UNIVERSITÀ DEGLI STUDI DI GENOVA

INSTITUT DE MECANIQUE DES FLUIDES DE TOULOUSE

# Numerical simulation of a droplet icing on a cold surface

Buccini Saverio
Peccianti Francesco

**Supervisors:**
Prof. Alessandro Bottaro
Prof. Dominique Legendre

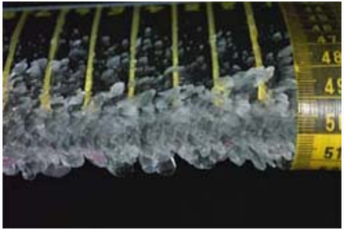
March 30, 2017

Università degli Studi di Genova

# Outline

# Introduction

# The icing problem - Effects

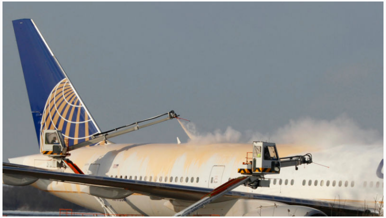


**Effects**

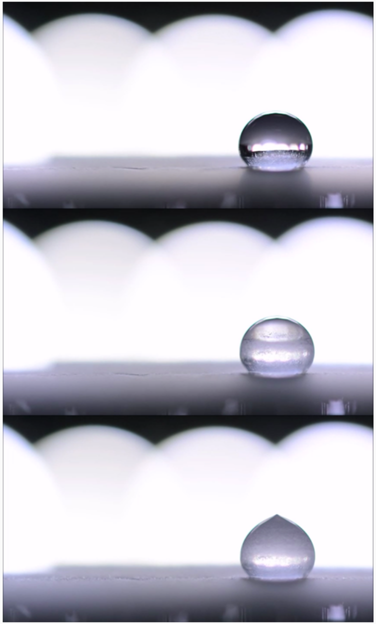
- Lift variation $\rightarrow$ stall
- Increased drag
- Instrumentation problems $\rightarrow$ AF447
- Increased weight

**Countermeasures**

- De-icing & anti-icing fluids
- De-icing boots
- Electrical resistances
- Hot air bleeding from engines



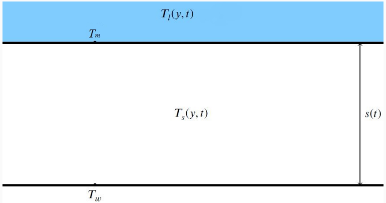
# Physical model and experiments



**Aim of the work**

- Freezing front evolution
- Final shape of the droplet and its causes
  1. Density variation
  2. Marangoni effect

# Stefan problem (1)




- solid phase: $0 \leq x < s(t)$

$$\frac{\partial T_s}{\partial t} = \alpha_s \frac{\partial^2 T_s}{\partial x_s^2}$$

- liquid phase: $s(t) < x < \infty$

$$\frac{\partial T_l}{\partial t} = \alpha_l \frac{\partial^2 T_l}{\partial x_l^2}$$

## Boundary conditions

- Constant temperatures $T\,(0,t) = T_w$ and $T\,(x \to \infty, t) = T_i$
- Interface condition at $x = s(t)$:

$$\begin{cases} -\lambda_l \frac{\partial T_l}{\partial x}\Big|_{s^+} + \lambda_s \frac{\partial T_s}{\partial x}\Big|_{s^-} = \rho_s L \frac{ds}{dt} \\ T_s\big|_{s^-} = T_l\big|_{s^+} = T_m \end{cases}$$

# Stefan problem (2)

**Assumptions**

- Heat transfer is driven by the sole conduction
- Thermophysical properties constant with temperature in each phase
- Phase change temperature fixed and known
- The entire domain initially at $T(x,0) = T_i$

**Analytical solution**

$$s = 2\delta\sqrt{\alpha_s t}$$

$$T_s(x,t) = T_{wall} + \frac{(T_m - T_{wall})}{erf(\delta)} erf\left(\frac{x}{2\sqrt{\alpha_s t}}\right)$$

