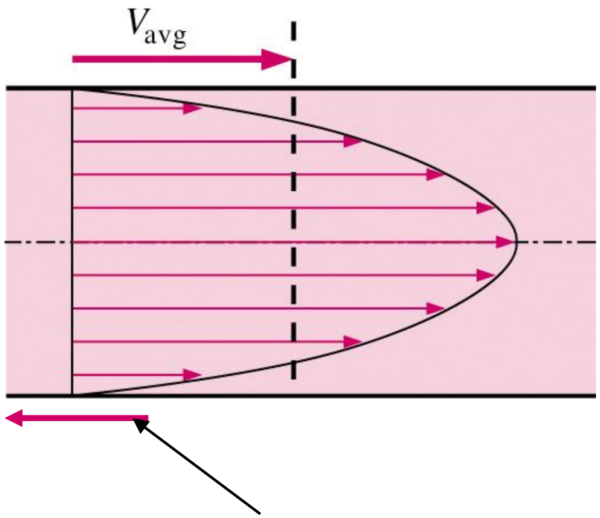
Chapter 8: Flow in Pipes

Objectives

1. Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow
2. Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements

Introduction



Friction force of wall on fluid

- Average velocity in a pipe
  - Recall - because of the no-slip condition, the velocity at the walls of a pipe or duct flow is zero
  - We are often interested only in V_{avg}, which we usually call just V (drop the subscript for convenience)
  - Keep in mind that the no-slip condition causes shear stress and friction along the pipe walls

Introduction

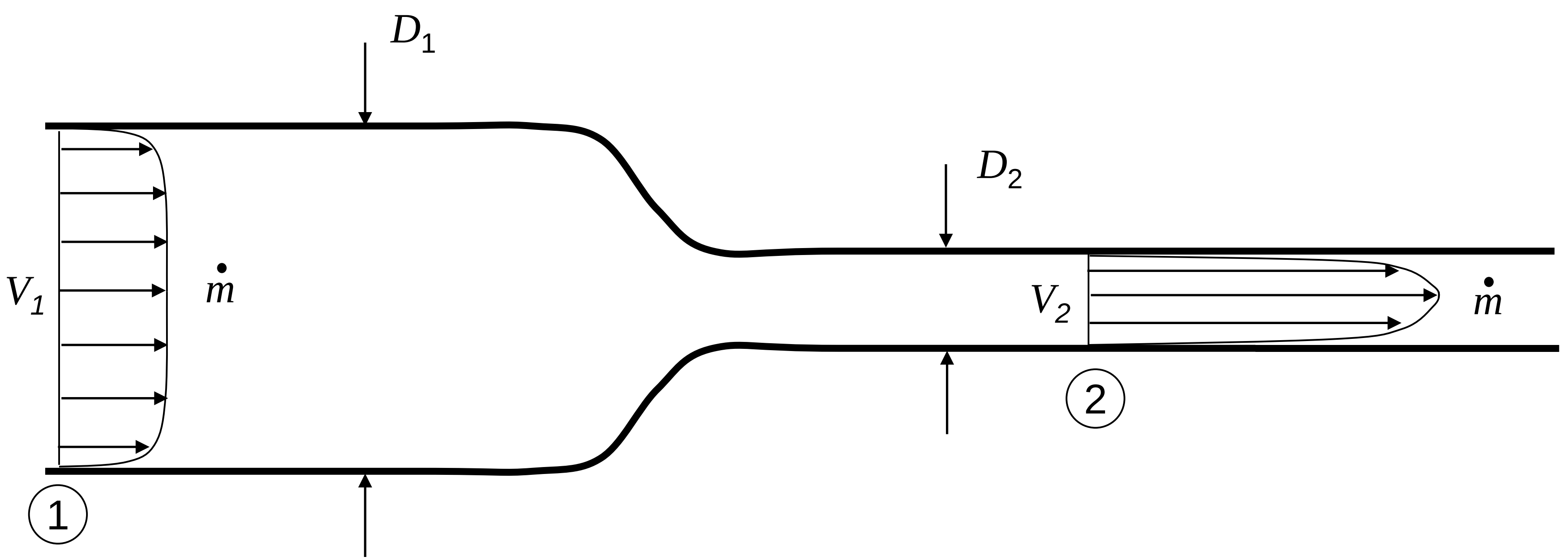
- For pipes of constant diameter and incompressible flow
  - V_{avg} stays the same down the pipe, even if the velocity profile changes
    - Why? Conservation of Mass

$$\dot{m} = \rho V_{avg} A = constant$$

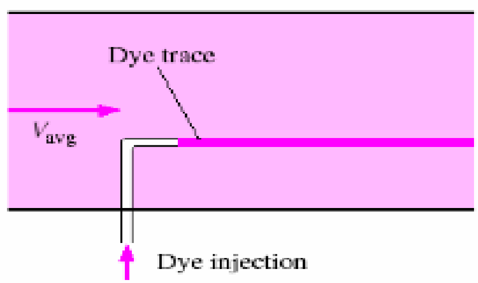
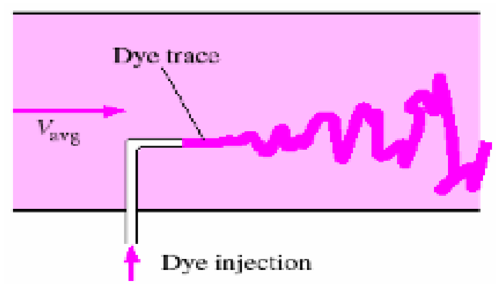
same same same

Introduction

- For pipes with variable diameter, $\dot{m}$ is still the same due to conservation of mass, but $V_1 \neq V_2$



Laminar and Turbulent Flows

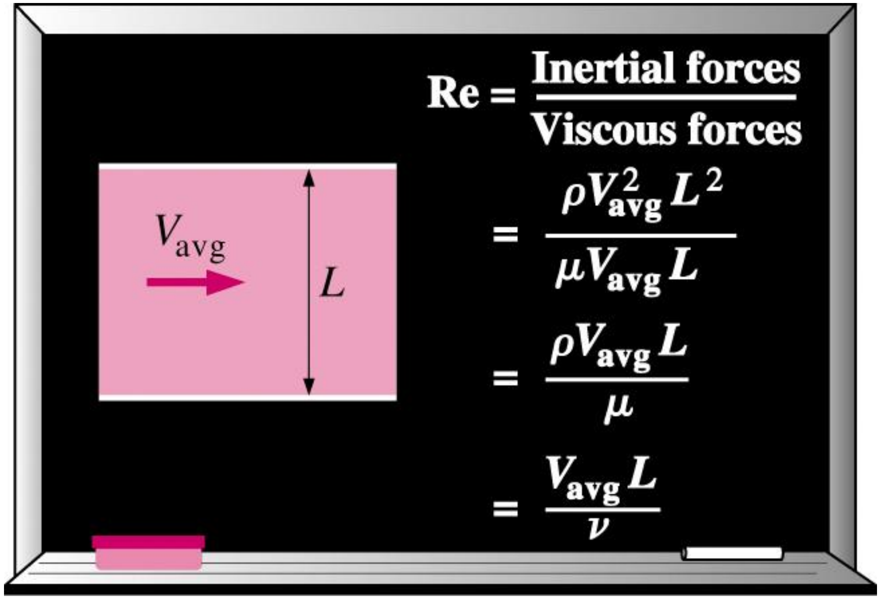
<u>Laminar Flow</u>	<u>Turbulent Flow</u>
Can be steady or unsteady.	Is always *unsteady*.
(Steady means that the flow field at any instant in time is the same as at any other instant in time.)	Why? There are always random, swirling motions (vortices or eddies) in a turbulent flow.
	Note: However, a turbulent flow can be steady *in the mean*. We call this a *stationary turbulent flow*.
Can be one-, two-, or three-dimensional.	Is always *three-dimensional*.
	Why? Again because of the random swirling eddies, which are in all directions.
	Note: However, a turbulent flow can be 1-D or 2-D *in the mean*.
Has regular, *predictable* behavior	Has irregular or *chaotic* behavior (cannot predict exactly – there is some randomness associated with any turbulent flow.
	
