Turbulence and CFD models: Theory and applications

Roadmap to Lecture 2

1. The turbulent world around us
2. Turbulence, does it matter?
3. Introduction to turbulence modeling – Basic concepts
4. Wall bounded flows and shear flows
5. A peek to the turbulence closure problem, some correlations in turbulence modeling, and the energy cascade

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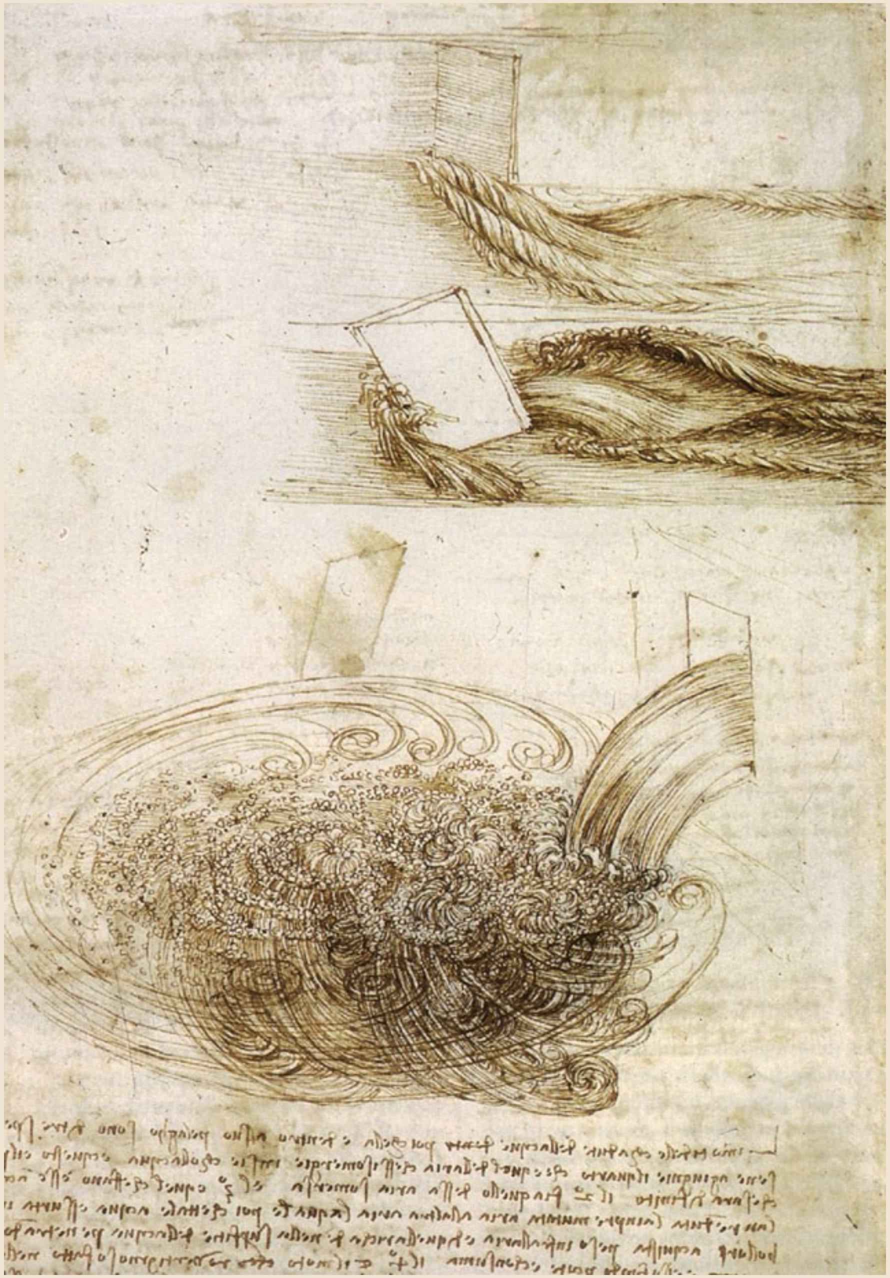
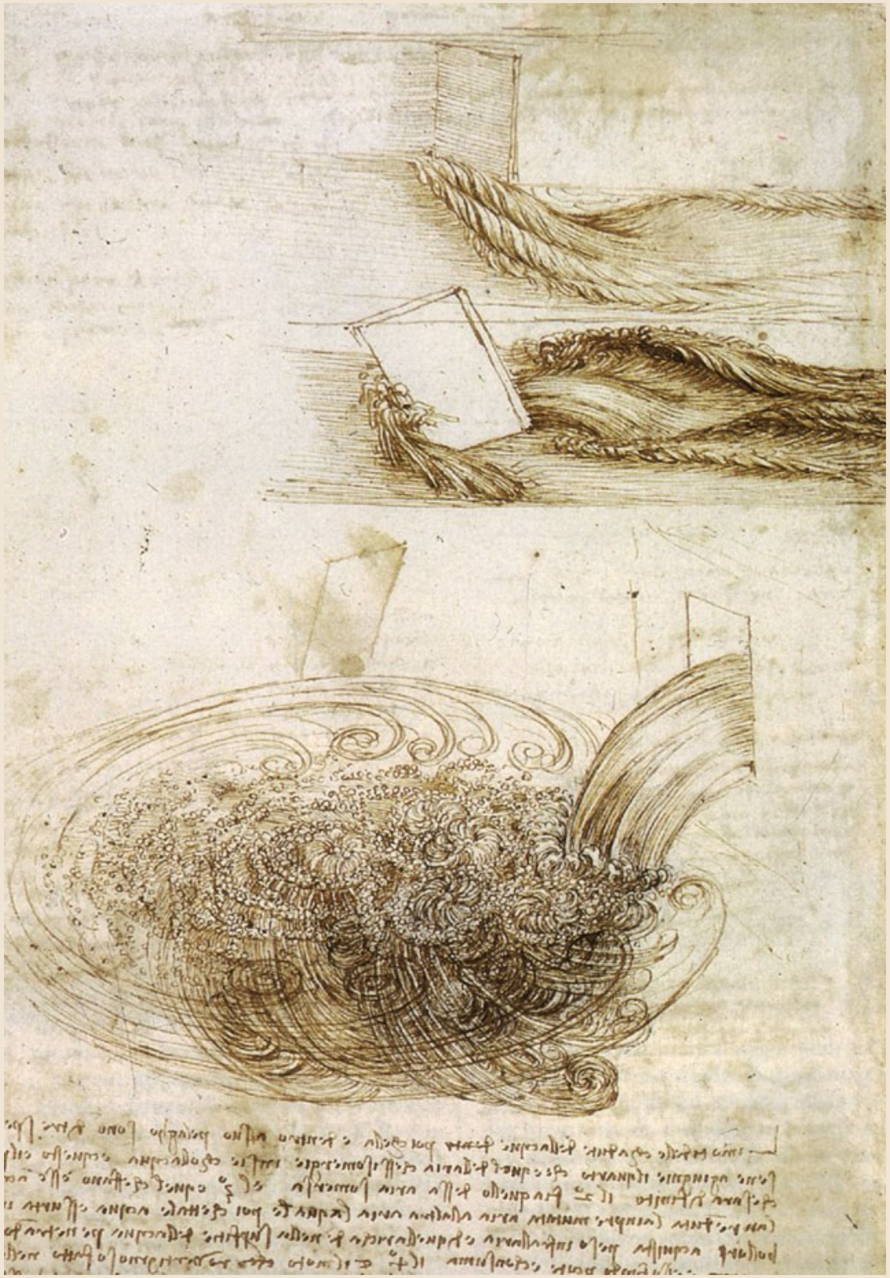


Leonardo da Vinci pioneered the flow visualization genre about 500 years ago. The illustrations to the left (*Studies of water passing obstacles and falling* c. 1508-1509) represents perhaps the world's first use of visualization as a scientific tool to study a turbulent flow.

The following da Vinci’s observation is close to the Reynold’s decomposition.

> ***“Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to the random and reverse motion.”***



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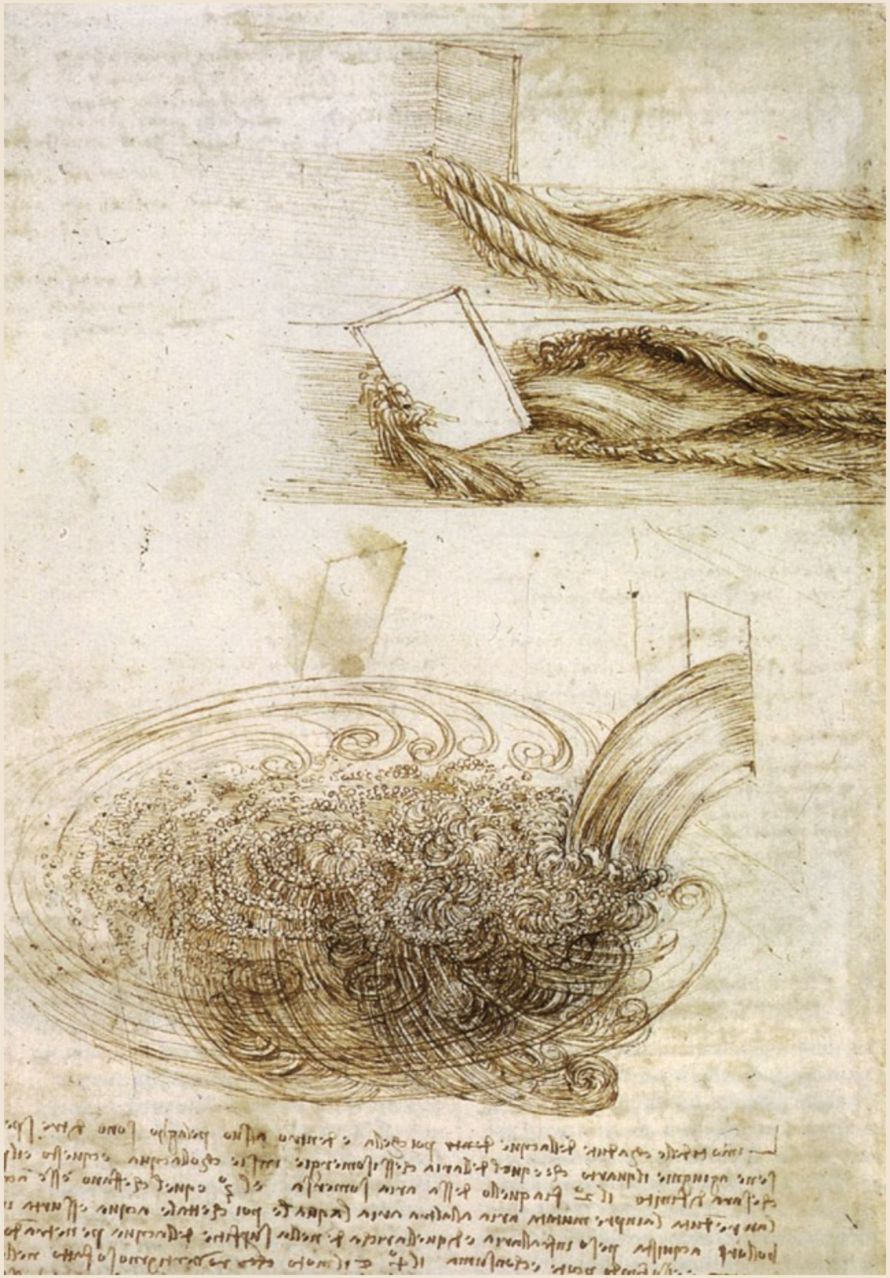
The following da Vinci's description is maybe the earliest reference to the importance of vortices in fluid motion.

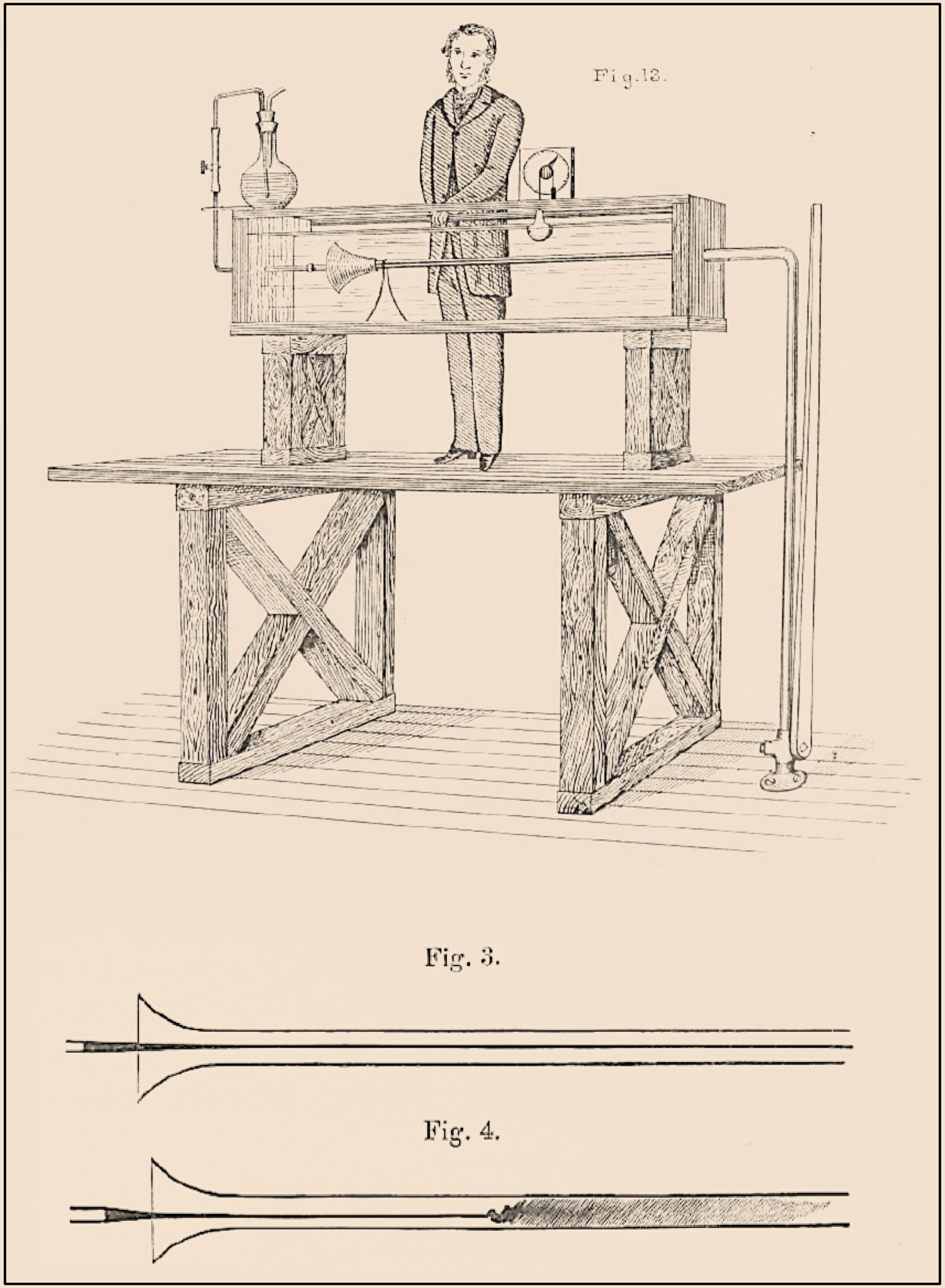
> ***"So moving water strives to maintain the course pursuant to the power which occasions it and, if it finds an obstacle in its path, completes the span of the course it has commenced by a circular and revolving movement."***



Leonardo da Vinci pioneered the flow visualization genre about 500 years ago. The illustrations to the left (*Studies of water passing obstacles and falling* c. 1508-1509) represents perhaps the world's first use of visualization as a scientific tool to study a turbulent flow.