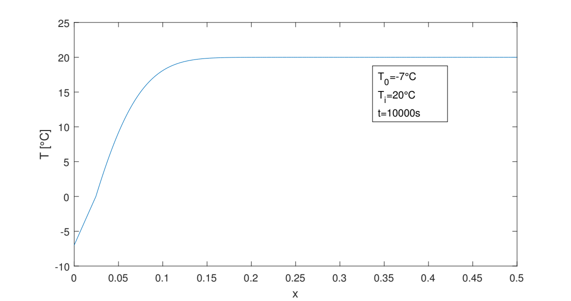
$$T_l(x,t) = T_i - \frac{(T_i - T_m)}{erfc(\delta\alpha)} erfc\left(\frac{x}{2\sqrt{\alpha_l t}}\right)$$

## Adimensional parameters

$$Ste = \frac{\rho_s c_{p,s} \left( T_m - T_{wall} \right)}{\rho_s L}$$

$$\phi = \frac{\rho_l c_{p,l} \left( T_i - T_m \right)}{\rho_s c_{p,s} \left( T_m - T_{wall} \right)}$$

$$\alpha = \sqrt{\frac{\alpha_s}{\alpha_l}}$$




## Equation for $\delta$

$$\frac{e^{\delta^2}}{erf\left(\delta\right)} - \frac{e^{-\delta^2 \alpha^2}}{erfc(\delta\alpha)} \frac{\phi}{\alpha} = \frac{\delta\sqrt{\pi}}{Ste}$$

# Velocity calculation

**Continuity**

$$\rho_{liq} V_{liq} + \rho_{sol} V_{sol} = const \qquad \rightarrow \qquad v_{liq} = \frac{dH}{dt} = \frac{ds}{dt} \cdot \underbrace{\left(1 - \frac{\rho_{sol}}{\rho_{liq}}\right)}_{r}$$

**Governing equation** $$\frac{\partial T_l}{\partial t} + v_{liq} \frac{\partial T_l}{\partial y} = \alpha_l \frac{\partial^2 T_l}{\partial x_l^2}$$

Being the parameter $r$ the indicator of the expansion:

$$T_l = T_0 + (T_m - T_0) \frac{erfc\left[\alpha\delta\left(\frac{x}{2\delta\sqrt{\alpha_s t}} - r\right)\right]}{erfc\left[\alpha\delta(1-r)\right]}$$

$$\frac{e^{-\delta^2}}{erf(\delta)} - \frac{\phi}{\alpha} \frac{e^{-(\alpha\delta)^2}}{erfc\left[\alpha\delta(1-r)\right]} \frac{e^{2r(\alpha\delta)^2}}{e^{(r\alpha\delta)^2}} = \frac{\delta\sqrt{\pi}}{Ste}$$

# Numerical method

Jadim is a research code developed by J. Magnaudet and D. Legendre's team in the Interface group at *Institut de Mécaniques des Fluides de Toulouse*. The code permits to describe in an accurate way physical mechanisms present in multiphasic flows.

- Volume of Fluid formulation is employed
- Thermal and Immersed Boundary Method routine are supported

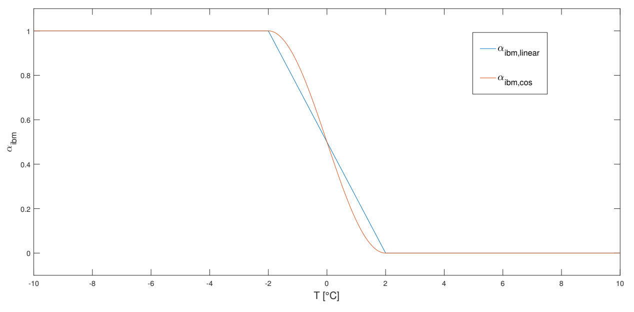
In the present work the objective is to couple the three of them and, in particular, to develop a thermal based IBM formulation

# Solid function fraction

A solid function fraction has been defined as follows, representing the amount of ice for each cell:

$$\alpha_{ibm,lin} = \tau \cdot \frac{T_{max} - T}{T_{max} - T_{min}}$$

$$\alpha_{ibm,cos} = 0.5 \cdot \tau \cdot \left[ cos \frac{\pi \cdot (T - T_{min})}{T_{max} - T_{min}} + 1 \right]$$




$$\phi = (1 - \tau)\,\phi_{air} + \tau\left[(1 - \alpha_{ibm})\,\phi_{liq} + \alpha_{ibm}\phi_{sol}\right]$$

