Turbulence and CFD models: Theory and applications

Roadmap to Lecture 6

Part 4

- 1. Near wall treatment
- 2. Incomplete list of turbulence models and references

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1. Near wall treatment

2. Incomplete list of turbulence models and references

Turbulence near the wall – Boundary layer



Actual profile – Physical velocity profile

- Walls are the main source of turbulence generation in flows.
- The presence of walls imply the existence of boundary layers.
 - In the boundary layer, large gradients exist (velocity, temperature, and so on).
 - To properly resolve these gradients, we need to use very fine meshes.
 - These gradients are larger if we are dealing with turbulent flows.

- The easiest way to resolve the steep gradients near the walls is by resolving the viscous sublayer.
 - To resolve the viscous sublayer, we need to cluster a lot of cells in the region where y⁺ is less than 5.
 - This can significantly increase the cell count.
 - And in the case of unsteady simulation, it can have a significant impact in the time-step, where very small time-steps are required for stability and accuracy reasons.



- A way around wall resolving simulations, is the use of wall functions.
- By using wall functions, we can use empirical correlations to bridge wall conditions to the log-law layer.
- The correlations provide a link between U and U_{τ} (or τ_w).





Viscous sublayer
$$y^+ < 5$$
 $u^+ = y^+$

Buffer layer

None of the previous correlations apply

Note: the range of y^+ values might change from reference to reference but roughly speaking they are all close to these values.

- The questions now are,
 - How do we transfer information from the empirical correlations to the walls and to the flow?
 - How do we compute the wall shear stresses?
- To answer these questions, let us summarize all the non-dimensional variables near the wall.



• Close to the walls we only know the wall shear stress, viscosity, and distance,

$$U = f\left(\tau_w, \rho, \mu, y\right)$$

Therefore, we use these quantities to create the non-dimensional groups.

- The velocity profile near the wall can be represented by using the previous non-dimensional quantities and correlations.
- By using non-dimensional quantities, the flow behavior near the wall is independent of the Reynolds number, geometry, or relevant physics (to some extent).
- The correlations take a very predictable behavior close to the walls for a wide variety of flows.
- The outer or mean flow, depends of the geometry, boundary conditions, physics, and so on.



Dimensionless mean velocity profile u⁺ as a function of the dimensionless wall distance y⁺ for turbulent pipe flow with Reynolds numbers between 4000 and 3600000 [1].



Mean velocity profiles in pipe flow showing the collective approach to a log law. The curves are for Reynolds numbers between Re = 31×10^3 and Re = 18×10^6 [2].

F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer, 2016.
 B. McKeon, J. Li, W. Jiang, J. Morrison, A. Smits. Further observations on the mean velocity distribution in fully developed pipe flow. 2004

 If we are dealing with globally laminar flows, or if we have a mesh fine enough to resolve the viscous sublayer, we can compute the wall shear stress as follows,





Note: the subscript p indicates values at the cell center and the subscripts w indicates values at the walls

- However, if we are dealing with turbulent flows and if we are using a coarse mesh such that y⁺ > 30 (let us use this limit for the moment), this approach is not accurate anymore.
- We are missing a lot of gradient information if we use this approach.
- By the way, some solvers use cell-centered quantities and some solvers use nodecentered quantities.
- Sometimes in this approach, damping functions are added to gain robustness.

- Wall resolving meshes allow for the accurate computation of steep gradients near the walls.
- The only drawback is that you will require a lot of cells close to the walls.
- The main idea behind wall functions, is to use coarser meshes without losing accuracy.
- In the cells next to the walls, the field quantities and wall shear stresses are approximated using correlations (*e.g.*, log-law layer).

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Wall resolving mesh

Wall modeling mesh

- Comparison of laminar and turbulent velocity profiles in a pipe.
- As it can be observed, close to the walls the velocity gradient is larger in the turbulent case.
- Therefore, fine meshes are required in order to properly resolve the steep gradients (velocity, temperature, etc.) close to the walls.



- If the first cell center is in log-law layer, we cannot use the previous approach because it is too inaccurate.
- Therefore, we need to use wall functions.
- That is, we bridge the wall conditions and cell centered values with the empirical correlations.
- The wall functions reduce the computational effort significantly because we do not need to resolve the viscous sublayer.
- Let us explain the standard wall functions using the method proposed by Launder and Spalding [1], which is probably the most widely used method.
- In this approach,

 $u^* = \begin{cases} y^* & \text{In the viscous sublayer} \\ \frac{1}{\kappa} \ln (Ey^*) & \text{In the log-law layer} \end{cases}$

- Notice that we are using u^* and y^* instead of u^+ and y^+ .
- Also, the log-law layer correlation is slightly different from what we have seen so far.

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Let us address these two issues.