The following da Vinci’s observation is an analogy to the energy cascade and coherent structures.

> ***“…the smallest eddies are almost numberless, and large things are rotated only by large eddies and not by small ones and small things are turned by small eddies and large”***



In 1883, Osborne Reynolds published his famous paper entitled,

> ***"An Experimental Investigation of the Circumstances which determine whether Motion of Water shall be Direct or Sinuous and of the Law of Resistance in Parallel Channels."***

This work had a profound effect on the development of fluid mechanics and hydrodynamic linear stability theory.

In this work, Reynolds introduces a dimensionless group, what we call today the Reynolds number.

> ***"It seemed, however, to be certain, if the eddies were owing to one particular cause, that integration would show the birth of eddies to depend upon some definite value of–"***

$$\frac{c\rho \mathrm{U}}{\mu}$$

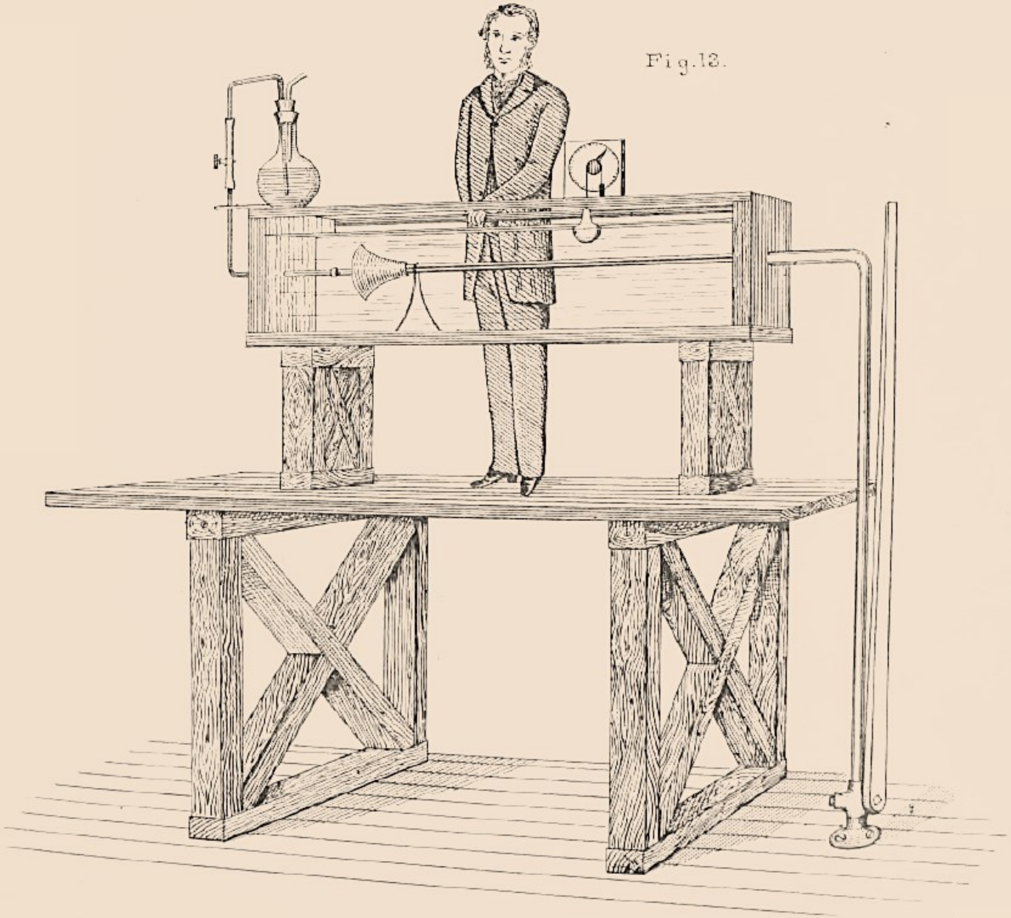


Fig.13.

Fig. 3.

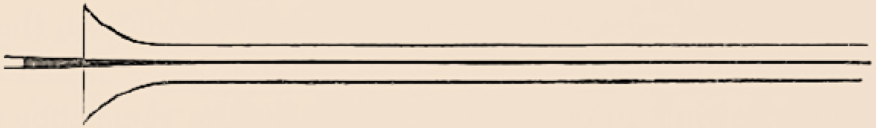


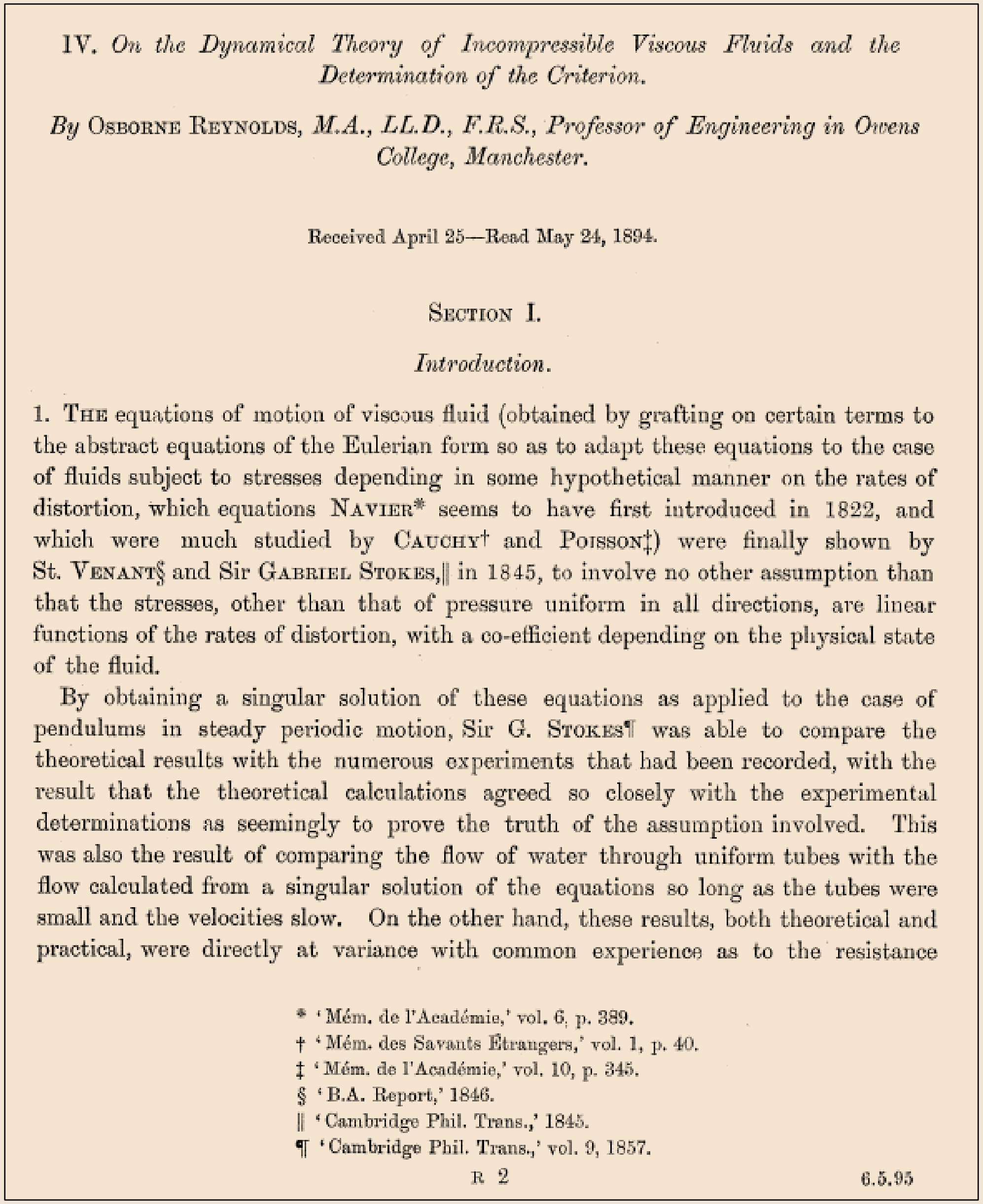
Fig. 4.

In his 1883 paper, Reynolds also writes,

> ***"Again, the internal motion of water assumes one or other of two broadly distinguishable forms – either the elements of the fluid follow one another along lines of motion which lead in the most direct manner to their destination, or they eddy about in sinuous paths, the most indirect possible."***

In present-day terminology we call direct motion laminar. This is a state in which the behavior of a fluid is very regular.

The sinuous paths Reynolds refers to are not sinusoidal (periodic) motions. Instead, they are the ones having many curves and turns, being very irregular. We refer to these flows as turbulent.



IV. *On the Dynamical Theory of Incompressible Viscous Fluids and the Determination of the Criterion.*

By Osborne Reynolds, *M.A., LL.D., F.R.S., Professor of Engineering in Owens College, Manchester.*

Received April 25—Read May 24, 1894.

Section I.

Introduction.

1. The equations of motion of viscous fluid (obtained by grafting on certain terms to the abstract equations of the Eulerian form so as to adapt these equations to the case of fluids subject to stresses depending in some hypothetical manner on the rates of distortion, which equations Navier[*] seems to have first introduced in 1822, and which were much studied by Cauchy[†] and Poisson[‡]) were finally shown by St. Venant[§] and Sir Gabriel Stokes,[‖] in 1845, to involve no other assumption than that the stresses, other than that of pressure uniform in all directions, are linear functions of the rates of distortion, with a co-efficient depending on the physical state of the fluid.

By obtaining a singular solution of these equations as applied to the case of pendulums in steady periodic motion, Sir G. Stokes[¶] was able to compare the theoretical results with the numerous experiments that had been recorded, with the result that the theoretical calculations agreed so closely with the experimental determinations as seemingly to prove the truth of the assumption involved. This was also the result of comparing the flow of water through uniform tubes with the flow calculated from a singular solution of the equations so long as the tubes were small and the velocities slow. On the other hand, these results, both theoretical and practical, were directly at variance with common experience as to the resistance

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[*] 'Mém. de l'Académie,' vol. 6, p. 389.

[†] 'Mém. des Savants Étrangers,' vol. 1, p. 40.

[‡] 'Mém. de l'Académie,' vol. 10, p. 345.

[§] 'B.A. Report,' 1846.

[‖] 'Cambridge Phil. Trans.,' 1845.

[¶] 'Cambridge Phil. Trans.,' vol. 9, 1857.

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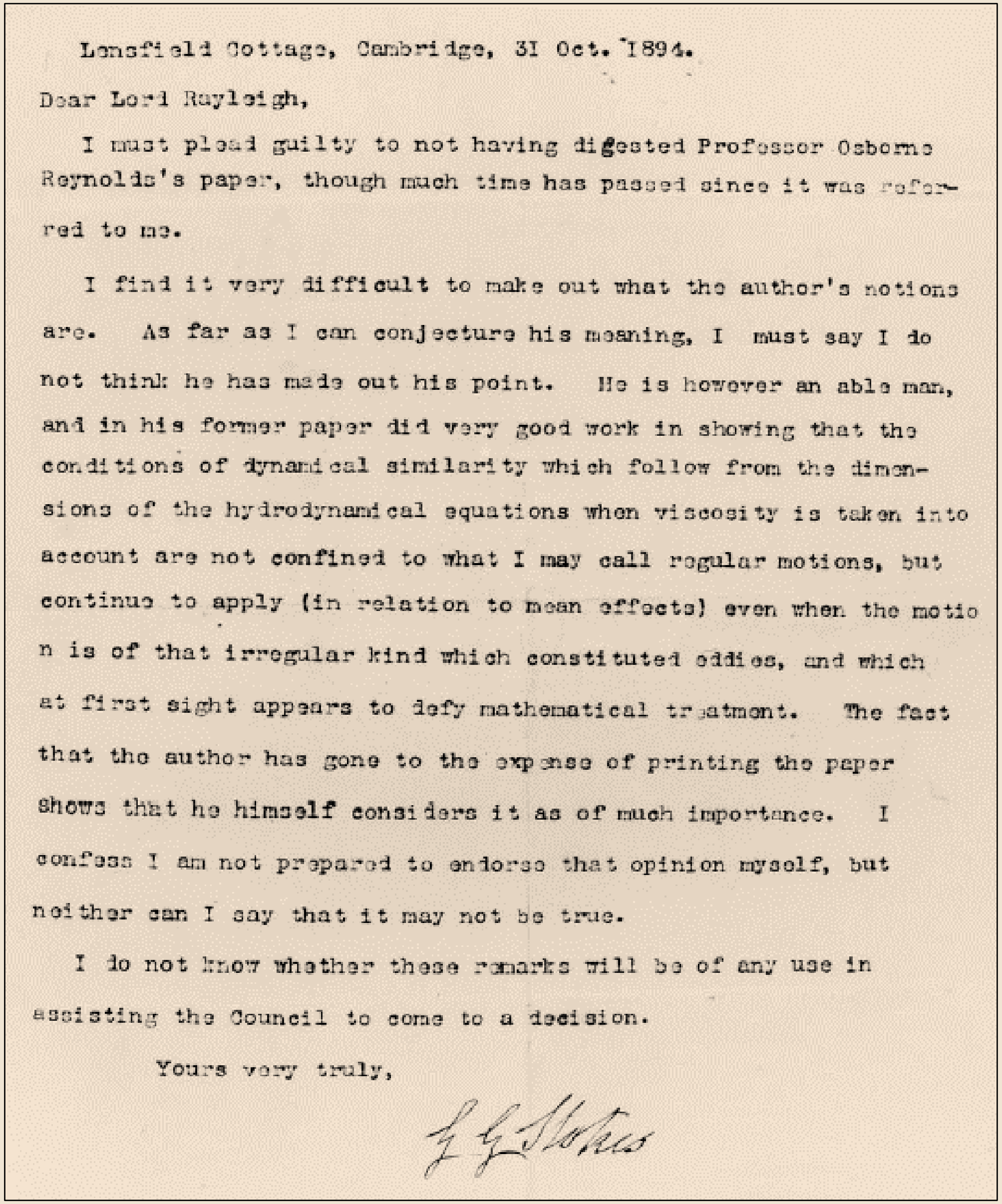
In a follow up paper published 1895 and titled,

> ***"On the Dynamical Theory of Incompressible Viscous Fluids and the Determination of the Criterion."***

Reynolds proposed to take the Navier-Stokes equations as a starting point to derive equations for the evolution of the mean (or average) velocity field. Furthermore, he obtained equations for the evolution of the kinetic energy contained in the mean velocity field, and for the energy contained in the small fluctuation motions.