# Implementation of velocity

## Velocity calculation

$$\boldsymbol{v}_{liq} = \left(1 - \frac{\rho_{ice}}{\rho_{water}}\right) \boldsymbol{V}_{front}$$

Scalar transport equation:

$$\frac{\partial \alpha_{ibm}}{\partial t} + \boldsymbol{V}_{front} \cdot \nabla \alpha_{ibm} = 0$$

$$\boldsymbol{V}_{front} \cdot \boldsymbol{n} = \frac{-\frac{\partial \alpha_{ibm}}{\partial T}\frac{\partial T}{\partial t}}{||\nabla \alpha_{ibm}||}$$

$$n_i = \frac{\frac{\partial \alpha_{ibm}}{\partial x_i}}{||\nabla \alpha_{ibm}||}$$

## Velocity imposition

$$\boldsymbol{f} = \alpha_{ibm} \frac{\boldsymbol{U} - \boldsymbol{U}^*}{\Delta t}$$

$$\begin{cases} U = 0 & \alpha_{ibm} \geq 0.95 \\ U = v_{liq} & 0 < \alpha_{ibm} < 0.95 \end{cases}$$

Being $\boldsymbol{U}^*$ a predictor velocity without considering the immersed object

**Pressure correction**

Jadim's SIMPLE algorithm is modified in order to take account of the calculated velocities

# The latent heat computation

$$\begin{cases} H = c_{p,l} T & \text{Liquid} \\ H = c_{p,s} T + L & \text{Solid} \end{cases}$$

$$\frac{\partial \rho H}{\partial t} = \nabla \cdot (k \nabla T)$$

**The source term method**

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) - S$$

$$S = \frac{\partial \alpha_{IBM}}{\partial T} \frac{\partial T}{\partial t} \frac{[L + T(c_{p,s} - c_{p,l})]}{c_p}$$

**The apparent heat capacity method**

$$c_{app} = \frac{dH}{dT}$$

$$c_{app} = c_p + L \frac{d\alpha_{IBM}}{dT}$$

$$\rho c_{app} \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T)$$

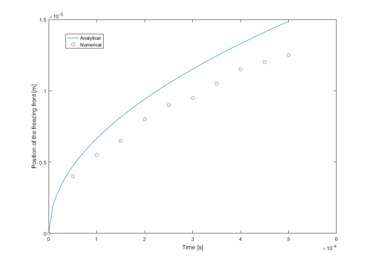
# 1D problem and validation

# IBM functions

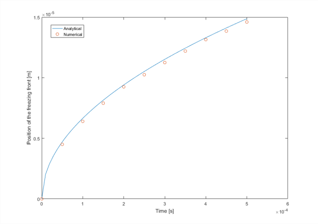
| | $err_{T,max}$ | $err_{T,avg}$ | $err_{int,max}$ | $err_{int,avg}$ |
|---|---|---|---|---|
| Linear $[-2, 2]$ | 6.1% | 5.7% | 27% | 17% |
| Linear $[-1, 1]$ | 4.3% | 4.1% | 14.2% | 9.2% |
| Linear $[0, 1]$ | 5.2% | 4.2% | 4.7% | 2.2% |
| Linear $[0, 0.1]$ | 2.8% | 2% | 3.5% | 1% |
| Cosine $[-1, 1]$ | 5.2% | 4.3% | 12.2% | 7.6% |
| Cosine $[0, 1]$ | 5.3% | 4.1% | 4.8% | 2.3% |

- For narrower solidification ranges, the results are more accurate
- The solidification front is well located if the inferior limit coincides to $0^\circ C$

