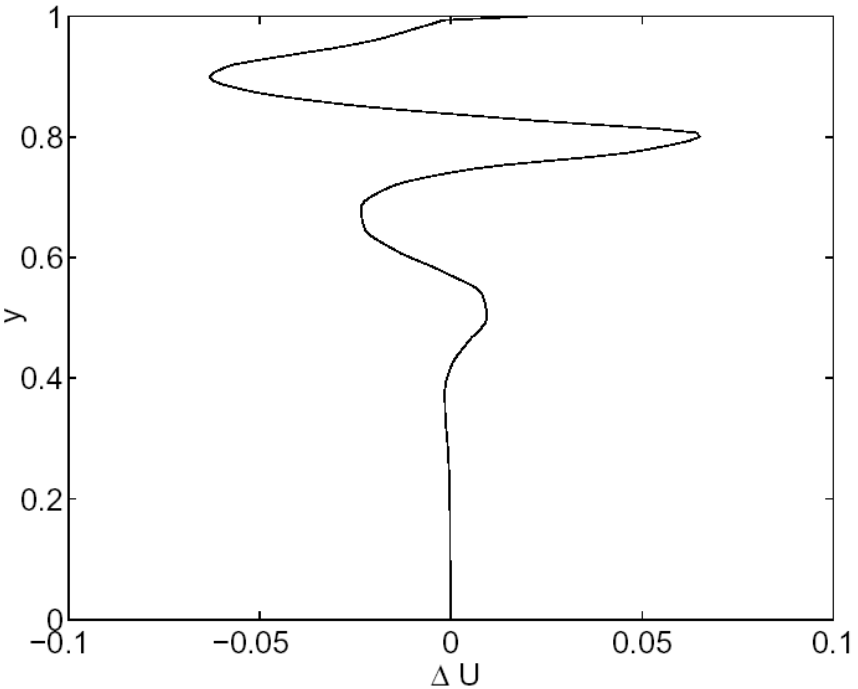
- OS eigenmodes are very sensitive to base flow variations (δU-pseudospectrum, the growth is less than for the ε-pseudospectrum since two-way - possibly *unphysical -* coupling between OS and Squire equations is not allowed)

CONCLUSIONS

- Exponential growth can take place in nominally subcritical conditions for minimal defects of very small norm



Poiseuille flow, Re = 3000, ω = 0.5



Minimal defect

CONCLUSIONS

- The initial stage of transition is likely to be influenced by the combined effect of algebraic and exponentially growing disturbances

CONCLUSIONS

- Two paths of transition have been identified:
  - *Oblique* transition: helical waves produce streaks which break down; nonlinearities destabilize the linearly growing state
  - TS-like transition, initiated by an axisymmetric, TS-like wave. Nonlinearities saturate the linearly growing wave, transition occurs via a secondary subharmonic instability (staggered patterns of Λ-vortices, and intermediate, short-lived, TW of the kind studied theoretically by Faisst & Eckhardt, *PRL* 2003 and Wedin & Kerswell, *JFM* 2004)