Without a doubt, the most important results of that paper is the procedure in which the velocity field is decomposed into a mean and a fluctuating part.

This procedure is now referred to as Reynolds averaging and it constitutes the basis for modern simulation techniques like RANS and LES models.



Lensfield Cottage, Cambridge, 31 Oct. 1894.

Dear Lord Rayleigh,

I must plead guilty to not having digested Professor Osborne Reynolds's paper, though much time has passed since it was referred to me.

I find it very difficult to make out what the author's notions are. As far as I can conjecture his meaning, I must say I do not think he has made out his point. He is however an able man, and in his former paper did very good work in showing that the conditions of dynamical similarity which follow from the dimensions of the hydrodynamical equations when viscosity is taken into account are not confined to what I may call regular motions, but continue to apply (in relation to mean effects) even when the motion is of that irregular kind which constituted eddies, and which at first sight appears to defy mathematical treatment. The fact that the author has gone to the expense of printing the paper shows that he himself considers it as of much importance. I confess I am not prepared to endorse that opinion myself, but neither can I say that it may not be true.

I do not know whether these remarks will be of any use in assisting the Council to come to a decision.

Yours very truly,

G G Stokes

It is interesting to mention that Reynolds 1895 paper received a hard review from Sir George Gabriel Stokes, one of scientist reviewing the paper.

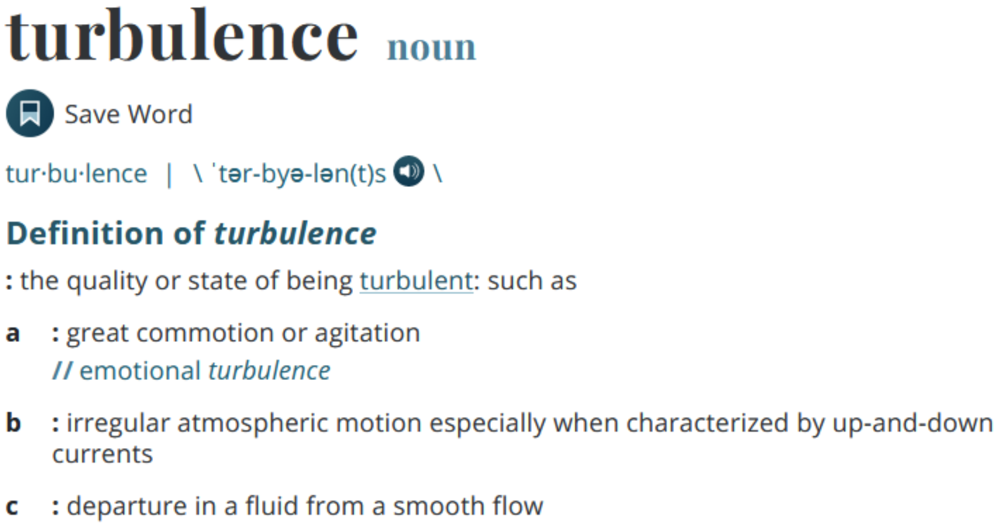
> ***"I must plead guilty to not having digested Professor Osborne Reynolds's paper, though much time has passed since it was referred to me."***
>
> ***"I find it very difficult to make out what the author's notions are. As far as I can conjecture his meaning, I must say I do not think he has made out his point."***

Despite this review, Reynolds' paper was published, and it is now seen as a landmark contribution to the field of fluid mechanics, and particularly to the research relating to the behavior of turbulent flows.

The turbulent world around us

What is turbulence?

- According to the Merriam-Webster dictionary, turbulence is defined as,



turbulence noun

Save Word

tur·bu·lence | \ ˈtər-byə-lən(t)s \

Definition of *turbulence*

: the quality or state of being turbulent: such as

a **:** great commotion or agitation
// emotional *turbulence*

b **:** irregular atmospheric motion especially when characterized by up-and-down currents

c **:** departure in a fluid from a smooth flow

- Similar definitions follow in different languages.
- Note that this turbulence definition is too general.
- Moving to technical grounds, namely, fluid dynamics, the use of the word **turbulent** to characterize a certain type of flow, namely, the counterpart of a streamline motion, smooth flow, or a laminar flow, is comparatively recent.

The turbulent world around us

What is turbulence?

- Not even Osborne Reynolds used the word turbulent in his two principal papers on turbulent flows,
    - O. Reynolds (1883). “An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels.”
    - O. Reynolds (1895). “On the dynamical theory of incompressible viscous fluids and the determination of the criterion.”
- On these two papers, Osborne Reynolds, defined turbulence (or turbulent flows) as eddying or sinuous motion (more on this later).
- Both papers were published in the Philosophical Transactions of the Royal Society, and essentially provided a marker for the direction of research in Engineering Fluid Mechanics for the next century.
- In fluid dynamics we prefer to use the terminology **turbulent flows** instead of **turbulence**.
- However, both terms are valid when dealing at a technical level.
- Hereafter, we will use both terminologies interchangeably.

What is turbulence?

- Due to the extremely complex nature of turbulence and its incomplete understanding, there is not a single accepted definition of turbulence.
    - Leonardo da Vinci was so intrigued by turbulence that he depicted it in many of his sketches (see previous slides).
    - While observing the flow of water, he gave one of the very first definitions of turbulence (if not the first one), this was more than 500 years ago.

> *"…the smallest eddies are almost numberless, and large things are rotated only by large eddies and not by small ones and small things are turned by small eddies and large"*

The turbulent world around us

What is turbulence?

- Due to the extremely complex nature of turbulence and its incomplete understanding, there is not a single accepted definition of turbulence.
  - Osborne Reynolds, one of the pioneers in the study of turbulent flows, in 1883, defined turbulent flows as eddying or sinuous motion [1].

> *"Again, the internal motion of water assumes one or other of two broadly distinguishable forms – either the elements of the fluid follow one another along lines of motion which lead in the most direct manner to their destination, or they eddy about in sinuous paths, the most indirect possible."*

> *"The small evidence which clear water shows as to the existences of internal eddies, not less than the difficulty of estimating the viscous nature of the fluid, appears to have hitherto obscured the very important circumstance that the more viscous a fluid is, the less prone is it to eddying or sinuous motion.*

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[1] O. Reynolds. "An Experimental Investigation of the Circumstances which determine whether Motion of Water shall be Direct or Sinuous and of the Law of Resistance in Parallel Channels." Philos. Trans. R. Soc. 174:935–82. 1883.

The turbulent world around us

What is turbulence?

- Due to the extremely complex nature of turbulence and its incomplete understanding, there is not a single accepted definition of turbulence.
  - Richardson, in 1922, introduces he idea of an energy cascade in turbulent flows [1],

> *"Big whorls have little whorls,*
> *which feed on their velocity,*
> *and little whorls have lesser whorls,*
> *and so on to viscosity"*

---

[1] L. F. Richardson. "Weather Prediction by Numerical Process". Cambridge University Press, 1922.

The turbulent world around us

What is turbulence?

- Due to the extremely complex nature of turbulence and its incomplete understanding, there is not a single accepted definition of turbulence.
  - T. von Karman who is known for his studies about Fluid Dynamics, quotes G. I. Taylor with the following definition of turbulence in 1937 [1],

> *"Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces or even when neighboring streams same fluid past or over one another."*

[1] T. Von Karman. "Some remarks on the statistical theory of turbulence". Proc. 5th Int. Congr. Appl. Mech, Cambridge, MA, 347, 1938.

The turbulent world around us

What is turbulence?

- Due to the extremely complex nature of turbulence and its incomplete understanding, there is not a single accepted definition of turbulence.
  - J.O. Heinz offers yet another definition for turbulence in 1959 [1],

> *"Turbulent fluid motion is an irregular condition of the flow in which quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned."*

[1] J. O. Hinze. "Turbulence". McGraw-Hill, New York, 1959.

What is turbulence?

- Due to the extremely complex nature of turbulence and its incomplete understanding, there is not a single accepted definition of turbulence.
  - Monin and Yaglom [1], in 1971, gave another definition of laminar and turbulent flows.
  - Notice that at this point, random variations in space and time are becoming a common in all definitions.

> *"It is known that all flows of liquids and gases may be divided into two sharply different types; the quiet smooth flows known as "laminar" flows, and their opposite, "turbulent" flows in which the velocity, pressure, temperature and other fluid mechanical quantities fluctuate in a disordered manner with extremely sharp and irregular space- and time-variations."*

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[1] A. S. Monin and A. M. Yaglom. Statistical Fluid Mechanics: The Mechanics of Turbulence, volume 1. M. I. T. Press, 1971.

The turbulent world around us

What is turbulence?

- Due to the extremely complex nature of turbulence and its incomplete understanding, there is not a single accepted definition of turbulence.
  - A more modern (in 1985), and highly specific definition of turbulence is given by G. T. Chapman and M. Tobak [1],

> *“Turbulence is any chaotic solution to the 3D Navier–Stokes equations that is sensitive to initial data and which occurs as a result of successive instabilities of laminar flows as a bifurcation parameter is increased through a succession of values.”*

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[1] G. T. Chapman and M. Tobak. “Observations, Theoretical Ideas, and Modeling of Turbulent Flows — Past, Present and Future, in Theoretical Approaches to Turbulence”. Dwoyeret al.(eds), Springer-Verlag, New York, pp. 19–49, 1985.

The turbulent world around us

What is turbulence?

- Due to the extremely complex nature of turbulence and its incomplete understanding, there is not a single accepted definition of turbulence.
  - S. Rodriguez, in 2019, gives an even more modern definition linked to the use of numerical approximations (and therefore computers), to deliver approximated solutions [1],

> *"Turbulent flows is the dynamic superposition of an extremely large number of eddies with random (irregular) but continuous spectrum of sizes and velocities that are interspersed with small, discrete pockets of laminar flow (as a result of the Kolmogorov eddies that decayed, as well as in the viscous laminar sublayer and in the intermittent boundary). In this sense, turbulent flows are intractable in its fullest manifestation; this is where good, engineering common sense and approximations can deliver reasonable solutions, albeit approximate."*

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[1] S. Rodriguez. "Applied Computational Fluid Dynamics and Turbulence modeling". Springer, 2019.

The turbulent world around us

What is turbulence?

- Due to its complexity, a definition does not work properly for turbulence, instead of it, it’s better to explain its characteristics.
- Tennekes and Lumley [1] in their book called “*A First Course in Turbulence”,* list the characteristics of turbulence:
    - Irregularity.
    - Diffusivity.
    - Large Reynolds numbers.
    - Three-Dimensional vorticity fluctuations.
    - Dissipation.
    - Continuum.
    - Feature of a flow, not fluid.

- These characteristics represent the fingerprint of turbulent motion

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[1] H. Tennekes and J. L. Lumley. “A first course in turbulence”. MIT Press, 1972.

The turbulent world around us

Turbulent flows have the following characteristics

- **Irregularity.** One characteristic of turbulent flows is their irregularity (or randomness). A fully deterministic approach to characterize turbulent flows is very difficult, if not impossible. Turbulent flows are usually described statically.

Photo credit: https://i.pinimg.com/originals/c7/8a/9b/c78a9b3bdc1e4689c371a8222f178fa2.jpg

Reference: H. Tennekes and J. L. Lumley. “A first course in turbulence”. MIT Press, 1972.

The turbulent world around us

Turbulent flows have the following characteristics

- **Diffusivity.** The diffusivity of turbulence causes rapid mixing and increased rates of momentum, heat, and mass transfer. A flow that looks random but does not exhibit the spreading of velocity fluctuations through the surrounding fluid is not turbulent.

Photo credit: https://flic.kr/p/BZumA

Reference: H. Tennekes and J. L. Lumley. “A first course in turbulence”. MIT Press, 1972.

The turbulent world around us

Turbulent flows have the following characteristics

- **Large Reynolds number.** Turbulent flows always occur at high Reynolds numbers. They are caused by a complex interaction between the viscous terms and nonlinear terms in the equations of motion. Randomness and nonlinearity combine to make the equations of turbulence nearly intractable.

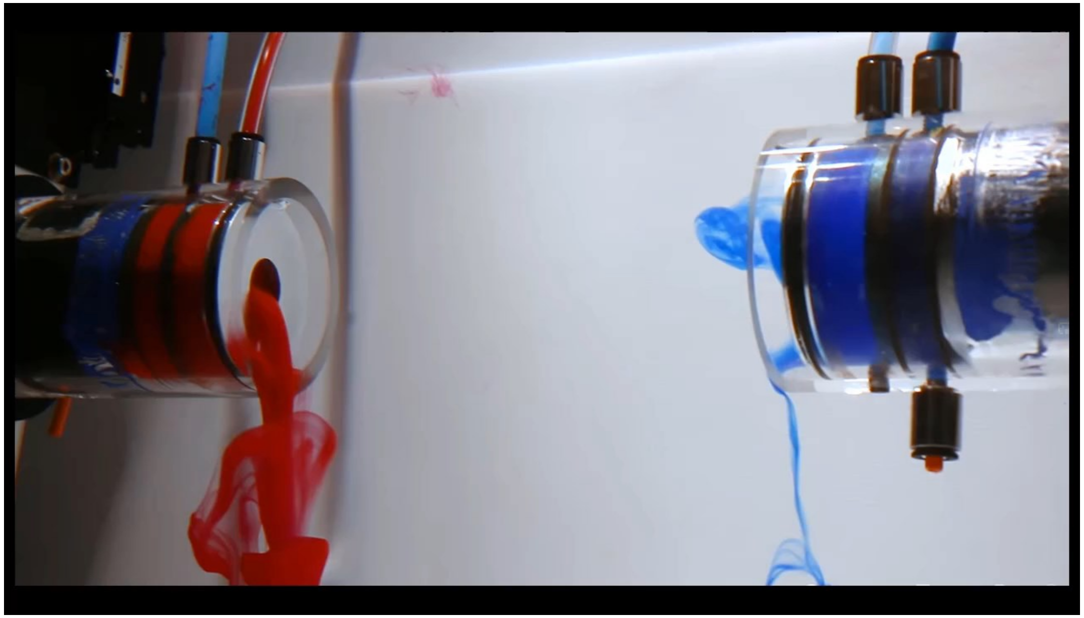
Photo credit: https://www.pinterest.it/pin/763993524266658708/

Reference: H. Tennekes and J. L. Lumley. “A first course in turbulence”. MIT Press, 1972.