Analytical solutions are possible (see Chapter 9).	No analytical solutions exist! (It is too complicated, again because of the 3-D, unsteady, chaotic swirling eddies.)
Occurs at *low* Reynolds numbers.	Occurs at *high* Reynolds numbers.

$$Re = \frac{\text{Inertial forces}}{\text{Viscous forces}}$$

Clay Institute Millennium Prize

Laminar and Turbulent Flows

Definition of Reynolds number



- Critical Reynolds number (Re_{cr}) for flow in a round pipe

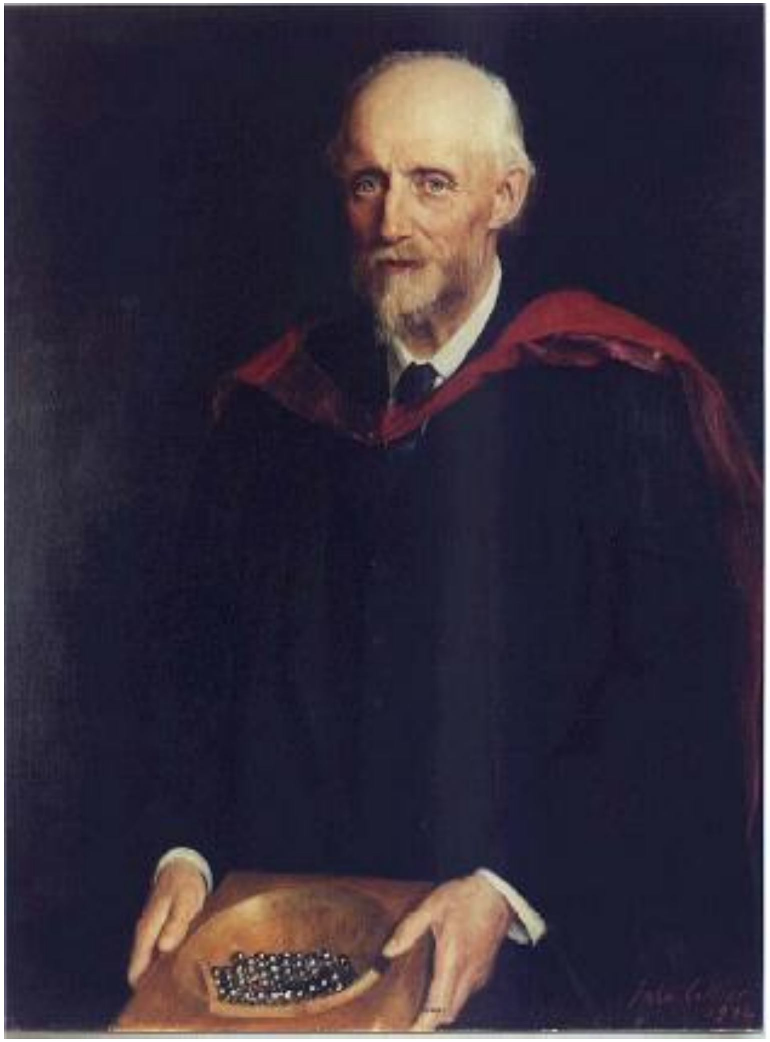
  $Re < 2300 \Rightarrow$ laminar

  $2300 \leq Re \leq 4000 \Rightarrow$ transitional

  $Re > 4000 \Rightarrow$ turbulent

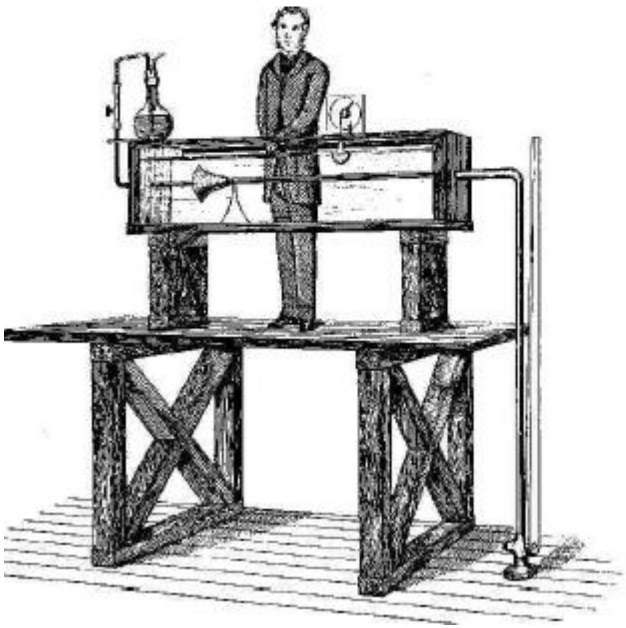
- Note that these values are approximate.
- For a given application, Re_{cr} depends upon
  - Pipe roughness
  - Vibrations
  - Upstream fluctuations and disturbances (valves, elbows, etc. that may perturb the flow)

Osborne Reynolds (1842-1912)

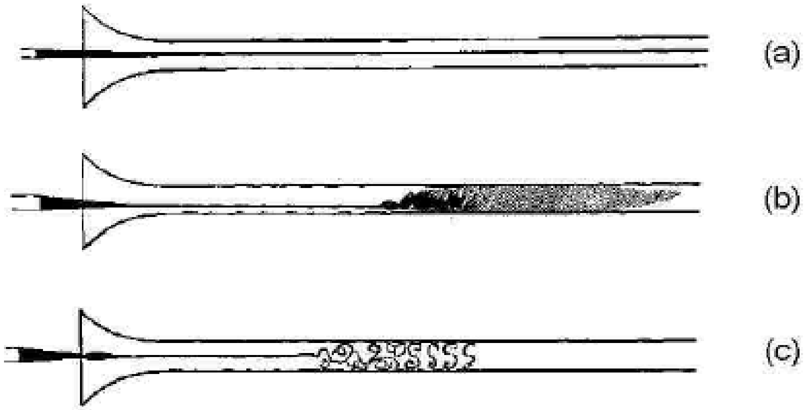


"An experimental investigation of the circumstances which determine whether motion of water shall be direct or sinuous and of the law of resistance in parallel channels", *Royal Society*, *Phil. Trans*. 1883

Osborne Reynolds 1880 Experiments

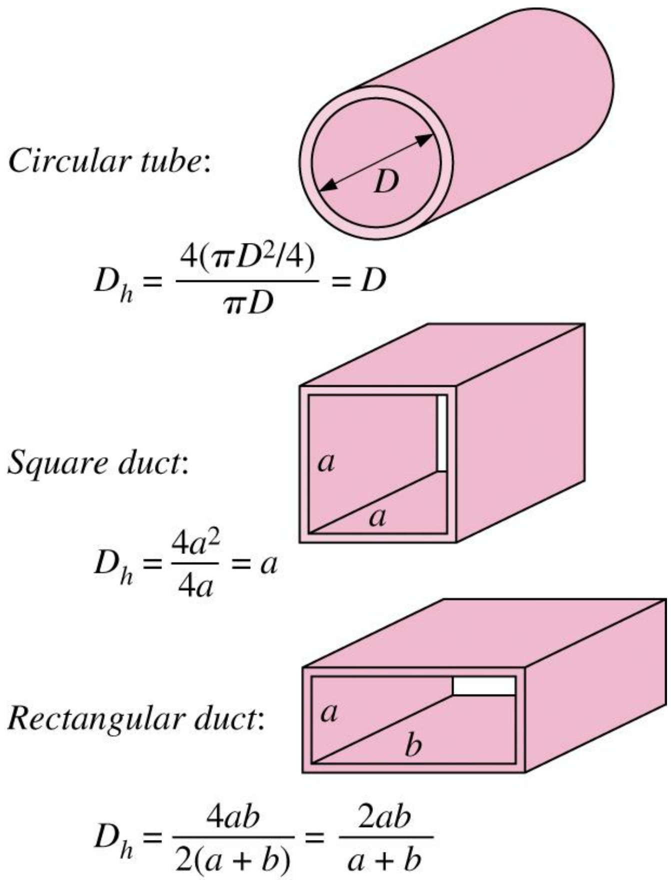


`the colour band would all at once mix up with the surrounding water, and fill the rest of the tube with a mass of coloured water ... On viewing the tube by the light of an electric spark, the mass of colour resolved itself into a mass of more or less distinct curls, showing eddies.'