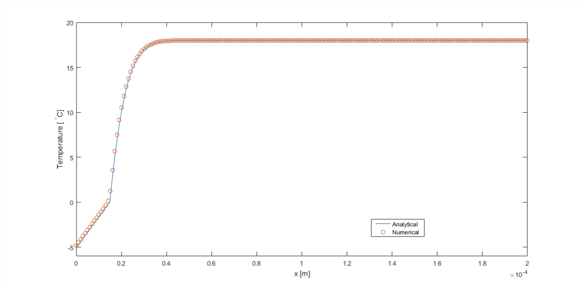
# IBM functions, figures




Freezing front, cos $[-1, 1]$



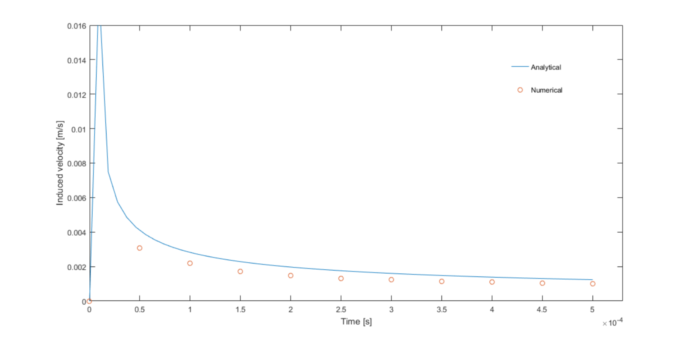

Freezing front, lin $[0, 0.1]$



# Velocity

| | $err_{vel,max}$ | $err_{vel,avg}$ |
|---|---|---|
| Lin $[-1, 1]$ | 16.5% | 8.1% |
| Lin $[0, 1]$ | 23.6% | 8.7% |
| Lin $[0, 0.1]$ | 100% | 53.8% |
| Cos $[-1, 1]$ | 13.7% | 7% |
| Cos $[0, 1]$ | 23.6% | 8.9% |

- Velocity is generally underestimated
- The chosen range has to be wide enough to properly calculate the velocity



# Grid convergence

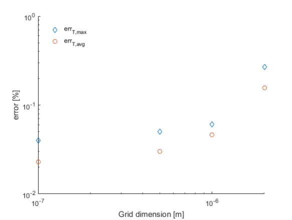



**Figure 1:** Temperature, cos $[-1, 1]$

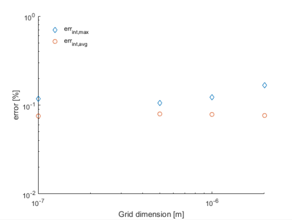



**Figure 3:** Interface position, cos $[-1, 1]$

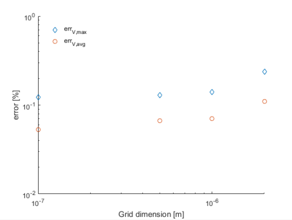



**Figure 2:** Velocity, cos $[-1, 1]$

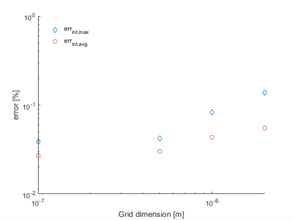



**Figure 4:** Interface position, lin $[0, 1]$

# Latent heat

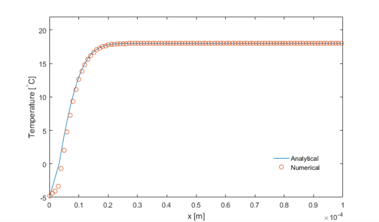



**Figure 5:** Apparent capacity method

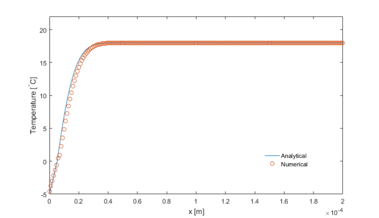
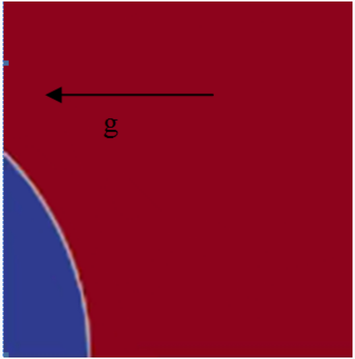



**Figure 6:** Source term method

- Average errors are comparable: $\sim 20\%$
- Source term method tends to become more accurate through time
- Apparent heat capacity worsens with time due to underestimation of the latent heat
- Source term method doesn't guarantee stability without additional loops