The turbulent world around us

Turbulent flows have the following characteristics

- **Three-Dimensional vorticity fluctuations.** Turbulence is rotational and three dimensional. Turbulence always exhibit high levels of fluctuating vorticity. The random vorticity fluctuations that characterize turbulence could not maintain themselves if the velocity fluctuations were two dimensional. Mechanisms such as the stretching of three-dimensional vortices play a key role in turbulence.



Video credit: https://www.youtube.com/watch?v=EVbdbVhzcM4&t. This video has been edited to fit this presentation.
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Reference: H. Tennekes and J. L. Lumley. “A first course in turbulence”. MIT Press, 1972.

The turbulent world around us

Turbulent flows have the following characteristics

- **Dissipation.** Turbulent flows are dissipative. Kinetic energy gets converted into heat due to viscous shear stresses. Turbulent flows die out quickly when no energy is supplied.

Photo credit: https://flic.kr/p/5qv1Bs

Reference: H. Tennekes and J. L. Lumley. “A first course in turbulence”. MIT Press, 1972.

The turbulent world around us

Turbulent flows have the following characteristics

- **Continuum.** Turbulence is a continuum phenomenon. Even the smallest eddies are significantly larger than the molecular scales.

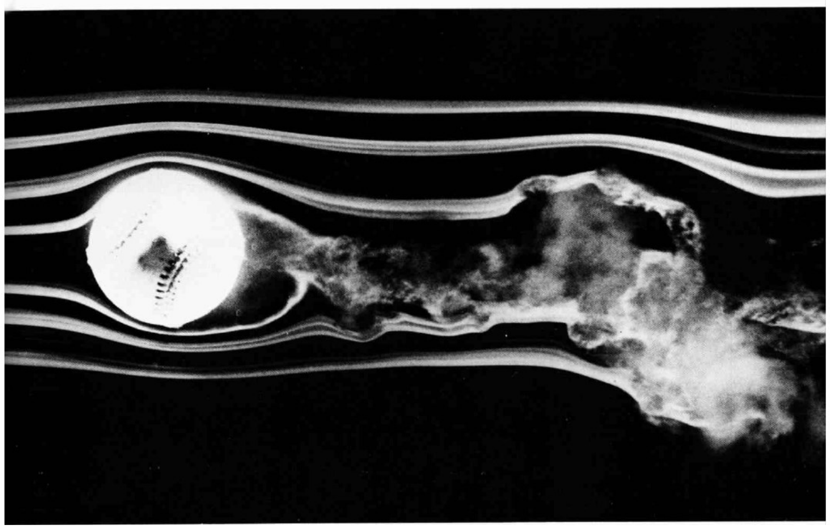
Photo credit: Sid Balachandran on Unsplash

Reference: H. Tennekes and J. L. Lumley. “A first course in turbulence”. MIT Press, 1972.

The turbulent world around us

Turbulent flows have the following characteristics

- **Feature of a flow, not fluid.** Turbulence is a feature of fluid flow and is not a property of the flow. A liquid or a gas at high Reynolds number will exhibit the same dynamics.



Air flow over a spinning baseball.
M. Van Dyke. An album of fluid motion.
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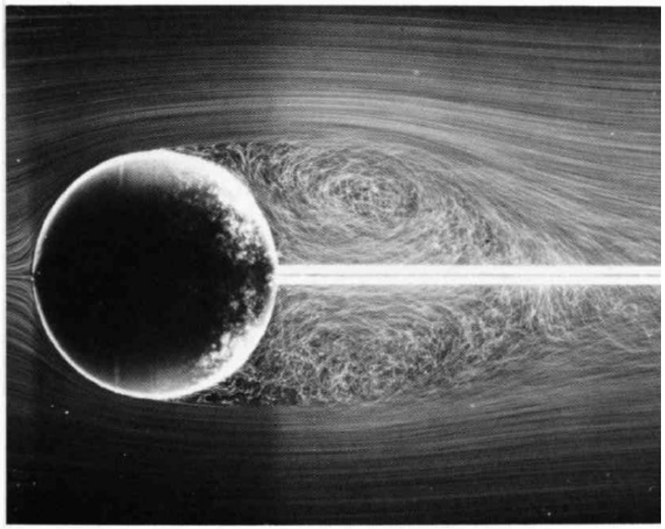

Water flow over a sphere.
M. Van Dyke. An album of fluid motion.
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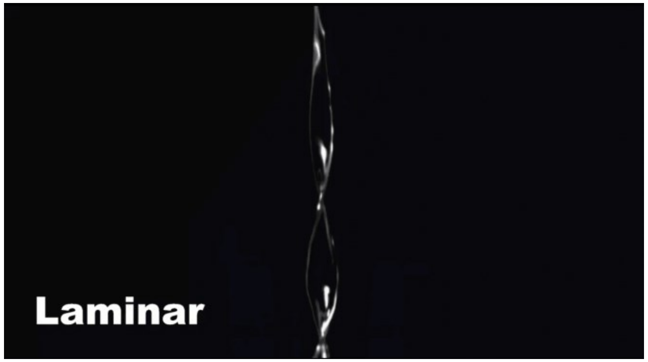
Reference: H. Tennekes and J. L. Lumley. “A first course in turbulence”. MIT Press, 1972.

Turbulent flows have the following characteristics

- In summary:
  - One characteristic of turbulent flows is their **irregularity** (or randomness). A fully deterministic approach to characterize turbulent flows is very difficult. Turbulent flows are usually described statically. Turbulent flows are always chaotic. But not all chaotic flows are turbulent. Magma flowing can be chaotic but not necessarily turbulent.
  - The **diffusivity** of turbulence causes rapid mixing and increased rates of momentum, heat, and mass transfer. A flow that looks random but does not exhibit the spreading of velocity fluctuations through the surrounding fluid is not turbulent.
  - Turbulent flows are **dissipative**. Kinetic energy gets converted into heat due to viscous shear stresses. Turbulent flows die out quickly when no energy is supplied.
  - Turbulent flows always occur at **high Reynolds numbers**. They are caused by a complex interaction between the viscous forces and convection.
  - Turbulent flows are **rotational**, that is, they have non-zero vorticity. Mechanisms such as the stretching of three-dimensional vortices play a key role in turbulence.
  - Turbulence is a **continuum** phenomenon. Even the smallest eddies are significantly larger than the molecular scales.
  - Turbulence is a **feature of fluid flow** and is not a property of the flow. A liquid or a gas at high Reynolds number will exhibit the same dynamics.



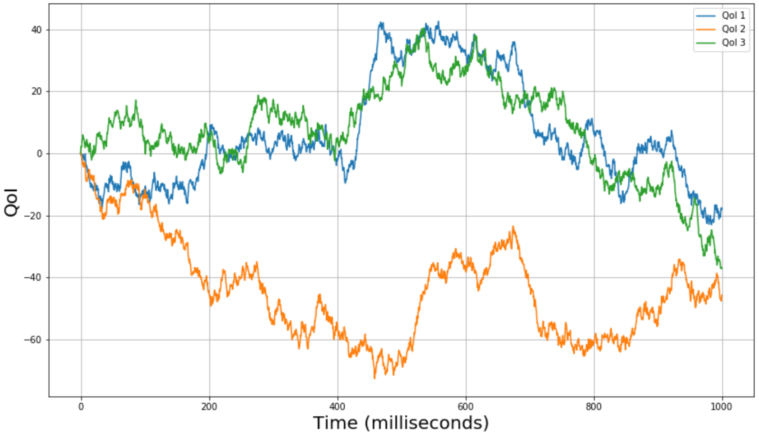
Video credit: https://www.youtube.com/watch?v=9dz7fUJBFOk
This video has been edited to fit this presentation.

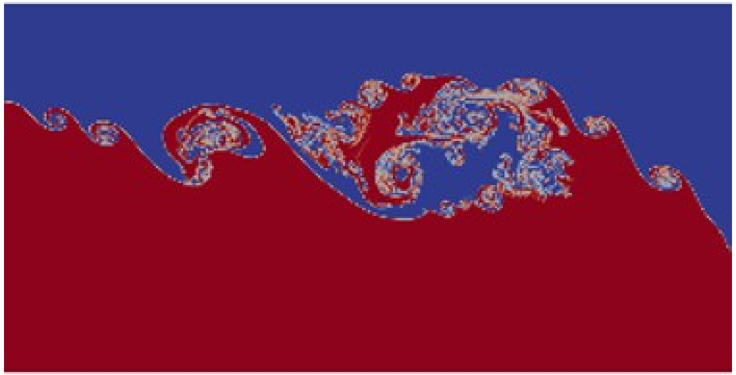
Reference: H. Tennekes and J. L. Lumley. “A first course in turbulence”. MIT Press, 1972.

The turbulent world around us

… so, after all, what is turbulence?

- Turbulent flows share all the previous characteristics.
- In addition, let us state the following:
  - Turbulence is an unsteady, aperiodic motion in which all three velocity components fluctuate in space and time.
  - Every transported quantity shows similar fluctuations (pressure, temperature, species, concentration, and so on)
  - Turbulent flows contains a wide range of eddy sizes (scales):
    - Large eddies derives their energy from the mean flow. The size and velocity of large eddies are on the order of the mean flow.
    - Large eddies are unstable and they break-up into smaller eddies.
    - The smallest eddies convert kinetic energy into thermal energy via viscous dissipation.
    - The behavior of small eddies is more universal in nature.





The turbulent world around us

Turbulence and computers

- In 1949 von Neumann predicted that the advent of digital computers would revolutionize the study of turbulence since it would become possible to simulate the Navier-Stokes equations in 3D in turbulent regimes.
- Von Neumann, convinced of the important role of computers for the study of turbulence made the following observations [1],

> *"… a considerable mathematical effort towards a detailed understanding of the mechanism of turbulence is called for …"*

- But that, given the analytic difficulties presented by the turbulence problem,

> *" … there might be some hope to 'break the deadlock' by extensive, but well-planned, computational efforts."*

- It took over 20 years until von Neumann's vision came to reality with the first genuine 3D simulations of turbulence conducted by Orszag and Patterson [2].

[1] J. von Neumann. 1963. Recent Theories of Turbulence. In Collected Works (1949–1963), Vol. 6, p. 437. Edited by A. H. Taub. Oxford, UK: Pergamon Press.
[2] S. A. Orszag and G. S. Patterson, Jr. Numerical Simulation of Three-Dimensional Homogeneous Isotropic Turbulence. Phys. Rev. Lett. 28, 76 – Published 10 January 1972.

Turbulence and computers

- Numerical simulations and the ability to analyze large ensemble of experimental data using computers have revolutionized our understanding of turbulent flows [1].

> *"From a fundamental perspective, the direct numerical simulation of idealized isotropic, homogeneous turbulence has been revolutionary in its impact on turbulence research because of the ability to simulate and display the full 3-D velocity field at increasingly large Reynolds number."*

> *"Similarly, experimentation on turbulence has advanced tremendously by using computer data acquisition; 20 years ago it was possible to measure and analyze time series data from single-point probes that totaled no more than 10 megabytes of information, whereas today statistical ensembles of thousands of spatially and temporally resolved velocity fields, taking 10 terabytes of storage space can be obtained and processed."*

[1] R. Ecke. The Turbulence Problem An Experimentalist's Perspective. Los Alamos Science Number 29, 2005.

The turbulent world around us

Turbulence and computers

- The numerical and experimental possibilities in turbulence research can only be enhanced even further by expected future increases in computational power.
- From an academic and research point of view, computers are used to improve the theoretical understanding of turbulent flows, developed new models, validate existent models, and analyze numerical and experimental data.
- From the industrial point of view, computers (and CFD models) have made accessible complex physics and large-scale simulations in relatively affordable times.
- Computers are an invaluable tool in the turbulence research field.
  - But, before conducting numerical simulations, we must understand the basis and apply this knowledge to reduce the uncertainties associated with turbulence models.
  - And this is the goal of this course.
    - Guide you, engineers, in computing turbulent flows.
  - We are going to elaborate on the theory.
  - And then we are going to confirm these observations using turbulence models or by directly resolving the governing equations (no turbulence models) under given conditions.

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[1] S. A. Orszag and G. S. Patterson, Jr. Numerical Simulation of Three-Dimensional Homogeneous Isotropic Turbulence. Phys. Rev. Lett. 28, 76 – Published 10 January 1972.