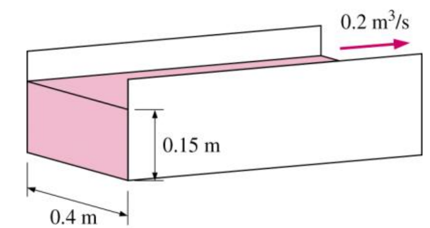


$Re_{cr} \approx 13000$ (??)

Laminar and Turbulent Flows



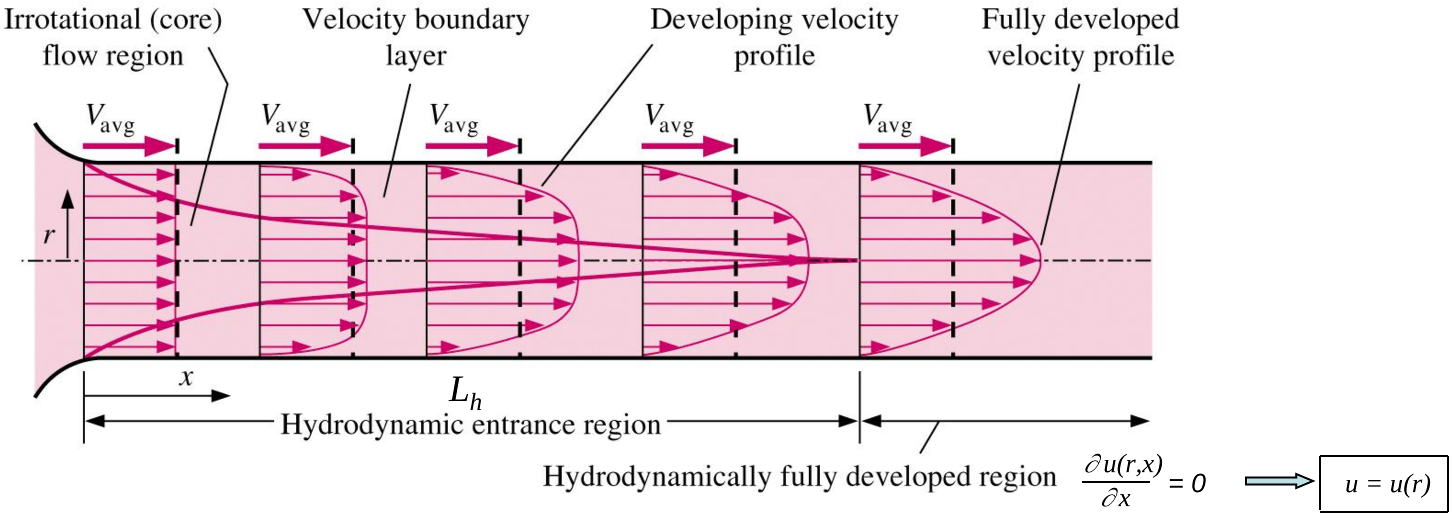
- For <u>non-round</u> pipes, define the hydraulic diameter
  $D_h = 4A_c/P$
  A_c = cross-section area
  P = wetted perimeter



- Example: open channel
  $A_c = 0.15 * 0.4 = 0.06\text{m}^2$
  $P = 0.15 + 0.15 + 0.4 = 0.7\text{m}$
  Don’t count free surface, since it does not contribute to friction along pipe walls!
  $D_h = 4A_c/P = 4*0.06/0.7 = 0.343\text{m}$
  <u>What does it mean?</u> This channel flow is equivalent to a round pipe of diameter 0.343m (approximately).

The Entrance Region

- Consider a round pipe of diameter D. The flow can be laminar or turbulent. In either case, the profile develops downstream over several diameters called the *entry length* L_h. L_h/D is a function of Re.



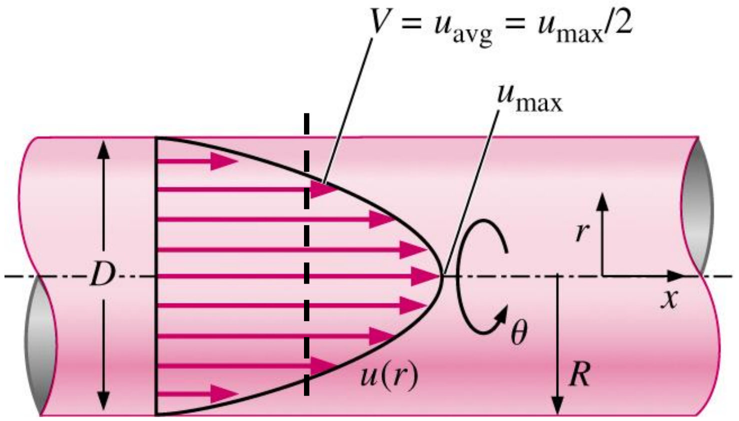
Fully Developed Pipe Flow

- Comparison of laminar and turbulent flow

  There are some major differences between laminar and turbulent fully developed pipe flows

  <u>Laminar</u>

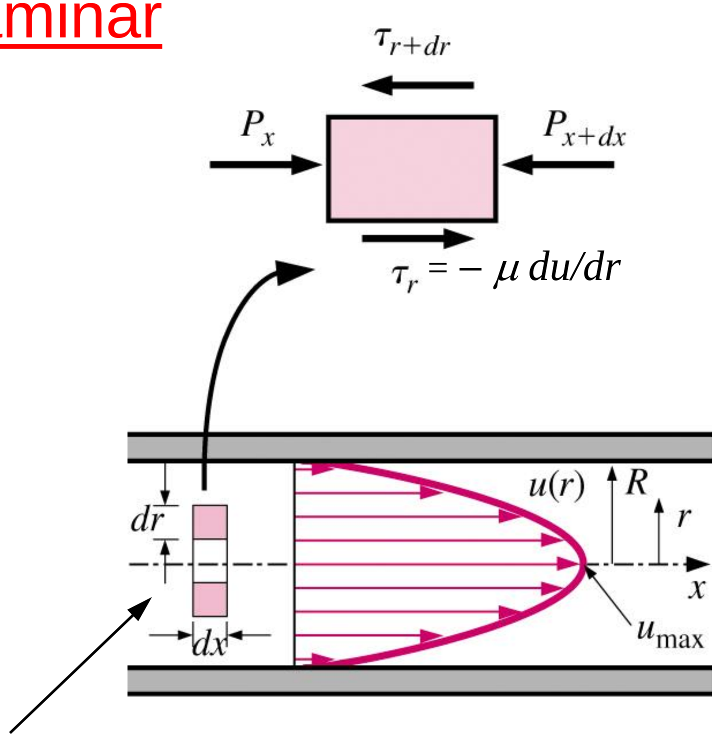
  - Can solve exactly (Chapter 9)
  - Flow is steady
  - Velocity profile is parabolic
  - Pipe roughness not important



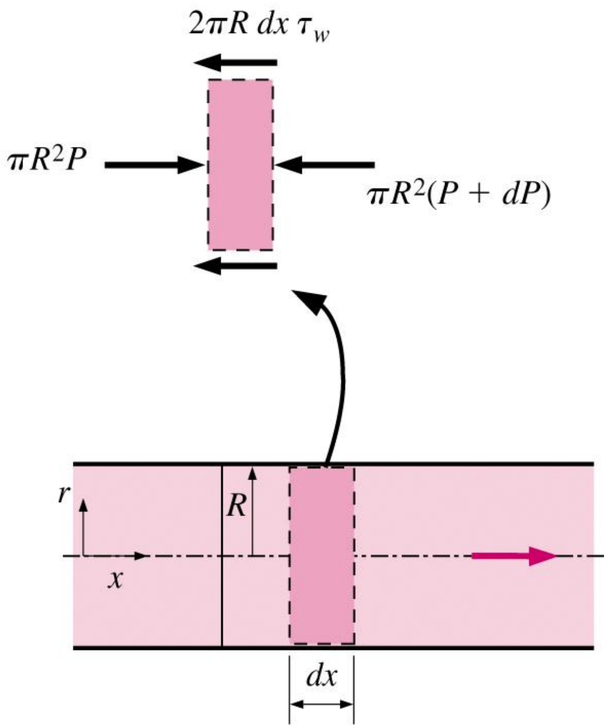
It turns out that $V_{avg} = ½\, u_{max}$ and $u(r) = 2V_{avg}(1 - r^2/R^2)$

Fully Developed Pipe Flow

Laminar



Ring-shaped differential volume element



$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} = constant$$

$$u(r) = \ldots,\ P_1 - P_2 = 32\,\mu\, L\, V_{avg}/D^2$$

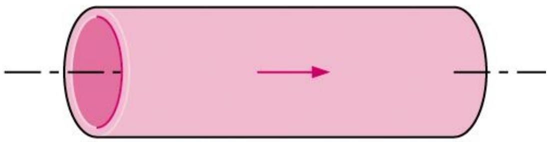
Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\, dx\, \tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R} = constant$$

Example



Oil at 20°C (ρ = 888 kg/m^3 and μ = 0.800 kg/m·s) flows steadily through a 5-cm-diameter 40-m-long pipe. The pressure at the pipe inlet and outlet are measured to be 745 and 97 kPa, respectively.