# Simulations of the droplet

# Parameters of the simulations




| | $\alpha \left[\frac{m^2}{s}\right]$ | $\rho \left[\frac{kg}{m^3}\right]$ |
|---|---|---|
| **Air** | $2.166 \cdot 10^{-5}$ | 1.2 |
| **Water** | $1.433 \cdot 10^{-7}$ | 1000 |
| **Ice** | $1.176 \cdot 10^{-6}$ | 917 |

| | |
|---|---|
| V | $1.35 \cdot 10^{-13}$ $[m^3]$ |
| $R_{90}$ | $4 \cdot 10^{-5}$ $[m]$ |
| dx | $10^{-6}$ $[m]$ |
| dt | $10^{-7}$ $[s]$ |

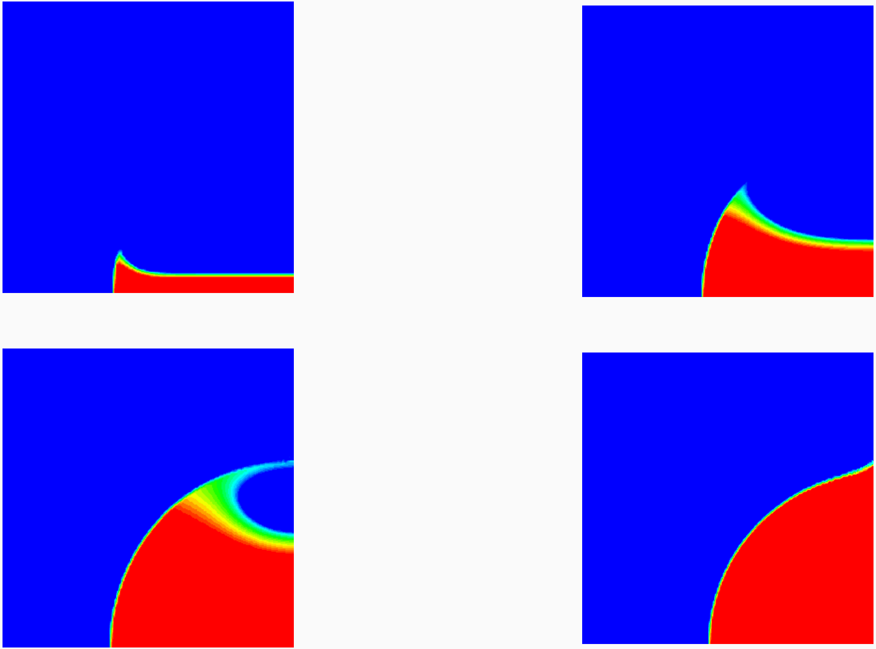
- West wall represents the cold plate and it is isothermal
- South is the symmetry axis
- North and east walls are adiabatic

$T_i$ $18^\circ C$

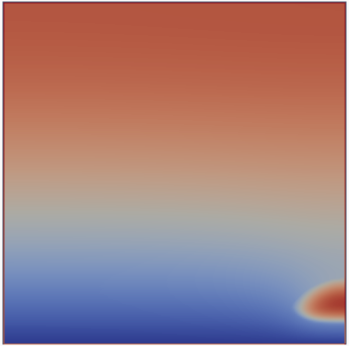
$T_w$ $-5^\circ C$

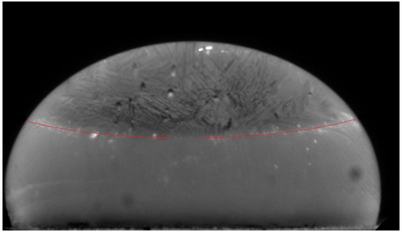
$$Bo = \frac{g \cdot \Delta\rho \cdot d^2}{\gamma} \ll 1$$

# The solidification process (1)



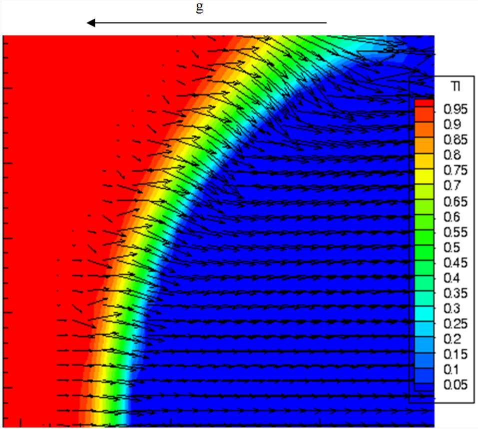
# The solidification process (2)