The turbulent world around us

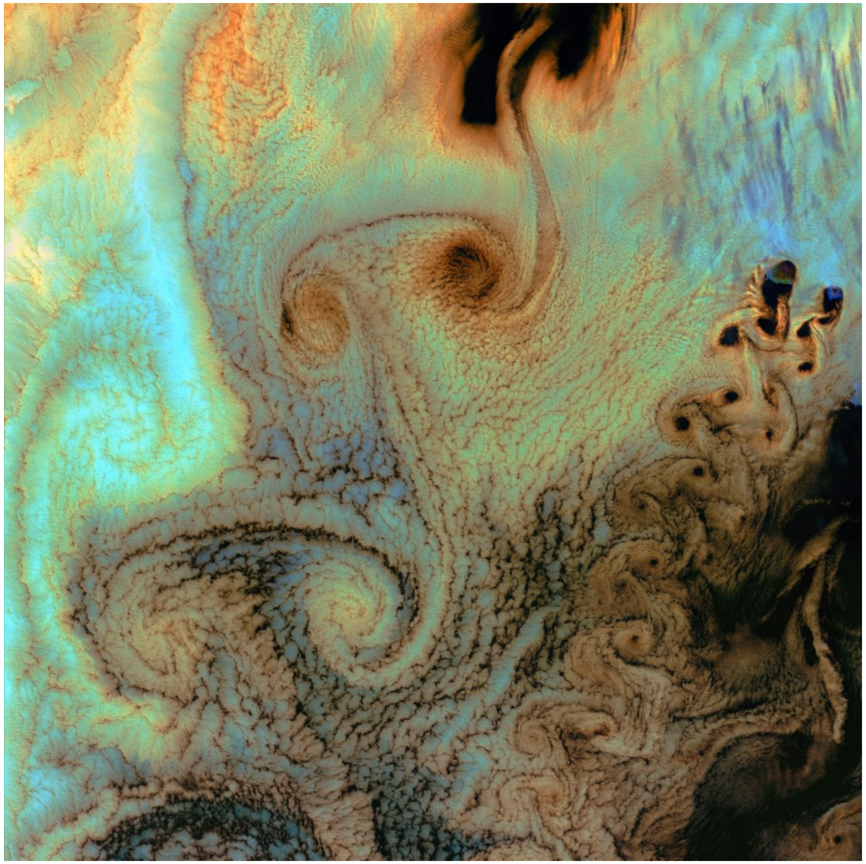
Before continuing, let me share a few more amazing images that show the beauty and complexity of turbulence in nature and engineering applications.

The turbulent world around us

Buoyant plume of smoke rising from a stick of incense

Photo credit: https://www.flickr.com/photos/jlhopgood/

The turbulent world around us



Von Karman vortices created when prevailing winds sweeping east across the northern Pacific Ocean encountered Alaska's Aleutian Islands
Photo credit: USGS EROS Data Center Satellite Systems Branch.
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Von Karman Vortex Streets in the northern Pacific Photographed from the International Space Station
Photo credit: NASA
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The turbulent world around us

Cirrus clouds - Kelvin-Helmholtz instability
Photo credit: https://www.pinterest.it/pin/8514686771867411/
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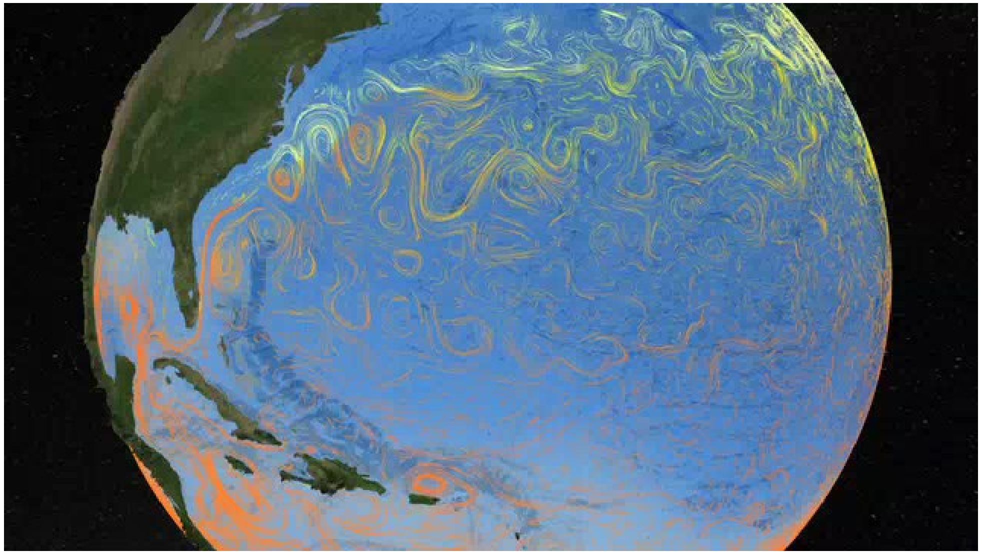

NASA Aquarius mission - Studies of ocean and wind flows

The Aquarius mission measured the salinity in the ocean, giving scientists the tools needed to improve predictions of future climate trends and events. Aquarius salinity data, combined with data from other sensors that measure sea level, rainfall, temperature, ocean color, and winds, gave us a much clearer picture of how the ocean works. Will higher temperatures intensify evaporation and alter sea surface salinity patterns? Will changes in salinity affect ocean circulation and how heat is distributed over the globe? Aquarius measurements provide a new perspective on the ocean, how it is linked to climate, and how it will respond to climate change

Video credit: https://svs.gsfc.nasa.gov/cgi-bin/details.cgi?aid=3829
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The turbulent world around us

Turbulent waters
Photo credit: https://www.flickr.com/photos/thepaegan
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Confluence of the Rio Negro and the Rio Solimoes near Manaus, Brazil
Photo credit:
https://en.wikipedia.org/wiki/File:Meeting_of_waters_from_the_air_manaus_brazil.JPG
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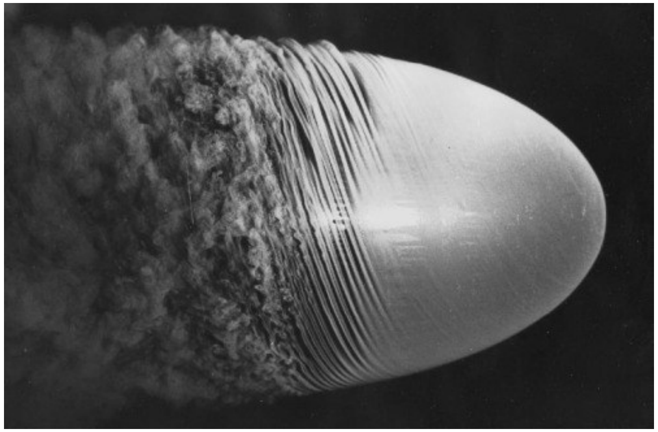
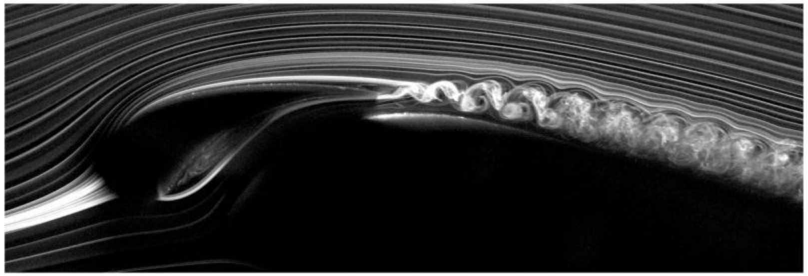
The turbulent world around us

Tugboat riding on the turbulent wake of a ship
Photo credit: https://www.flickr.com/photos/oneeighteen/

Trailing vortices
Photo credit: Steve Morris. AirTeamImages.

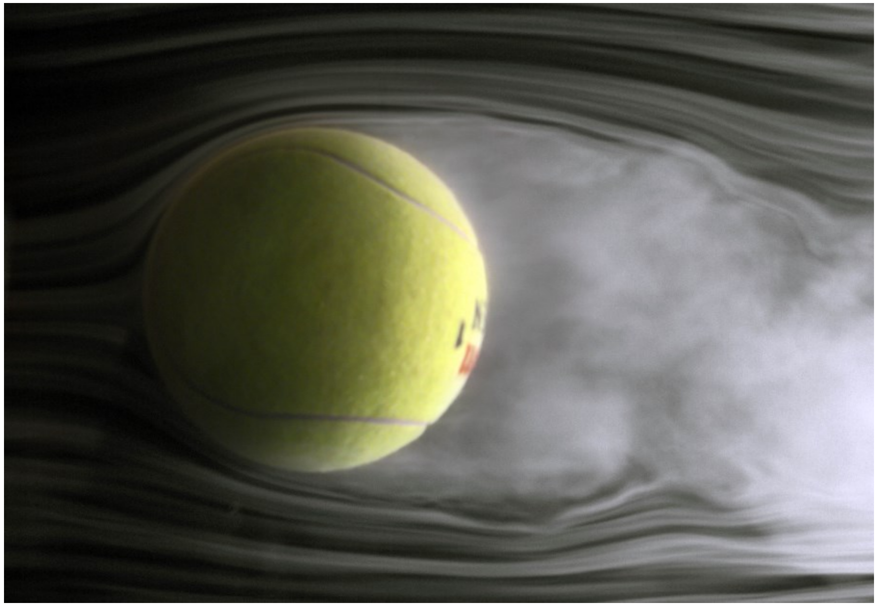


Flow visualization over a spinning spheroid
Photo credit: Y. Kohama.



Flow around an airfoil with a leading-edge slat
Photo credit: S. Makiya et al.

The turbulent world around us



Wind Tunnel Test of New Tennis Ball
Photo credit: NASA
http://tennisclub.gsfc.nasa.gov/tennis.windtunnelballs.html
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Wake turbulence behind individual wind turbines
Photo credit: NREL's wind energy research group.
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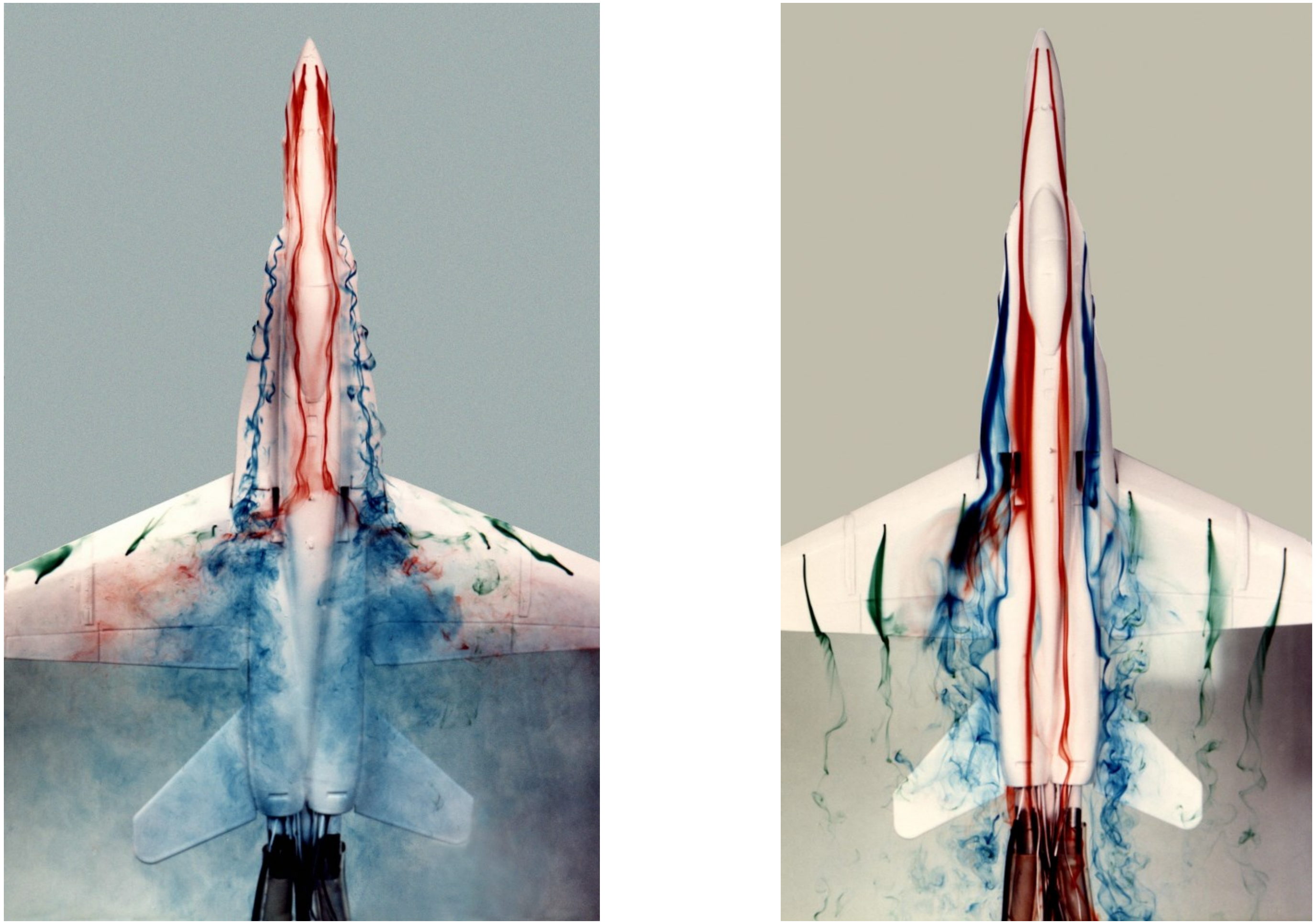
The turbulent world around us



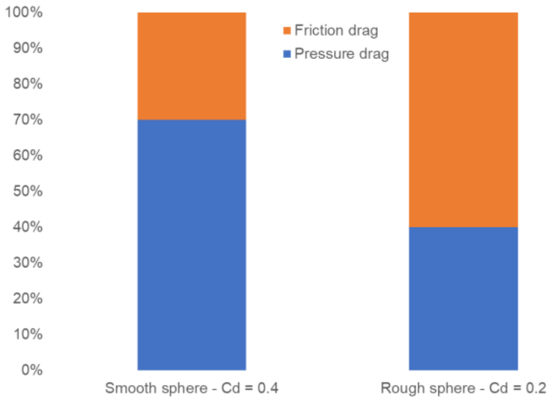
Vortices on a 1/48-scale model of an F/A-18 aircraft inside a Water Tunnel

Photo credit: NASA Dryden Flow Visualization Facility. http://www.nasa.gov/centers/armstrong/multimedia/imagegallery/FVF