1) Determine the average velocity and the flow rate through the pipe;
2) Verify that the flow through the pipe is laminar;
3) Determine the value of the Darcy friction factor f;
4) Determine the pumping power required to overcome the pressure drop.

Definition: $$\boxed{\Delta P_L = f\ \frac{L}{D}\ \frac{\rho V_{avg}^2}{2}}$$

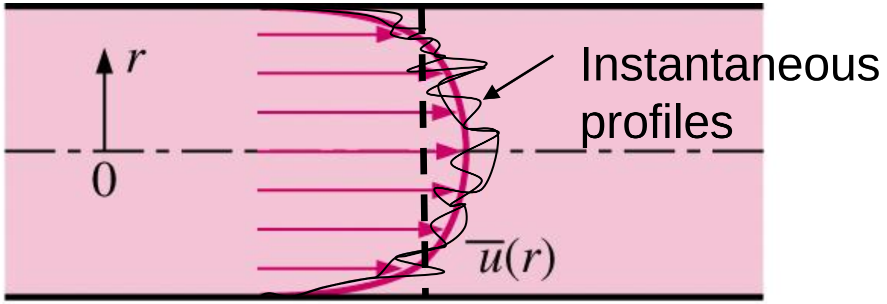
f: Darcy friction factor

(this definition applies to both laminar and turbulent flows)

Fully Developed Pipe Flow

Turbulent

- *Cannot* solve exactly (too complex)
- Flow is unsteady (3D swirling eddies), but it is steady in the mean
- Mean velocity profile is fuller (shape more like a top-hat profile, with very sharp slope at the wall)
- Pipe roughness is very important



- V_{avg} 85% of u_{max} (depends on Re a bit)
- No analytical solution, but there are some good semi-empirical expressions that approximate the velocity profile shape. See text

  Logarithmic law (Eq. 8-46)

  Power law (Eq. 8-49)

Fully Developed Pipe Flow
Wall-shear stress

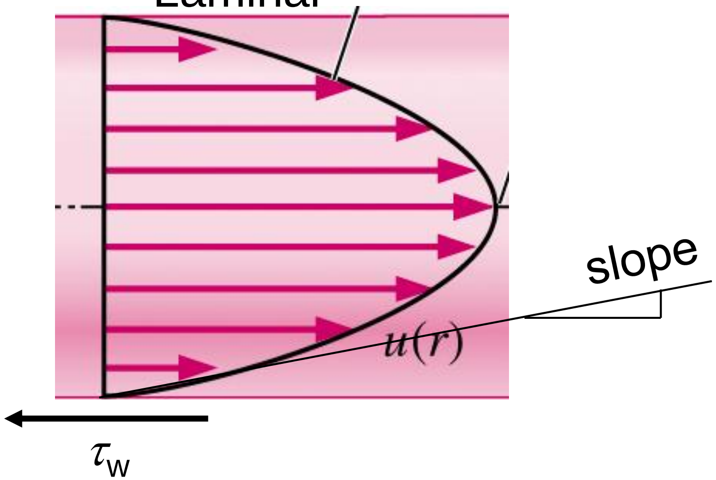
- Recall, for simple shear flows $u=u(y)$, we had

$$\tau = \mu \, du/dy$$

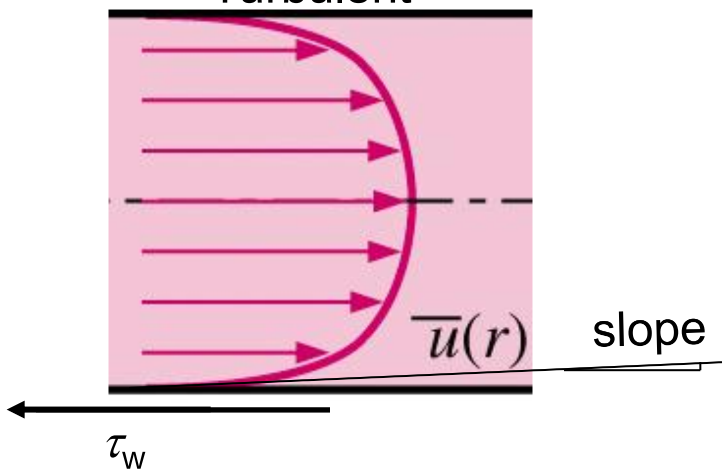
- In fully developed pipe flow, it turns out that

$$\tau = -\mu \, du/dr$$

Laminar



Turbulent

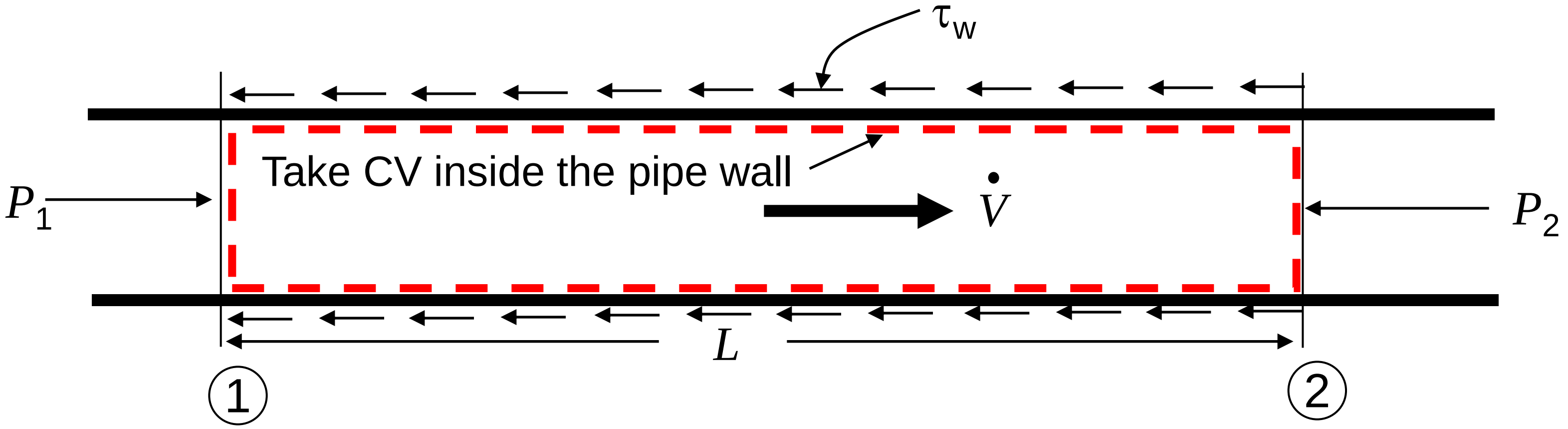


τ_w = shear stress at the wall, acting on the fluid

$$\boxed{\tau_{w,turb} > \tau_{w,lam}}$$

Fully Developed Pipe Flow
Pressure drop

- There is a direct connection between the pressure drop in a pipe and the shear stress at the wall
- Consider a <u>horizontal pipe</u>, fully developed, and incompressible flow