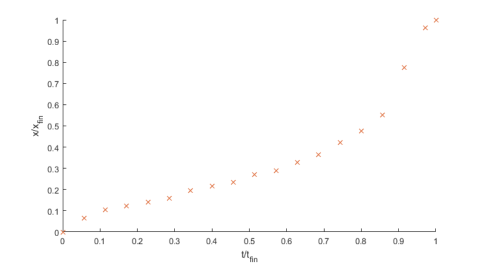
- Higher thermal diffusivity of the air causes the droplet outer layer to solidify
- Thermally treated as a mixture between ice and water
- Dynamically considered as liquid

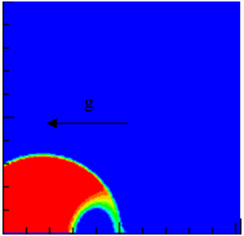
# Velocity field




- Inside the solid velocity is zero
- Composition of $V_x$ and $V_y$ returns a vector perpendicular to the interface
- Velocity field in the water comes from the combination of incompressibility and the axis of symmetry

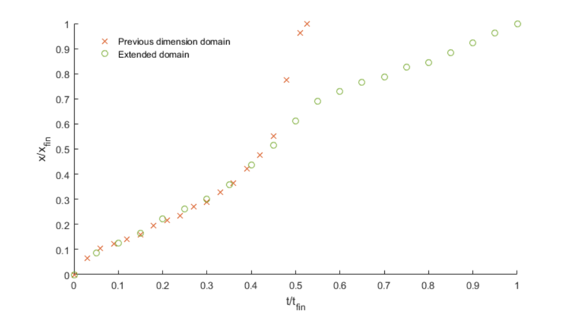
# Evolution of the freezing front (1)

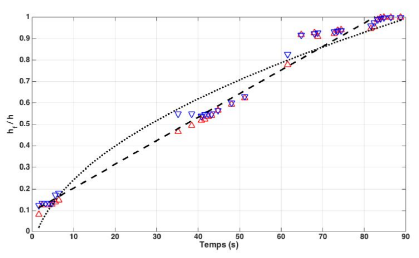



- Quasi-linear trend till the outer layer of ice is thin
- Acceleration due to the increased thermal diffusivity

# Evolution of the freezing front (2)

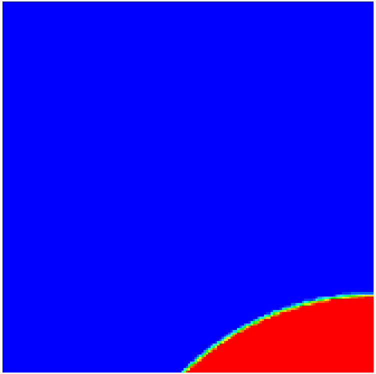


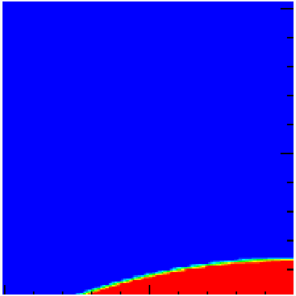



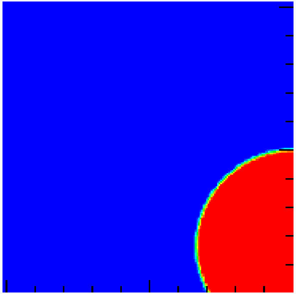

- Experimentally the trend seems to be linear differently from the Stefan problem
- An expanded domain guarantees a more accurate result, minimum size depends on the contact angle
- Numerical results confirm the global linear behaviour