• The idea of introducing the new quantity u^* , is to avoid the singularity that occurs when the wall shear stress is equal to zero in u^+ (*i.e.*, in a separation point).

$$U_{\tau} = \sqrt{\frac{\tau_w}{\rho}}$$
 $U_{\tau} = 0$ if $\tau_w = 0$ $u^+ = \frac{U}{U_{\tau}}$

• The new quantities u^* are y^* defined as follows,

$$U_{\tau}^{*} = C_{\mu}^{1/4} k_{p}^{1/2} \qquad y^{*} = \frac{C_{\mu}^{1/4} k_{p}^{1/2} y_{p}}{\nu} \qquad u^{*} = \frac{1}{\kappa} \ln \left(Ey^{*}\right)$$

- It is worth noting that y^+ is equal to y^* in equilibrium conditions.
- Recall the concept of equilibrium from the derivation of the C_{μ} coefficient.
- Also recall the equation of the ratio of Reynolds stress to turbulent kinetic energy.

$$C_{\mu} = \frac{\tau_{uv}^2}{k^2} \qquad \qquad u_{\tau}^2 = \frac{\tau_w}{\rho}$$

 In the standard wall functions formulation of Launder and Spalding [1], the correlation for the log-law layer is given as follows,

$$u^* = \frac{1}{\kappa} \ln\left(Ey^*\right)$$

• Whereas the traditional correlation is given as follows,

$$u^{+} = \frac{1}{\kappa} \ln\left(y^{+}\right) + C^{+}$$

- These two correlations are approximately the same, as shown in the figure.
- Any difference is due to the values of the constants used.



 All the relations of the standard wall functions formulation of Launder and Spalding [1], can be summarized as follows,

The only unknown quantity is the wall shear stress

Note: the subscript p indicates values at the cell center and the subscripts w indicates values at the walls

$$P_k \approx \tau_w \frac{\partial \overline{u}}{\partial y} = \tau_w \frac{\tau_w}{\kappa \rho C_\mu^{1/4} k_p^{1/2} y_p}$$

 $u^* = \frac{U_p C_{\mu}^{1/4} k_p^{1/2}}{\tau_w / \rho} \quad \longleftarrow$

$$\epsilon_p = \frac{C_\mu^{3/4} k_p^{3/2}}{\kappa y_p}$$

• The boundary condition for TKE at the walls is,

$$\frac{\partial k}{\partial n} = 0$$

• And recall that,

$$u^* = \frac{1}{\kappa} \ln \left(Ey^* \right) \qquad y^* = \frac{\rho C_{\mu}^{1/4} k_p^{1/2} y_p}{\mu} \qquad \kappa = 0.4187 \qquad E = 9.793$$

These are the values used in Fluent

These relations apply only to the cells adjacent to the walls.

You can have an automatic wall treatment just by simply adding a conditional clause,

$$u^* = \begin{cases} y^* & y^* < 11.225\\ \frac{1}{\kappa} \ln(Ey^*) & y^* > 11.225 \end{cases}$$

- The value of 11.225 (which is the one used in Fluent), comes from the intersection of the two correlations.
- This value might change depending on the constant used.
- If you recall, we found a value of approximately 10.8, of course, we used different values for the constants.
- In this approach, we should avoid to place the first cell center in the buffer layer, as errors are large in this region.
- Remember, is very difficult (if not impossible) to have a uniform y⁺ value.
- Therefore, you should monitor the average y⁺ value at the walls.
- It is also recommended to monitor the maximum and minimum values of y⁺ and verify that they do not cover more that 10% of the surface or are located in critical areas.

- We just presented the wall functions for the momentum and turbulence variables.
- Similar wall functions can be derived for temperature, species, and so on.
- There are many wall functions implementations.
 - Standard wall functions (the approach we just presented).
 - Scalable wall functions.
 - Non-equilibrium wall functions.
 - Enhanced wall treatment.
 - Two-layer approach.
 - y⁺ insensitive wall treatment.
- In the literature, you can find viscosity-based approaches, and so on.
- The approach presented, is also known as a log-law based approach.
- In Fluent, the wall boundary conditions for the field variables are all taken care of by the wall functions.
- You do not need to be concerned about the boundary conditions at the walls.

- It is also possible to formulate y⁺ insensitive wall functions.
- That is, formulations that cover viscous sublayer, buffer region, and log-law region.
- This can be achieved by using a blending function between the viscous sublayer and the log-law layer [1].
- To use this approach you need to use turbulence models able to deal with wall resolving meshes and wall modeling meshes.
- The $k \omega$ family of turbulence models are y⁺ insensitive.
- Kader [1] proposed the following blending function to obtain a y⁺ insensitive formulation,

$$u^{+} = e^{\Gamma} u_{lam}^{+} + e^{1/\Gamma} u_{turb}^{+}$$

$$\Gamma = -\frac{a (y^{+})^{4}}{1 + by^{+}} \qquad a = 0.01 \qquad b = 5$$

 This formula guarantees the correct asymptotic behavior for large and small values of y⁺ and reasonable representation of velocity profiles in the cases where y⁺ falls inside the buffer region.

[1] B. Kader. Temperature and Concentration Profiles in Fully Turbulent Boundary Layers. 1981.

- Plot of Kader's [1] blending function.
- In the plot, the Spalding function [2] is also represented.
- The Spalding function is another alternative to obtain a y⁺ insensitive treatment.
- It is essentially a fit of the laminar, buffer and logarithmic regions of the boundary layer.