- Let's apply conservation of mass, momentum, and energy to this CV

Fully Developed Pipe Flow
Pressure drop

- <u>Conservation of Mass</u>

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\rho \dot{V}_1 = \rho \dot{V}_2 \rightarrow \dot{V} = const$$

$$V_1 \frac{\pi D^2}{4} = V_2 \frac{\pi D^2}{4} \rightarrow \boxed{V_1 = V_2}$$

- <u>Conservation of x-momentum</u>

$$\sum F_x = \cancel{\sum F_{x,grav}} + \sum F_{x,press} + \sum F_{x,visc} + \cancel{\sum F_{x,other}} = \sum_{out} \beta \dot{m} V - \sum_{in} \beta \dot{m} V$$

$$P_1 \frac{\pi D^2}{4} - P_2 \frac{\pi D^2}{4} - \tau_w \pi D L = \underbrace{\cancel{\beta_2 \dot{m} V_2} - \cancel{\beta_1 \dot{m} V_1}}$$

Terms cancel since $\beta_1 = \beta_2$ and $V_1 = V_2$

Fully Developed Pipe Flow
Pressure drop

- Thus, *x*-momentum reduces to

$$(P_1 - P_2)\frac{\pi D^2}{4} = \tau_w \pi D L \quad \text{or} \quad \boxed{P_1 - P_2 = 4\tau_w \frac{L}{D}}$$

- Energy equation (in head form)

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} + \cancel{h_{pump,u}} = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2} + \cancel{h_{turbine,e}} + h_L$$

cancel (horizontal pipe)

Velocity terms cancel again because $V_1 = V_2$

$$\boxed{P_1 - P_2 = \rho g h_L}$$

h_L = irreversible head loss; it is felt as a pressure drop in the pipe

Fully Developed Pipe Flow
Head Loss

- From momentum CV analysis $$\boxed{P_1 - P_2 = 4\tau_w \frac{L}{D}}$$
- From energy CV analysis $$\boxed{P_1 - P_2 = \rho g h_L}$$
- Equating the two gives

$$4\tau_w \frac{L}{D} = \rho g h_L \qquad \boxed{h_L = \frac{4\tau_w}{\rho g}\frac{L}{D}}$$

- To predict **head loss**, we need to be able to calculate τ_w. How?
  - Laminar flow: solve exactly
  - Turbulent flow: rely on empirical data (experiments)
  - In either case, we can benefit from dimensional analysis!

Fully Developed Pipe Flow
Darcy Friction Factor

- τ_w = func(ρ, V, D, μ, ε)

ε = average roughness of the inside wall of the pipe

- Π-analysis gives

$$\Pi_1 = f$$

$$\boxed{f = \frac{8\tau_w}{\rho V^2}}$$

$$\boxed{Re = \frac{\rho V D}{\mu}}$$

$$\Pi_2 = Re$$

$$\boxed{\epsilon/D = \text{roughness factor}}$$

$$\Pi_3 = \frac{\epsilon}{D}$$

$$\Pi_1 = func(\Pi_2, \Pi_3)$$

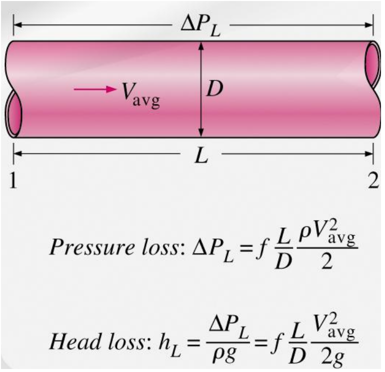
$$\boxed{f = func(Re, \epsilon/D)}$$

Fully Developed Pipe Flow Friction Factor

- Now go back to equation for h_L and substitute f for τ_w

$$h_L = \frac{4\tau_w}{\rho g}\frac{L}{D} \qquad f = \frac{8\tau_w}{\rho V^2} \rightarrow \tau_w = f\rho V^2/8$$

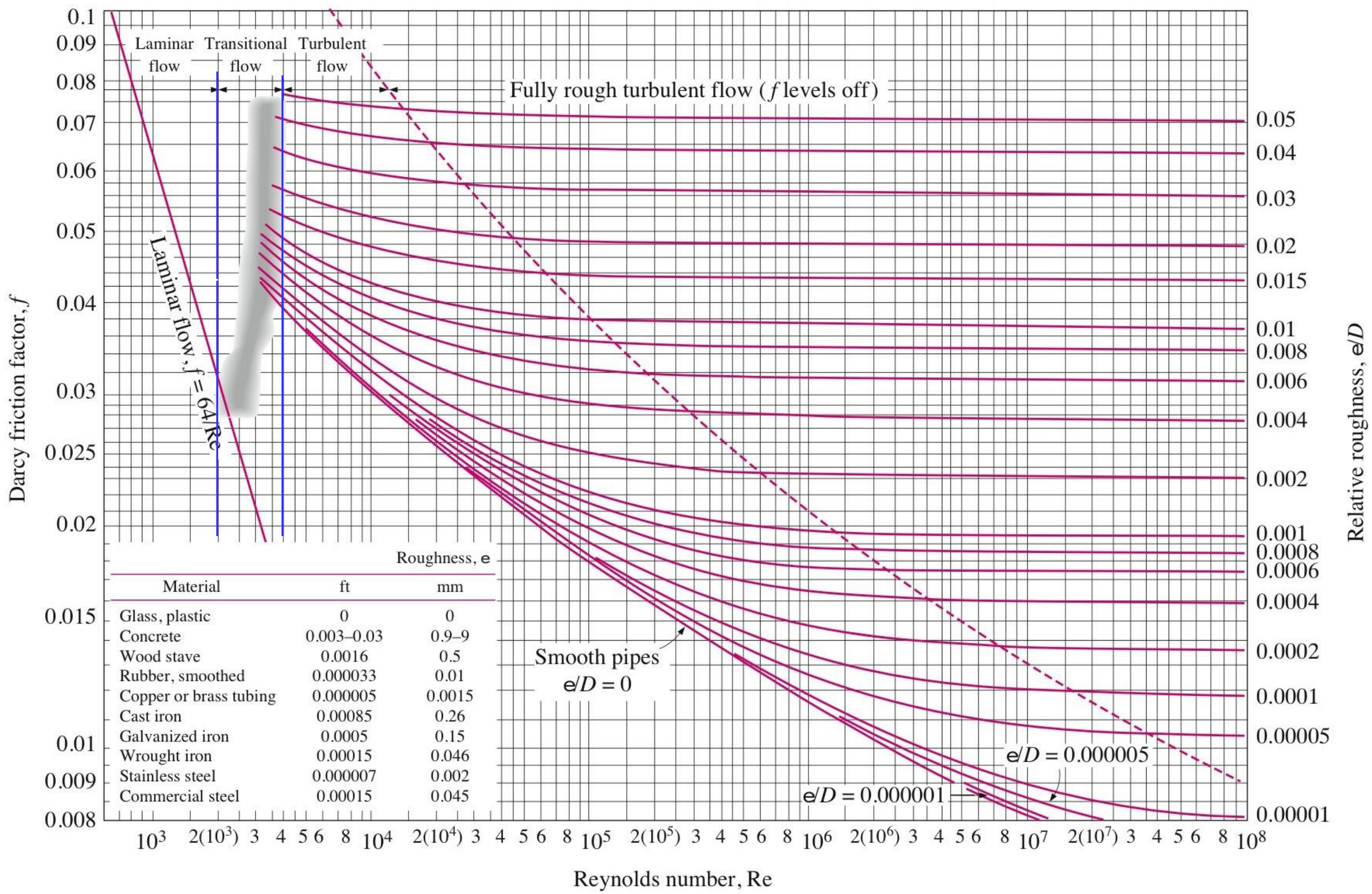
$$\boxed{h_L = f\frac{L}{D}\frac{V^2}{2g}}$$



- Our problem is now reduced to solving for Darcy friction factor f
  - Recall $f = func(Re, \epsilon/D)$
  - Therefore
    - Laminar flow: **f = 64/Re** (exact)
    - Turbulent flow: Use charts or empirical equations (Moody Chart, a famous plot of f vs. Re and ε/D)

But for laminar flow, roughness does not affect the flow unless it is huge

The Moody Chart



Fully Developed Pipe Flow
Friction Factor

- Moody chart was developed for circular pipes, but can be used for non-circular pipes using hydraulic diameter
- Colebrook equation is a curve-fit of the data which is convenient for computations

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

Implicit equation for f which can be solved with an iterative numerical method

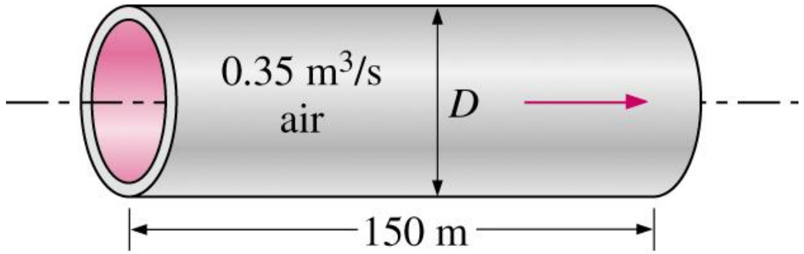
- Both Moody chart and Colebrook equation are accurate to ±15% due to roughness size, experimental error, curve fitting of data, etc.