# Contact angle influence (1)

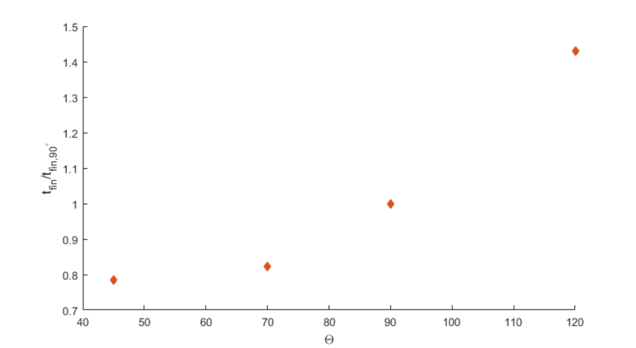
- Wall temperature
- Initial temperature
- Droplet's volume
- Mesh size





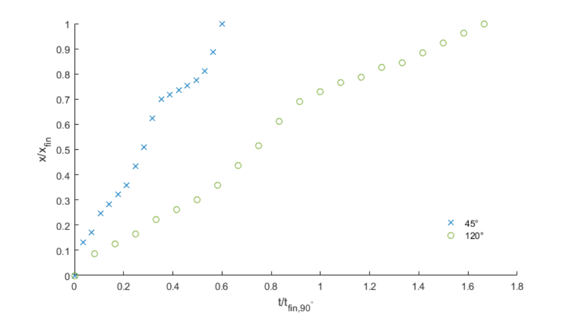


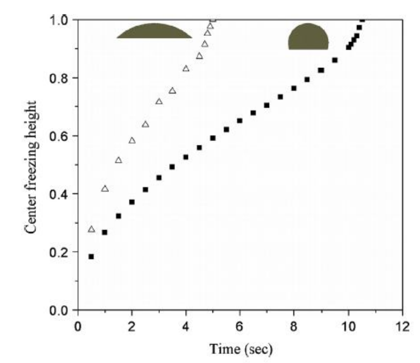
# Contact angle influence (2)




- Hydrophobic surfaces tends to delay the ice accretion, commonly employed as constructive materials or coatings
- We observed no formation of the protrusion in correspondence of a threshold of about $\Theta \simeq 30°$

# Contact angle influence (3)



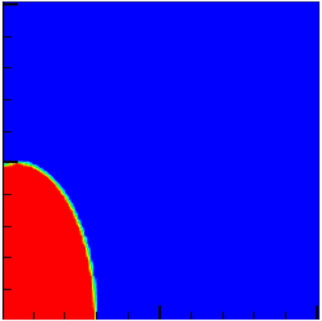




- Results similar to other numerical works
- Quasi-linear behaviour independent from the contact angle, the difference is the total solidification time

# Elliptical droplet (1)

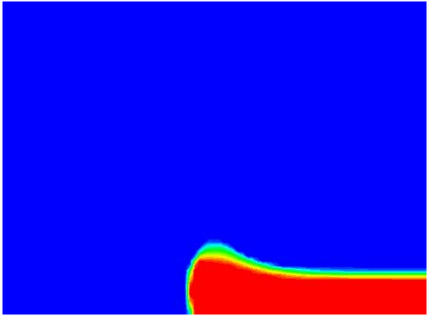
It is not a physical case ⇒ the droplet's volume and consequentially Bond number are too small to cause an elliptical shape

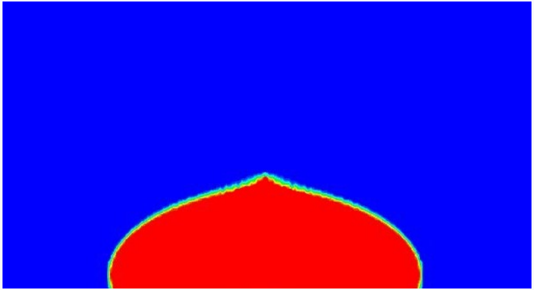
A qualitative study to investigate if new dynamics appear



# Elliptical droplet (2)

- The freezing front shape resemble the spheric cap case and the experimental results
- The pointy protrusion continues to appear





# Conclusions

# Conclusions and future developments

**Conclusions**

- Successfully coupled VoF, thermics and IBM
- The characteristic pointy tip has been reproduced
- Evolution of the freezing front confirms experimental results over analytical ones
- Parametric study of the contact angle influence

**Future developments**

- Ameliorate the computation of the latent heat of solidification
- Tracking the freezing front for a better calculation of the velocity
- Experimentally confirm the influence of the contact angle

# Thank you for your attention