The turbulent world around us

(a) $C_D \approx 0.4$

(b) $C_D \approx 0.2$

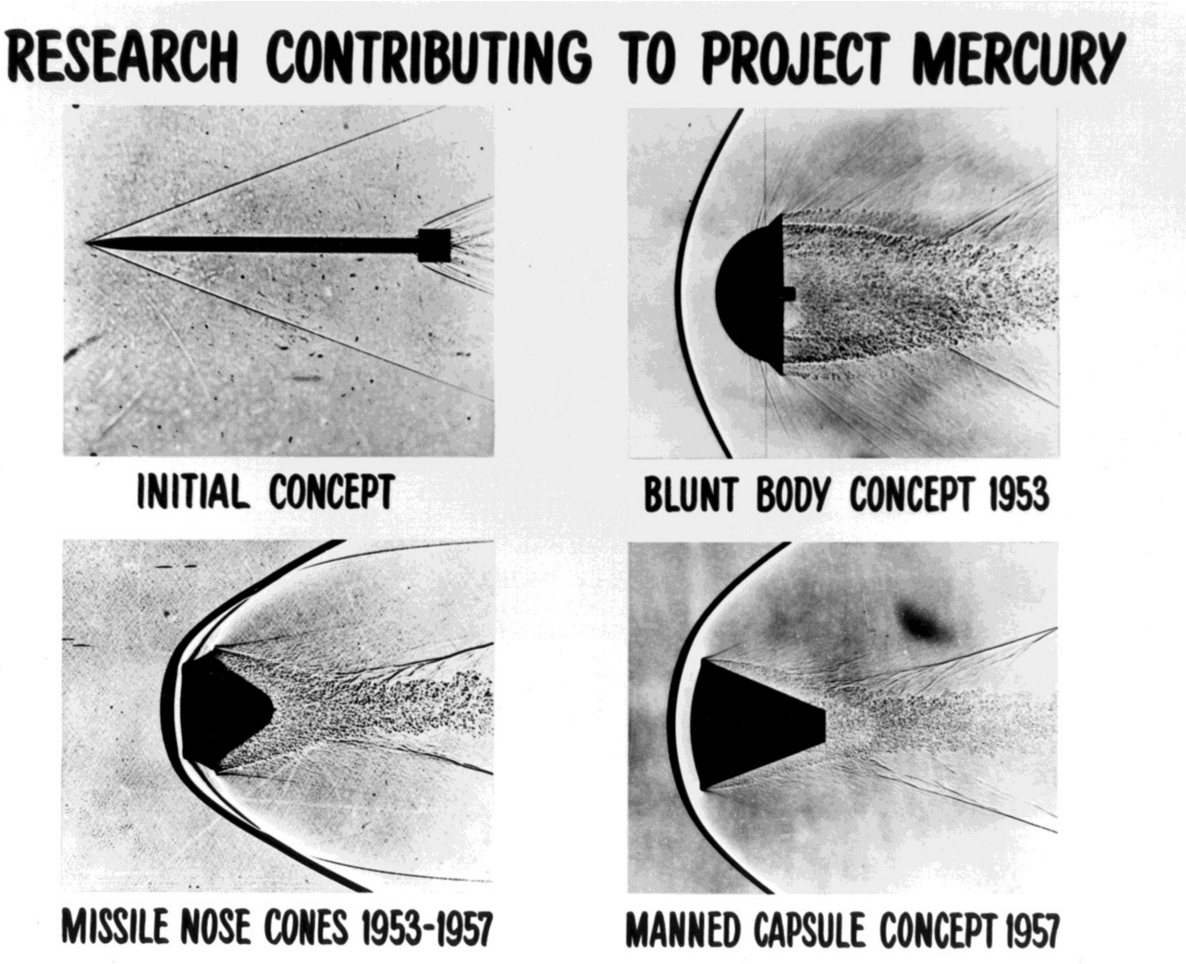


Abstract representation of the drag decomposition

Flow around two spheres. Left image: smooth sphere. Right image: sphere with rough surface at the nose

Photo credit: http://www.mhhe.com/engcs/civil/finnemore/graphics/photos/AuthorRecommendedImages/index.html

The turbulent world around us



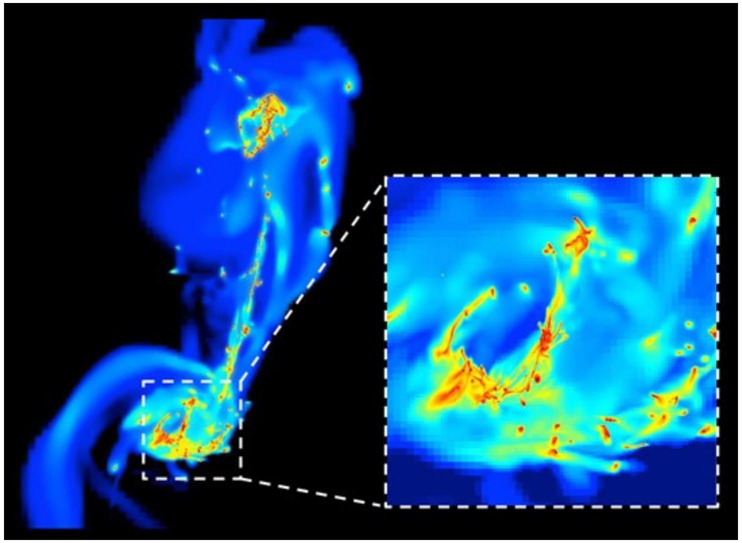
Shadowgraph Images of Re-entry Vehicles

Photo credit: NASA on the Commons. https://www.flickr.com/photos/nasacommons/

The turbulent world around us

Astrophysical, plasma, planetary and quantum turbulence

M8: The Lagoon Nebula
Photo credit: Steve Mazlin, Jack Harvey, Rick Gilbert, and Daniel Verschatse. Star Shadows Remote Observatory, PROMPT, CTIO



A frame from the simulation of the two colliding Antennae galaxies.
Photo credit: F. Renaud / CEA-Sap.

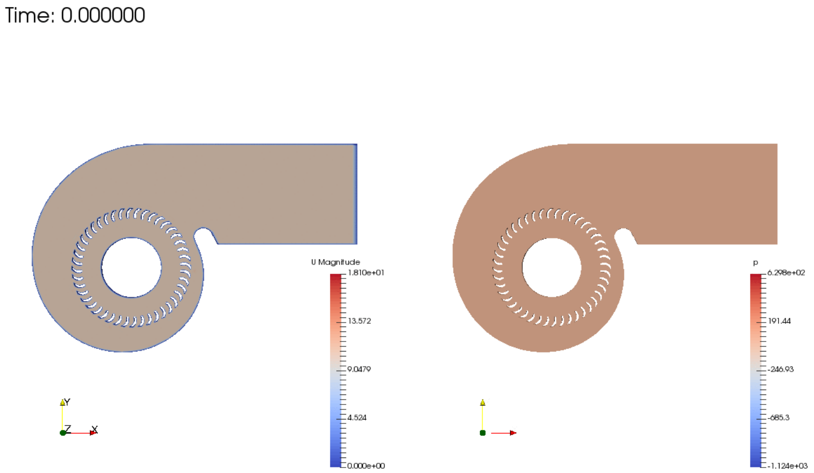
Jupiter photo taken by Juno's cam.
Photo credit: NASA / JPL / SwRI / MSSS / David Marriott

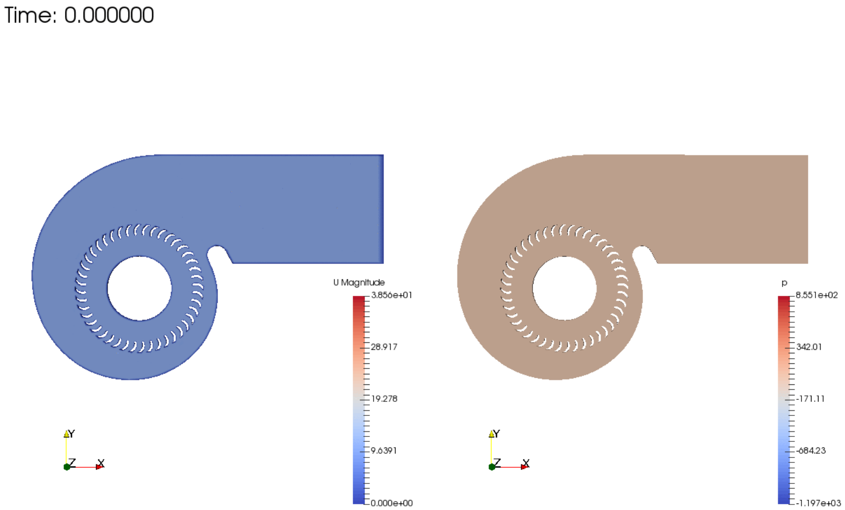
Roadmap to Lecture 2

1. ~~The turbulent world around us~~
2. **Turbulence, does it matter?**
3. Introduction to turbulence modeling – Basic concepts
4. Wall bounded flows and shear flows
5. A peek to the turbulence closure problem, some correlations in turbulence modeling, and the energy cascade

Turbulence, does it matter?

Blower simulation using sliding grids





No turbulence model used (laminar, no turbulence modeling, DNS, unresolved DNS, exact Navier-Stokes, name it as you want)

http://www.wolfdynamics.com/training/turbulence/image1.gif

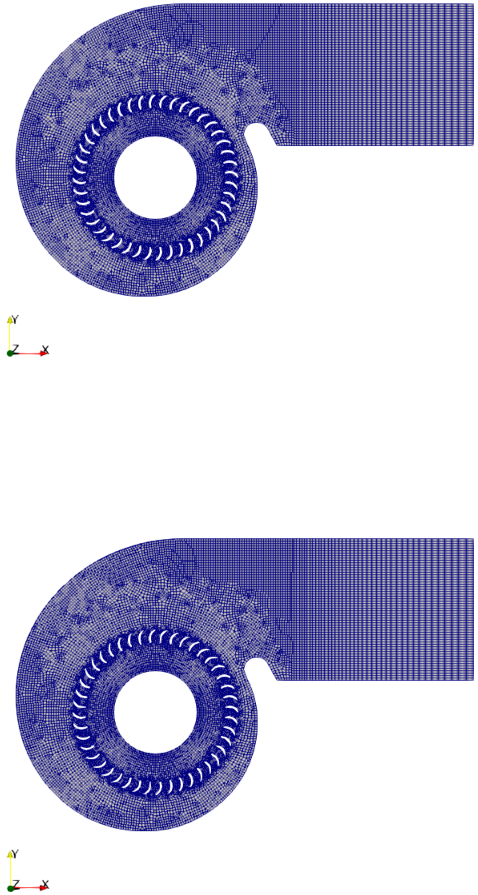
K-epsilon turbulence model

http://www.wolfdynamics.com/training/turbulence/image2.gif

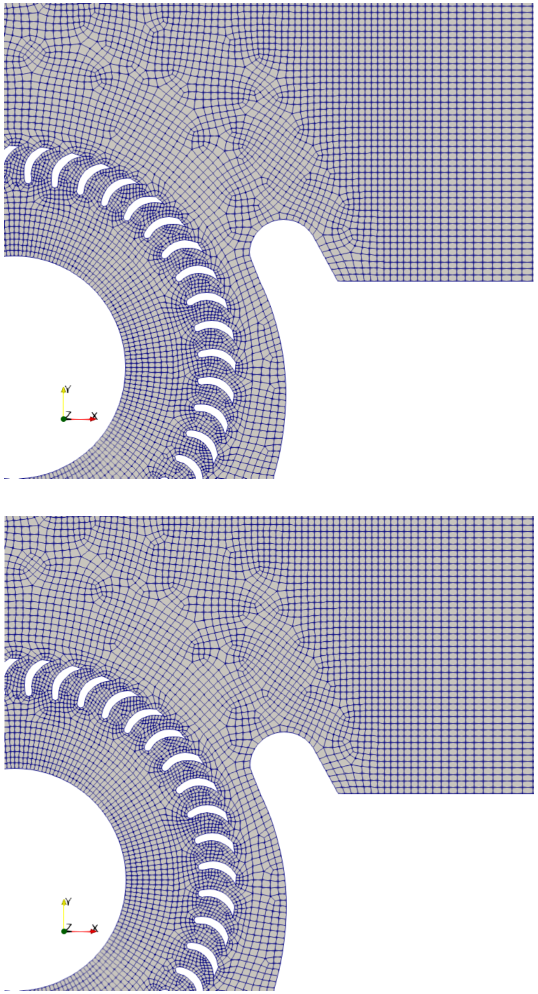
Turbulence, does it matter?

Blower simulation using sliding grids

- Even if the mesh is coarse, thanks to the help of the turbulence model, we managed to capture the right physics.



http://www.wolfdynamics.com/training/turbulence/blower1.gif

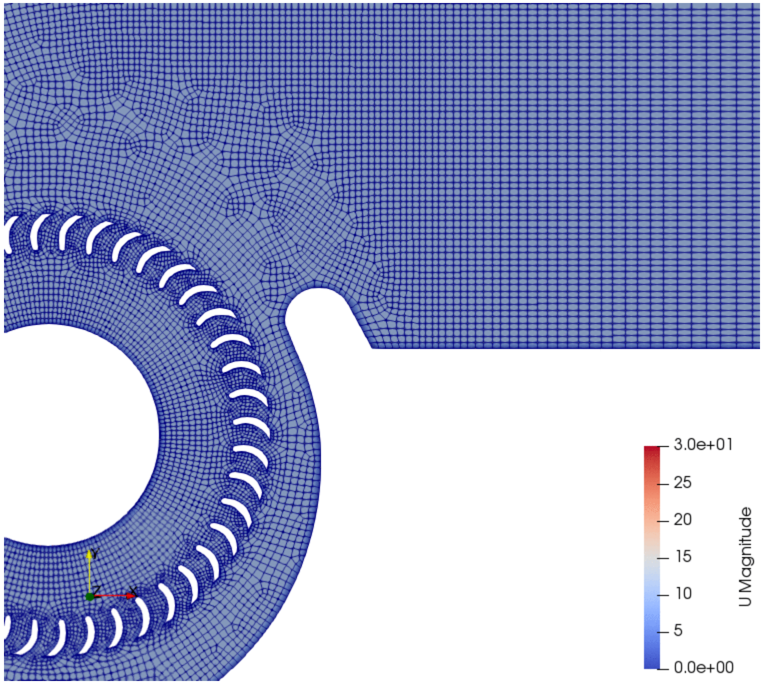


http://www.wolfdynamics.com/training/turbulence/blower2.gif

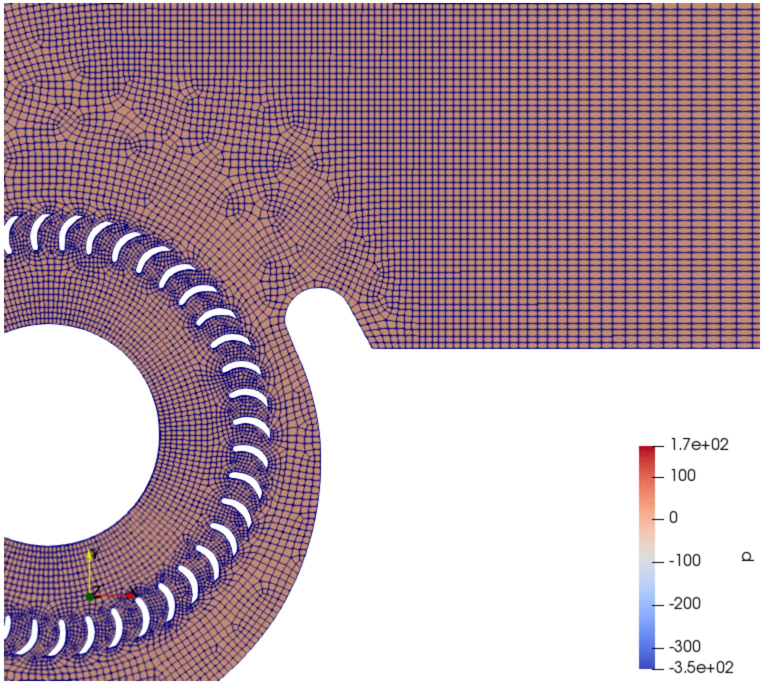
Turbulence, does it matter?