Types of Fluid Flow Problems

- In design and analysis of piping systems, 3 problem types are encountered
  1. Determine Δp (or h_L) given L, D, V (or flow rate)
     Can be solved directly using Moody chart and Colebrook equation
  2. Determine V, given L, D, Δp
  3. Determine D, given L, Δp, V (or flow rate)
- Types 2 and 3 are common engineering design problems, i.e., selection of pipe diameters to minimize construction and pumping costs. However, iterative approach required since both V and D are in the Reynolds number.

Example

Heated air at 1 atm and 35°C is to be transported in a 150-m–long circular plastic duct at a rate of 0.35 m^3/s. If the head loss in the pipe is not to exceed 20 m, determine the maximum required pumping power, the minimum diameter of the duct, average velocity, the Reynolds number and the Darcy friction factor.



ρ = 1.145 kg/m^3, ν = 1.655 10^{-5} m^2/s

$$D = 0.66\left[\epsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \qquad \begin{array}{c} 10^{-6} < \epsilon/D < 10^{-2} \\ 5000 < Re < 3\times 10^8 \end{array}$$

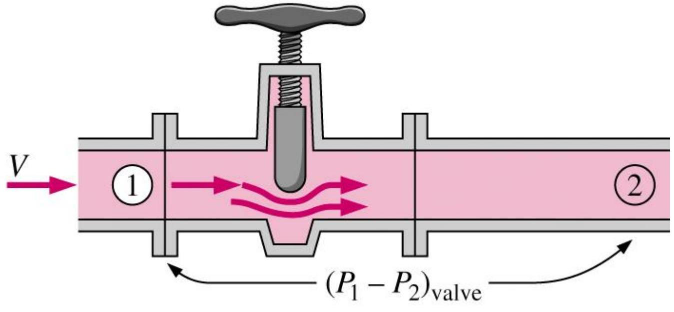
Minor Losses

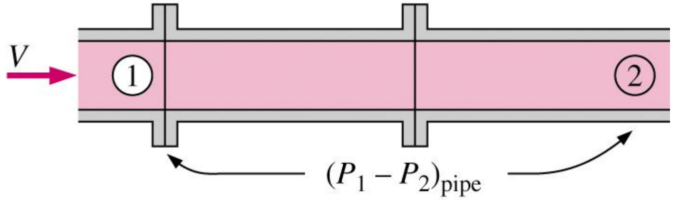
- Piping systems include fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions.
- These components interrupt the smooth flow of fluid and cause additional losses because of flow separation and mixing
- We introduce a relation for the minor losses associated with these components

$$\boxed{h_L = K_L \frac{V^2}{2g}}$$

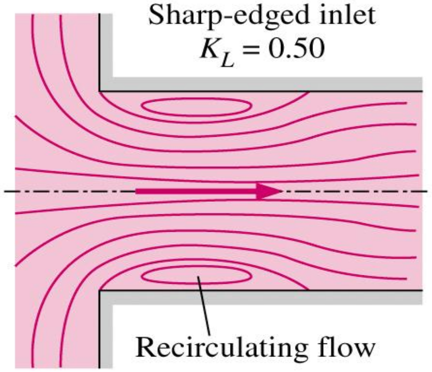
- K_L is the loss coefficient.
- It is different for each component.
- It is assumed to be independent of Re.
- Typically provided by manufacturer or generic table (e.g., Table 8-4 in text).

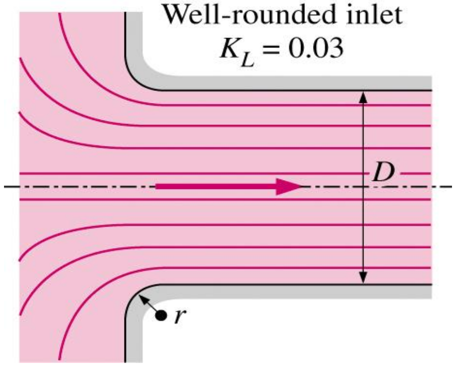
Minor Losses





$$\Delta P_L = (P_1 - P_2)_{valve} - (P_1 - P_2)_{pipe}$$





The loss coefficient K_L is determined by measuring the additional pressure loss the component causes, and dividing it by the dynamic pressure in the pipe

The head loss at the inlet of a pipe is almost negligible for well rounded inlets

Minor Losses

- Total head loss in a system is comprised of major losses (in the pipe sections) and the minor losses (in the components)

$$h_L = h_{L,major} + h_{L,minor}$$

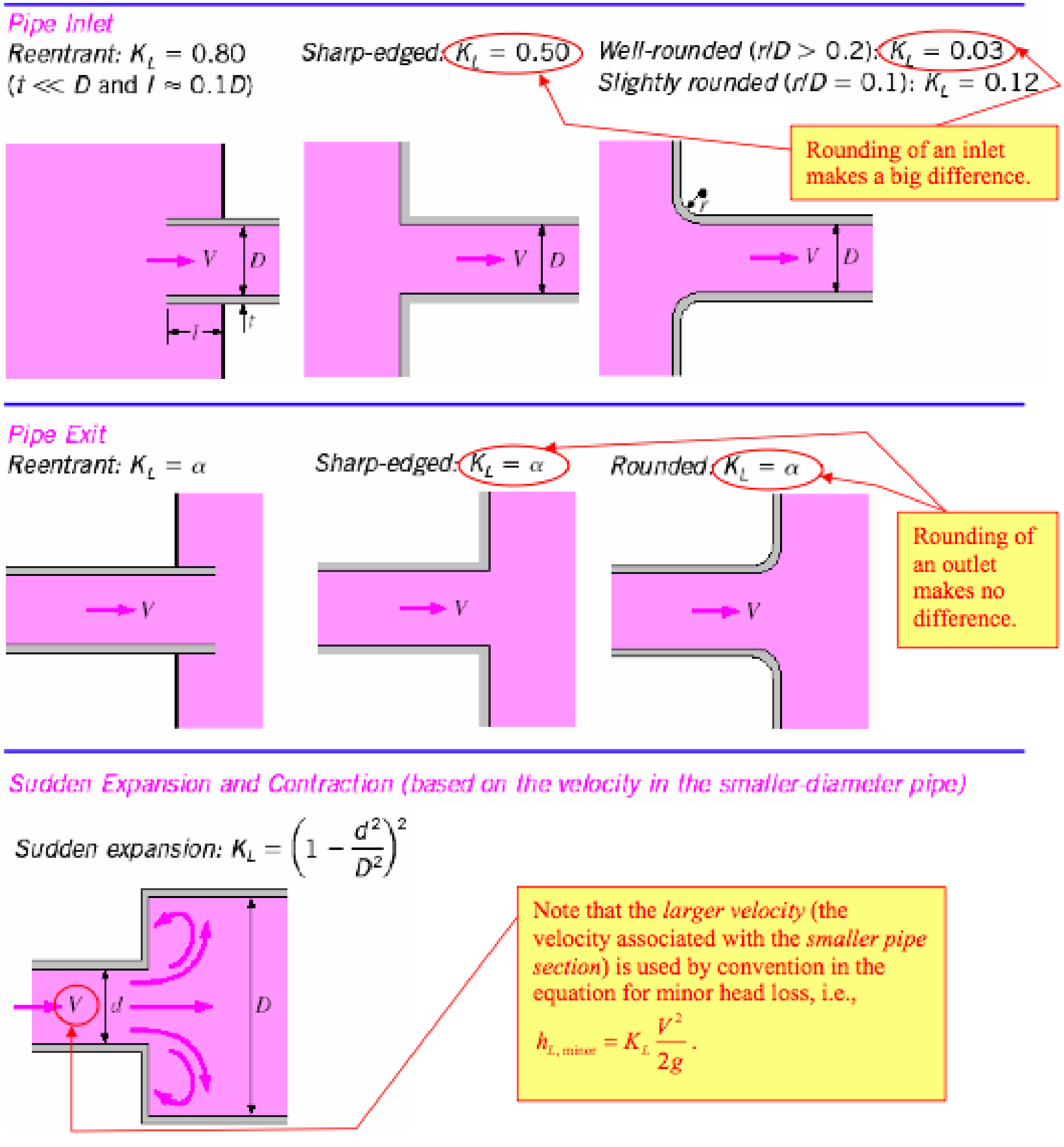
$$h_L = \underbrace{\sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g}}_{i \text{ pipe sections}} + \underbrace{\sum_j K_{L,j} \frac{V_j^2}{2g}}_{j \text{ components}}$$