Final remarks

- If you want good accuracy, use a wall resolving approach.
- This approach is relatively affordable if you are running steady simulations.
- If you have massive separation, have in mind that wall functions are not very accurate.
- Heat transfer and non-equilibrium applications requires high accuracy (wall resolving treatment). This requirement is not compulsory; however, it is strongly recommended.
- Using wall functions is not about putting one single cell in the log-law layer. You need to put enough cells in the log-law region to resolve the velocity, temperature, and turbulence variables profiles.
- In the wall resolving approach, try to get an average y⁺ value close to 1 or lower.
- Values of y⁺ lower that 0.1 will not give you large improvement.
- And as a matter of fact, pushing the mesh to values of y⁺ below 0.1 can results in low quality meshes for industrial applications.

Final remarks

- As for the wall modeling approach, in the wall resolving treatment you need to cluster enough cells to resolve the viscous sublayer profiles (velocity, temperature, turbulence quantities, and so on).
- It is recommended to use at least 15 boundary layer cells with a low expansion ratio (1.15 or less) to properly resolve the profiles.
- No need to mention it, but hexahedral cells are preferred over any other type of cells in the boundary layer region.
- Do not use mesh refinement with standard wall functions as the solution tends to deteriorate.
- The use of wall functions limits the grid resolution of the boundary layer for low to moderate Reynolds number.
- The absolute minimum of boundary layer cells when using wall functions is five.
- Avoid as much as possible to put your first cell center in the buffer layer.

2D Zero pressure gradient flat plate



2D Zero pressure gradient flat plate



• Velocity profile at the sampling location.

2D Zero pressure gradient flat plate



• Velocity profile at the sampling location – Detailed view.

2D Zero pressure gradient flat plate



• Cell center clustering toward the walls.

2D Zero pressure gradient flat plate



Non-dimensional velocity profiles.

2D Zero pressure gradient flat plate



• P_k and E_k profiles.

2D Zero pressure gradient flat plate

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• Mesh comparison – Wall resolving mesh vs. Wall modeling mesh.

2D Zero pressure gradient flat plate

0.00	0.25	Velocity magnitude (m/s) 0.50	0.75	1.00	0.00	0.25	Velocity magnitude (m/s) 0.50	0.75	1.00
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Wall resolving mesh.

Wall modeling mesh.

• Plot of velocity magnitude contours.

2D Zero pressure gradient flat plate



Wall resolving mesh – k = 0 at the wall

Wall modeling mesh – $\frac{\partial k}{\partial n} = 0$ at the wall

• Plot of turbulent kinetic energy contours.

Roadmap to Lecture 6

Part 4

- **1. Near wall treatment**
- 2. Incomplete list of turbulence models and references

- The following is an incomplete list of RANS/URANS/RSM turbulence models.
- Have in mind that some of the models have many variants.
 - Cebeci-Smith.
 - Baldwin-Lomax.
 - Johnson-King.
 - Bradshaw-Ferris-Atwell.
 - L-VEL.
 - Prandtl mixing length.
 - Van-driest mixing length.
 - Prandtl one equation.
 - Nee-Kovasznay.
 - Baldwin-Barth.
 - Spalart-Allmaras.
 - Secundov Nut-92.
 - Wolfshtein.

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 - Norris-Reynolds.
 - Wray-Agarwal.
 - Rotta k-kl.
 - Standard K-Epsilon.
 - RNG K-Epsilon.
 - Realizable K-Epsilon.
 - Myong-Kasagi K-Epsilon.
 - Launder-Sharme K-Epsilon.
 - Lam-Bremhorst K-Epsilon.
 - Jones-Launder K-Epsilon.
 - Chien K-Epsilon.
 - Lien-Leschziner K-Epsilon.

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 - Speziale K-Epsilon.
 - Rubinstein-Barton.
 - Gatski-Speziale.
 - Lien-Chien-Leschziner.
 - Apsley-Leschziner.
 - Saffman-Spalding k-Omega.
 - Kolmogorov 1942 K-Omega.
 - Wilcox 1988 K-Omega.
 - Wilcox 1998 K-Omega.
 - Wilcox 2006 K-Omega.
 - Menter 2003 K-Omega SST.
 - Langtry-Menter K-Omega.

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- Have in mind that some of the models have many variants.
 - K-e-Rt.
 - K-e-zeta-F.
 - Q-Zeta.
 - Pope EARSM.
 - Walin-Johansson EARSM.
 - Mishra-Girimaji.
 - Wilcox RSM.
 - LRR RSM.
 - SSG RSM.
 - GLVY RSM.
 - Craft cubic model.

- The following is an incomplete list of RANS/URANS/RSM turbulence models.
- Have in mind that some of the models have many variants.
 - Gibson-Launder.
 - Craft-Launder.
 - Shima.
 - V2-f
 - Gamma-Re-Theta.
 - LCTM.
 - K-KI-Omega.
 - Transition SST.
 - GEKO.