Blower simulation using sliding grids

- Even if the mesh is coarse, thanks to the help of the turbulence model, we managed to capture the right physics.



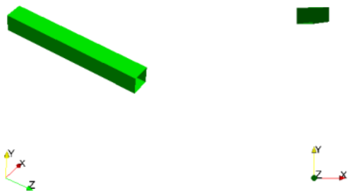
K-epsilon turbulence model – Velocity magnitude field (m/s)
http://www.wolfdynamics.com/training/turbulence/blower3.gif



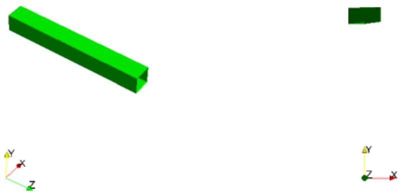
K-epsilon turbulence model – Relative pressure field (Pa)
http://www.wolfdynamics.com/training/turbulence/blower4.gif

Turbulence, does it matter?

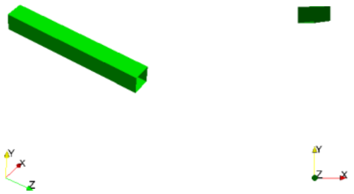
Vortex shedding past square cylinder



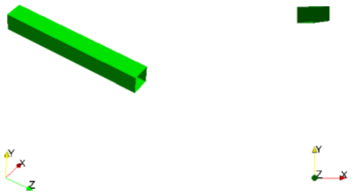
URANS (K-Omega SST with no wall functions) – Vortices visualized by Q-criterion
www.wolfdynamics.com/wiki/squarecil/urans2.gif



LES (Smagorinsky) – Vortices visualized by Q-criterion
www.wolfdynamics.com/wiki/squarecil/les.gif



Laminar (no turbulence model) – Vortices visualized by Q-criterion
www.wolfdynamics.com/wiki/squarecil/laminar.gif



DES (SpalartAllmarasDDES) – Vortices visualized by Q-criterion
www.wolfdynamics.com/wiki/squarecil/des.gif

Turbulence, does it matter?

Vortex shedding past square cylinder

Turbulence model	Drag coefficient	Strouhal number	Computing time (s)
Laminar	2.81	0.179	93489
LES	2.32	0.124	77465
DES	2.08	0.124	70754
SAS	2.40	0.164	57690
URANS (WF)	2.31	0.130	67830
URANS (No WF)	2.28	0.135	64492
RANS	2.20	-	28246 (10000 iter)
Experimental values	2.05-2.25	0.132	-

Note: all simulations were run using 4 cores.

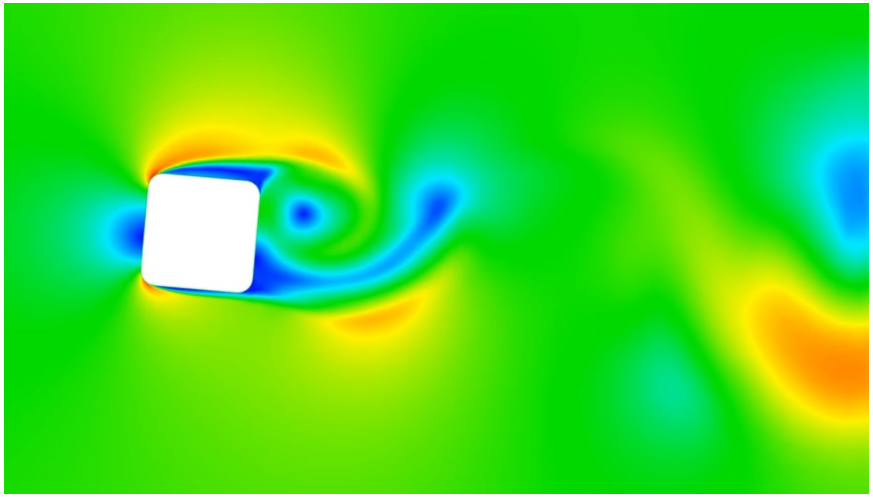
References:

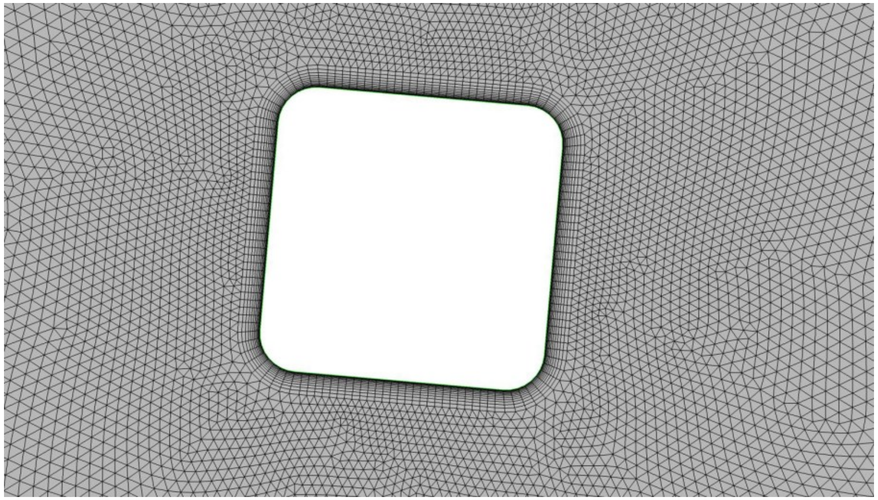
D. A. Lyn and W. Rodi. “The flapping shear layer formed by flow separation from the forward corner of a square cylinder”. *J. Fluid Mech., 267, 353, 1994.*

D. A. Lyn, S. Einav, W. Rodi and J. H. Park. “A laser-Doppler velocimetry study of ensemble-averaged characteristics of the turbulent near wake of a square cylinder”. *Report. SFB 210 /E/100.*

Turbulence, does it matter?

Transitional flow past square cylinder with rounded corners – Re = 54000





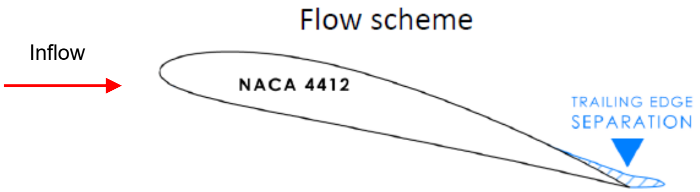
🎞 Velocity magnitude

www.wolfdynamics.com/wiki/turb/media1.mp4

Turbulence model	Drag coefficient	Lift coefficient
DNS	0.06295	0.07524
LES	0.1146	0.03269
SAS	0.1058	0.0258
URANS (No WF)	0.1107	0.00725
Transition K-KL-Omega	0.059	-0.0104
Transition K-Omega SST	0.0987	-0.0143
Experimental values	0.045 to 0.075	-0.011 to -0.015

Turbulence, does it matter?

Separated flow around a NACA-4412 airfoil



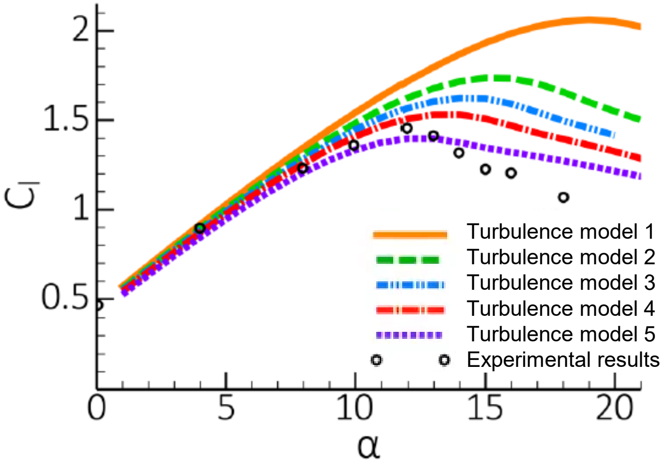
Incompressible flow

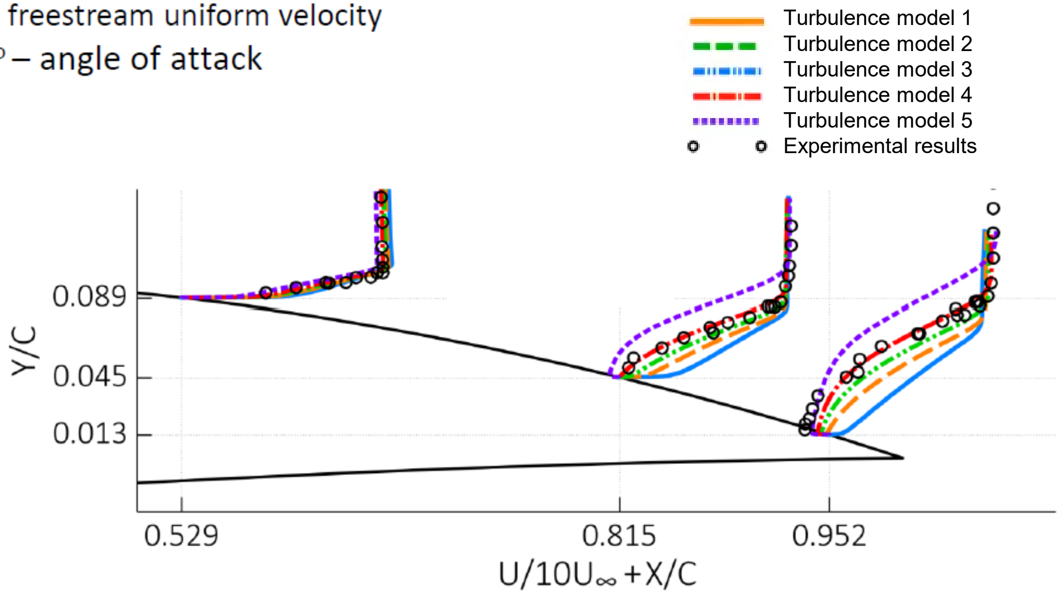
Re = $U_\infty \cdot C/\nu = 1.64\cdot 10^6$

C - airfoil chord

U_∞ - freestream uniform velocity

$\alpha = 12^\circ$ – angle of attack





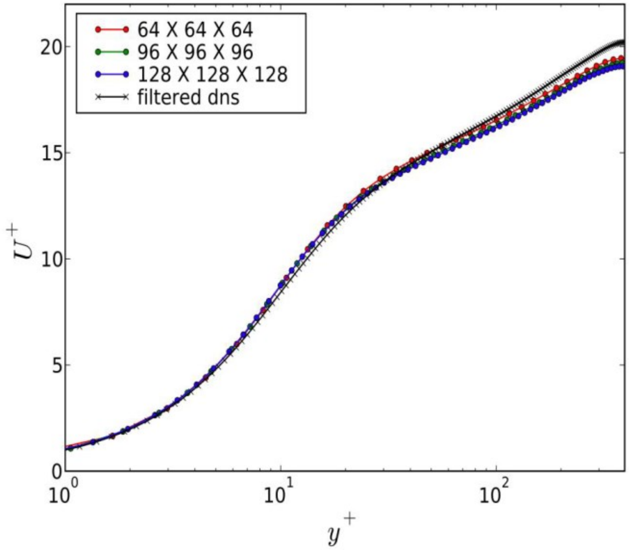
- CFD has been around since the late 1970s, and after all these years is not that easy to compute the flow around 2D airfoils.
- In particular, predicting the maximum lift and stall characteristics is not trivial.

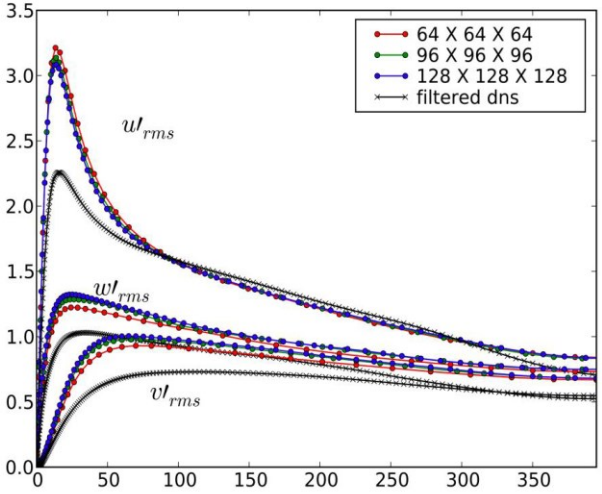
References:

F. Menter. “A New Generalized k-omega model. Putting flexibility into Turbulence models (GEKO)”, Ansys Germany
A. J. Wadcock. “Investigation of Low-Speed Turbulent Separated Flow Around Airfoils”, NASA Contractor Report 177450

Turbulence, does it matter?

Grid independent solutions and modeling errors





- CFD has been around since the late 1970s. Since then, a lot of progress has been done in hardware, software, algorithms and turbulence models.
- But many times, even if we get converged, grid-independent solutions we fail to get a good match with the experiments (not that the experiments are always right) or a reference solution due to modeling errors.

> *"The multiscale nature of turbulence creates unique challenges for numerical simulations. Discretization methods must preserve the physical processes, reducing or eliminating artificial dissipation and dispersion ...*
> *… How do you establish confidence in the numerical simulations of turbulent flows?"*

References:

P. Moin. "Turbulence: V&V and UQ Analysis of a Multi-scale Complex System". Center for Turbulence Research Stanford University

Turbulence, does it matter?

Turbulence is not a trivial problem

> *"Turbulence is the most important unresolved problem of classical physics"*
>
> Richard Feynman

> *"Turbulence was probably invented by the devil on the seventh day of creation when the good lord was not looking"*
>
> Peter Bradshaw

> *"Turbulence is the graveyard of theories"*
>
> Hans W. Liepmann

Turbulence, does it matter?

Turbulence is not a trivial problem

- This is probably my favorite quote, as it covers the largest elephants in CFD, mesh and turbulence.