- If the piping system has constant diameter

$$h_L = \left(f \frac{L}{D} + \sum K_L \right) \frac{V^2}{2g}$$

Minor Losses

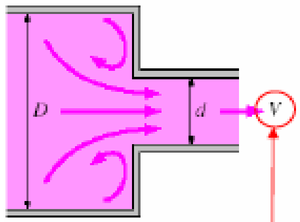
Here are some sample loss coefficients for various minor loss components. More values are listed in Table 8-4, page 350 of the Çengel-Cimbala textbook:

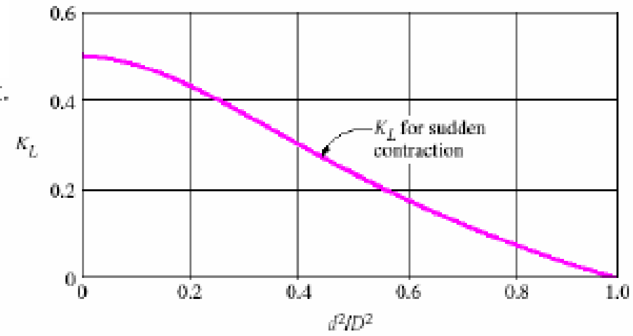


$\alpha = 2$ for fully developed laminar flow

$\alpha \approx 1$ for fully developed turbulent flow

Sudden contraction: See chart.





Note: These are backwards. The K_L values listed for Expansion should be those for Contraction, and vice-versa.

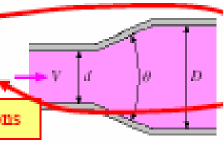
Note again that the *larger velocity* (the velocity associated with the *smaller pipe section*) is used by convention in the equation for minor head loss, i.e., $h_{L,\text{minor}} = K_L \dfrac{V^2}{2g}$.

Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Expansion:

$K_L = 0.02$ for $\theta = 20°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$

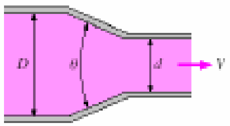
These are for contractions



Contraction (for $\theta = 20°$):

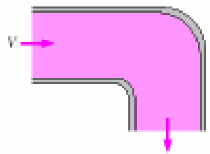
$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$

These are for expansions

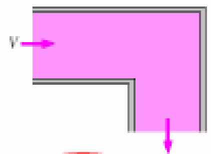


Bends and Branches

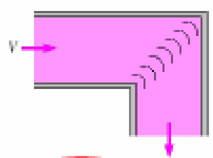
90° smooth bend:
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$



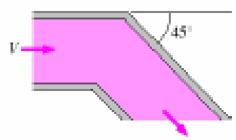
90° miter bend
(without vanes): $K_L = 1.1$



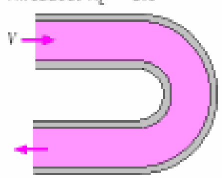
90° miter bend
(with vanes): $K_L = 0.2$



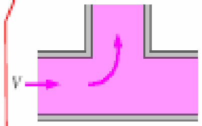
45° threaded elbow:
$K_L = 0.4$



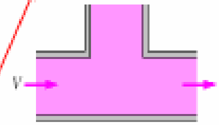
180° return bend:
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$



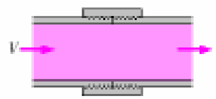
Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$



Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$

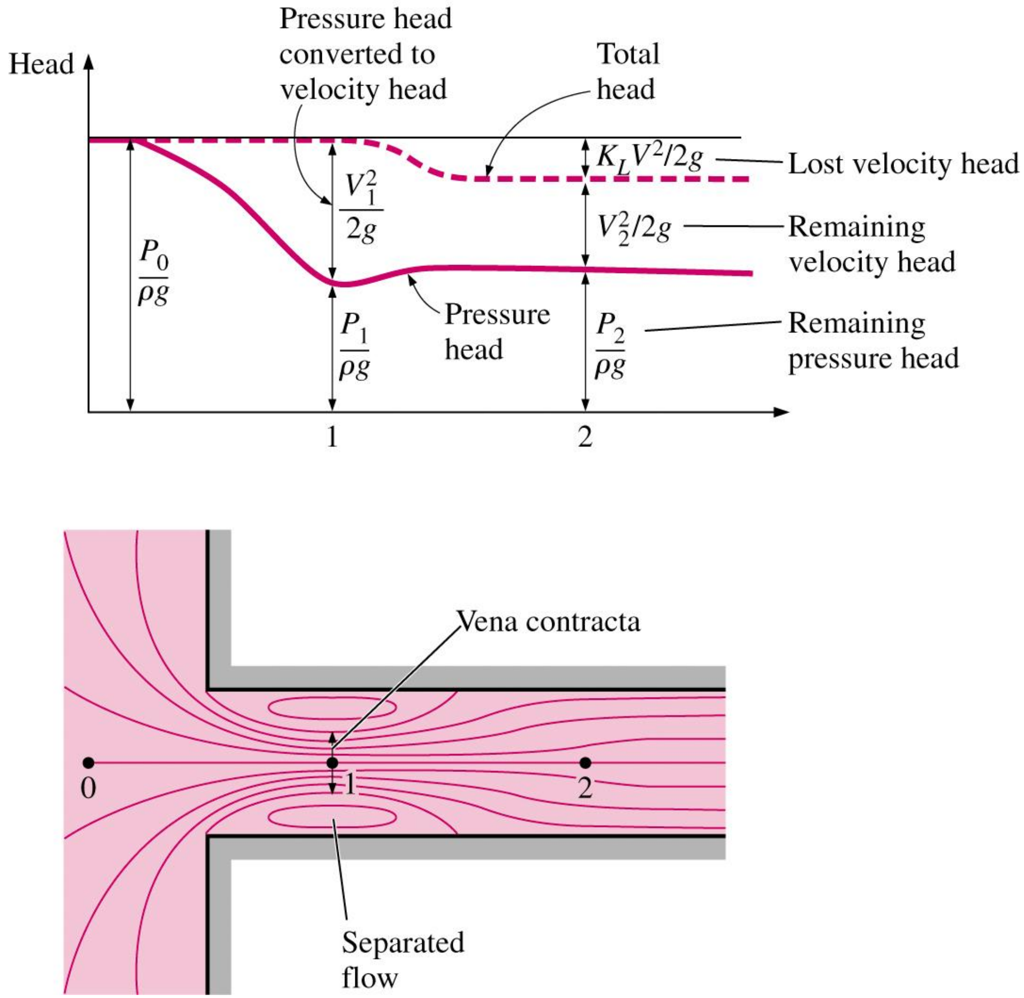


Threaded union:
$K_L = 0.08$



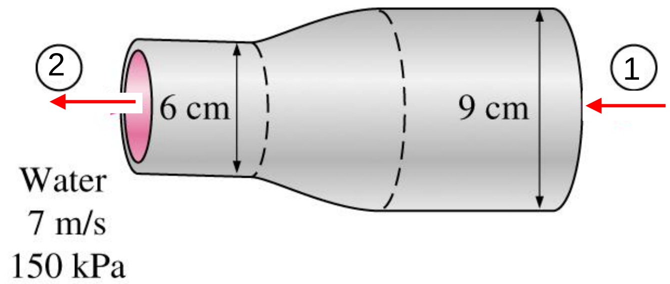
For tees, there are two values of K_L, one for *branch flow* and one for *line flow*.

Head Loss at a Sharp-Edge Inlet



Example

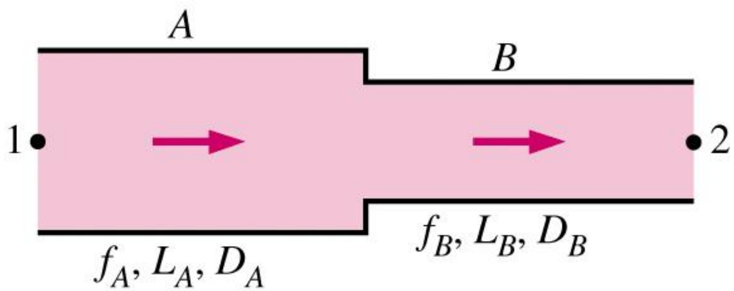
A 9-cm-diameter horizontal water pipe contracts gradually to a 6-cm-diameter pipe. The walls of the contraction section are angled 30° from the horizontal. The average velocity and pressure of water at the exit of the contraction section are 7 m/s and 150 kPa, respectively. Determine the head loss in the contraction section and the pressure in the larger-diameter pipe. In the case of plastic pipes, determine also the friction factor for both pipes in series.



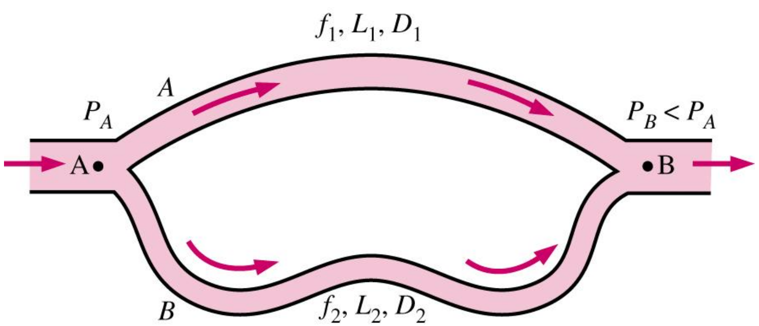
Turbulent fully developed flow at sections 1 and 2 (?), ρ = 998 kg/m^3, μ = 1.002 x 10^{-3} kg/(m s), K_L ?

Piping Networks and Pump Selection

- Two general types of networks
  - Pipes in series
    - Volume flow rate is constant
    - Head loss is the summation of parts
  - Pipes in parallel
    - Volume flow rate is the sum of the components
    - Pressure loss across all branches is the same



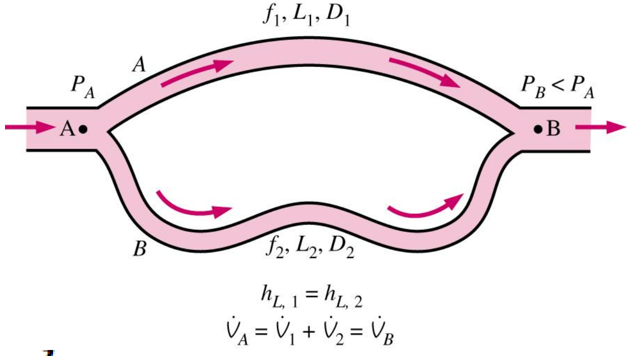
$$\dot{V}_A = \dot{V}_B$$
$$h_{L,\,1\text{-}2} = h_{L,\,A} + h_{L,\,B}$$



$$h_{L,\,1} = h_{L,\,2}$$
$$\dot{V}_A = \dot{V}_1 + \dot{V}_2 = \dot{V}_B$$

Piping Networks and Pump Selection

- For parallel pipes, perform CV analysis between points A and B



$$V_A = V_B$$

$$\frac{P_A}{\rho g} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_A} = \frac{P_B}{\rho g} + \cancel{\frac{V_B^2}{2g}} + \cancel{z_B} + h_L$$

$$h_L = \frac{\Delta P}{\rho g}$$

- Since ΔP is the same for all branches, head loss in all branches is the same

$$h_{L,1} = h_{L,2} \Longrightarrow f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$

Piping Networks and Pump Selection

- Head loss relationship between branches allows the following ratios to be developed

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{\frac{1}{2}} \qquad \frac{\dot{\mathcal{V}}_1}{\dot{\mathcal{V}}_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{\frac{1}{2}}$$

  so that the relative flow rates in parallel pipes are established from the requirements that the head loss in each pipe is the same

- Real pipe systems result in a system of non-linear equations.
- Note: the analogy with electrical circuits should be obvious
  - Flow rate ($\dot{V}$): current (I)
  - Pressure gradient (Δp): electrical potential (V)
  - Head loss (h_L): resistance (R), however h_L is very nonlinear

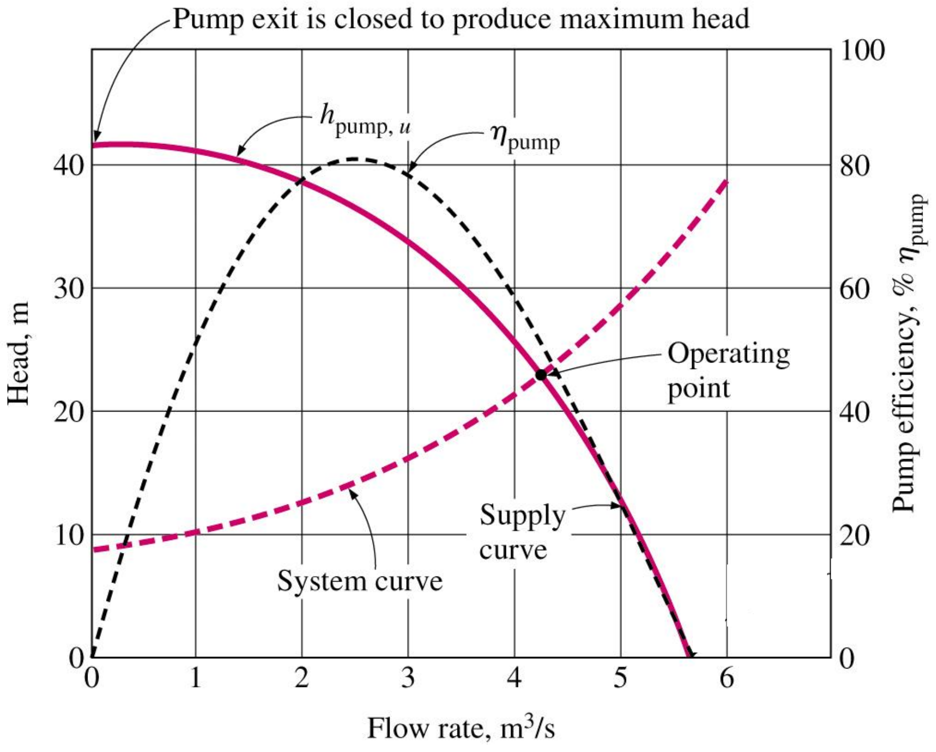
Piping Networks and Pump Selection

- When a piping system involves pumps and/or turbines, pump and turbine head must be included in the energy equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + \boxed{h_{pump,u}} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \boxed{h_{turbine,e}} + h_L$$

- The useful head of the pump ($h_{pump,u}$) or the head extracted by the turbine ($h_{turbine,e}$), are functions of volume flow rate, i.e., they are not constants.
- Operating point of system is where the system is in balance, e.g., where pump head is equal to the head loss (plus elevation difference, velocity head difference, etc.)

Pump and systems curves



- Supply curve for $h_{pump,u}$: determined experimentally by manufacturer. It is possible to build a functional relationship for $h_{pump,u}$.
- System curve determined from analysis of fluid dynamics equations
- Operating point is the intersection of supply and demand curves
- If peak efficiency is far from operating point, pump is wrong for that application.