> *"Geometry modeling is to meshing what turbulence modeling is to computational fluid dynamics (CFD) – a mathematically complex model of something important that we try to treat as the proverbial black box."*
>
> John Chawner - Pointwise

Roadmap to Lecture 2

1. ~~The turbulent world around us~~
2. ~~Turbulence, does it matter?~~
3. **Introduction to turbulence modeling – Basic concepts**
4. Wall bounded flows and shear flows
5. A peek to the turbulence closure problem, some correlations in turbulence modeling, and the energy cascade

Introduction to turbulence modeling – Basic concepts

“Essentially, all models are wrong,
but some are useful”

G. E. P. Box

George Edward Pelham Box
18 October 1919 – 28 March 2013. Statistician, who worked in the areas of quality control, time-series analysis, design of experiments, and Bayesian inference. He has been called *“one of the great statistical minds of the 20th century”*.

Turbulence modeling in engineering

- Most natural and engineering flows are turbulent, hence the necessity of modeling turbulence.
- The goal of turbulence modeling is to develop equations that predict the time averaged velocity, pressure, temperature fields without calculating the complete turbulent flow pattern as a function of time.
- Turbulence can be wall bounded or free shear, and depending on what you want to simulate, you will need to choose an appropriate turbulence model.
- There is no universal turbulence model, hence you need to know the capabilities and limitations of the turbulence models.
- Due to the multi-scale and unsteady nature of turbulence, modeling it is not an easy task.
- Simulating turbulent flows in any general CFD solver (*e.g.*, OpenFOAM®, SU2, Fluent, CFX, Star-CCM+) requires selecting a turbulence model, providing initial conditions and boundary conditions for the closure equations of the turbulent model, selecting a near-wall modeling, and choosing runtime parameters and numerics.

Why turbulent flows are challenging?

- Unsteady aperiodic motion.
- All fluid properties and transported quantities exhibit random spatial and temporal variations.
- They are intrinsically three-dimensional due to vortex stretching.
- Strong dependence from initial conditions.
- Contains a wide range of scales (eddies).
- Therefore, in order to accurately model/resolve turbulent flows, the simulations must be three-dimensional, time-accurate, and with fine enough meshes such that all spatial scales are properly captured.
- Additional physics that makes turbulence modeling even harder:
  - Buoyancy, compressibility effects, heat transfer, multiphase flows, transition to turbulence, surface finish, combustion, and so on.

Reynolds number and Rayleigh number

- It is well known that the Reynolds number characterizes if the flow is laminar or turbulent.
- So before doing a simulation or experiment, check if the flow is turbulent.
- The Reynolds number is defined as follows,

Convective effects

$$Re_L = \frac{\rho U L}{\mu} = \frac{U L}{\nu}$$

Viscous effects

$$\nu = \frac{\mu}{\rho}$$

- Where *U* is a characteristic velocity, *e.g.*, free-stream velocity.
- And *L* is representative length scale, *e.g.*, length, height, diameter, etc.

Reynolds number and Rayleigh number

- If you are dealing with natural convection, you can use the Rayleigh number (Ra), Grashof number (Gr), and Prandtl number (Pr) to characterize the flow.

Specific heat

Thermal expansion coefficient

Buoyancy effects

$$Ra = \frac{g\beta L^3 \Delta T}{\nu\alpha} = \frac{\rho^2 c_p \beta g L^3 \Delta T}{\mu k} = Gr \times Pr$$

Viscous effects

Thermal conductivity

Momentum diffusivity

$$Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

Thermal diffusivity

$$Gr = \frac{g\beta(T_S - T_\infty)L^3}{\nu^2}$$

Reynolds number and Rayleigh number

- Turbulent flow occurs at large Reynolds number.
  - For external flows,

$$Re_x \geq 500000$$

Around slender/streamlined bodies (surfaces)

$$Re_d \geq 20000$$

Around an obstacle (bluff bodies)

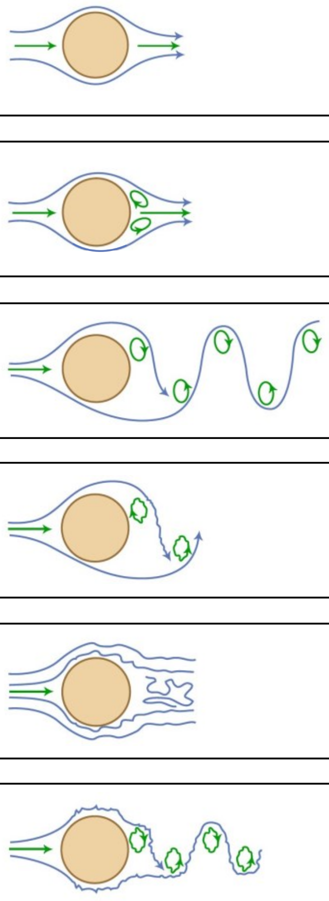
  - For internal flows,

$$Re_{d_h} \geq 2300$$

- Notice that other factors such as free-stream turbulence, surface conditions, blowing, suction, roughness and other disturbances, may cause transition to turbulence at lower Reynolds number.
- If you are dealing with natural convection and buoyancy, turbulent flows occurs when

$$\frac{Ra}{Pr} \geq 10^9$$

What happens when we increase the Reynolds number?

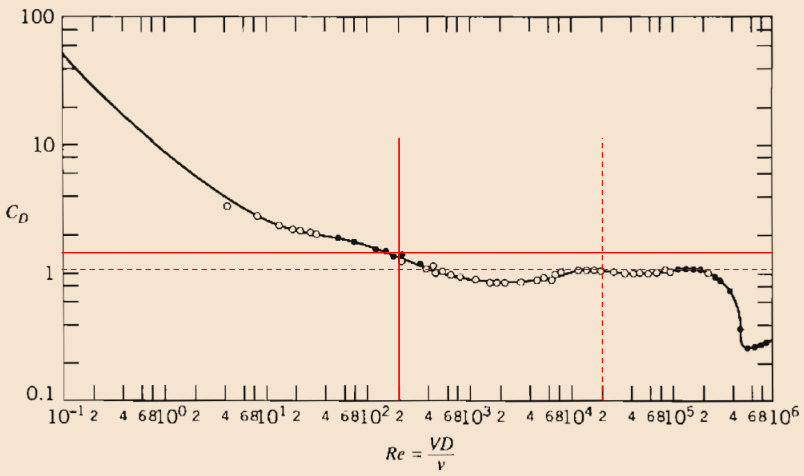


Creeping flow (no separation) Steady flow	$Re < 5$	- Easy to simulate - Steady
A pair of stable vortices in the wake Steady flow	$5 < Re < 40 - 46$	
Laminar vortex street (Von Karman street) Unsteady flow	$40 - 46 < Re < 150$	- Relatively easy to simulate. - It becomes more challenging when the boundary layer transition to turbulent - Unsteady
Laminar boundary layer up to the separation point, turbulent wake Unsteady flow	$150 < Re < 300$ Transition to turbulence $300 < Re < 3 \text{ x } 10^5$	
Boundary layer transition to turbulent Unsteady flow	$3 \text{ x } 10^5 < Re < 3 \text{ x } 10^6$	- Challenging to simulate - Unsteady
Turbulent vortex street, but the wake is narrower than in the laminar case Unsteady flow	$3 \text{ x } 10^6 > Re$	

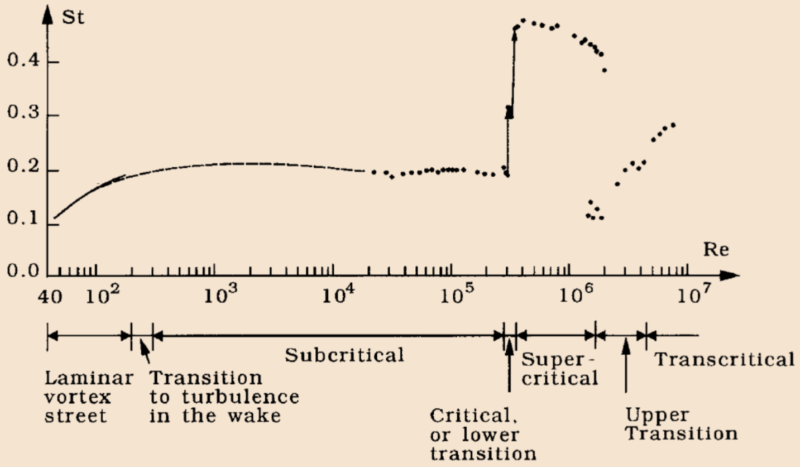
Vortex shedding behind a cylinder and Reynolds number

Introduction to turbulence modeling – Basic concepts

What happens when we increase the Reynolds number?



Drag coefficient as a function of Reynolds number for a smooth cylinder [1]



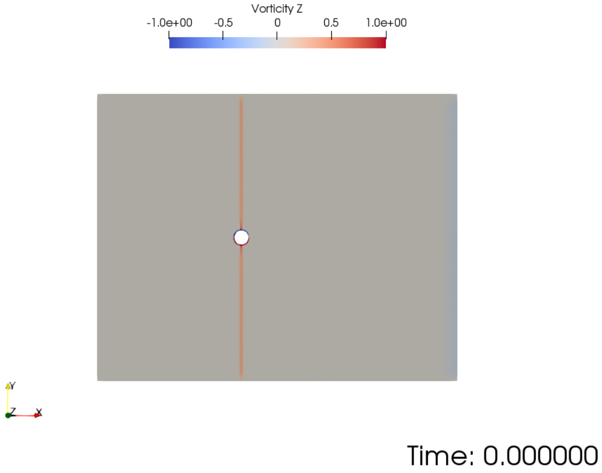
Strouhal number $St = \frac{fL}{U}$ for a smooth cylinder [2]

References:

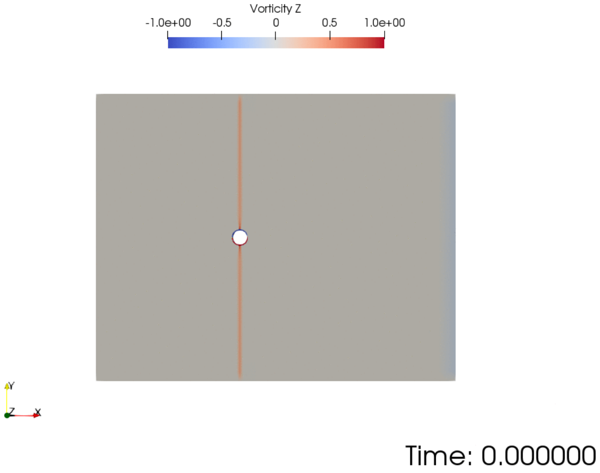
1. Fox, Robert W., et al. Introduction to Fluid Mechanics. Hoboken, NJ, Wiley, 2010
2. Sumer, B. Mutlu, et al. Hydrodynamics Around Cylindrical Structures. Singapore, World Scientific, 2006

Introduction to turbulence modeling – Basic concepts

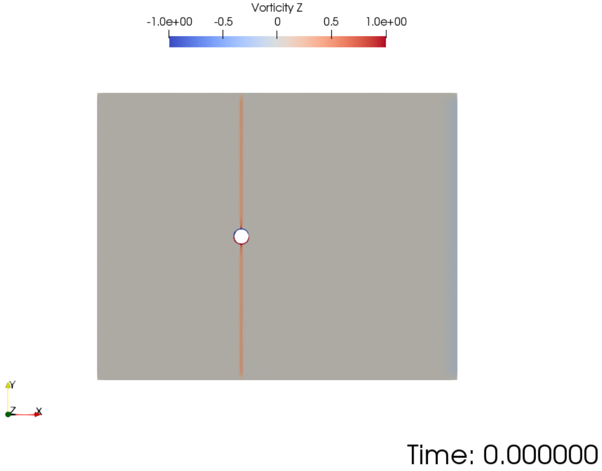
What happens when we increase the Reynolds number?



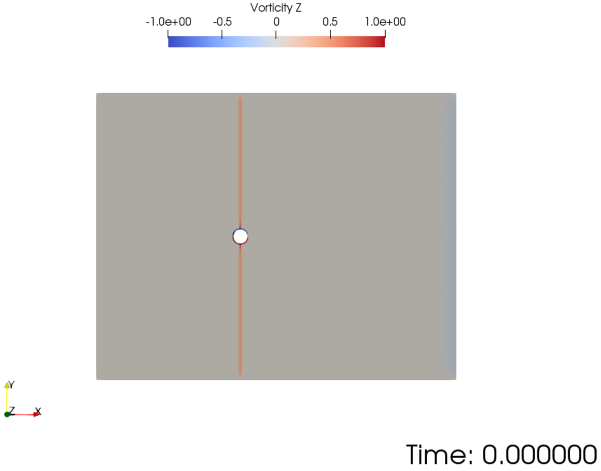
Reynolds = 20 – Non-uniform initialization
Laminar flow with separation
http://www.wolfdynamics.com/training/turbulence/unscyl1.gif



Reynolds = 200 – Non-uniform initialization
Laminar flow with vortex shedding
http://www.wolfdynamics.com/training/turbulence/unscyl3.gif

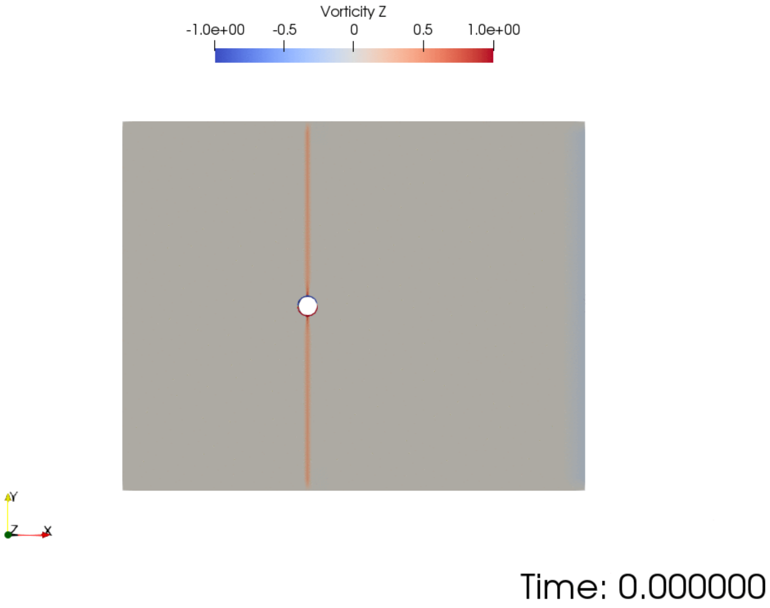


Reynolds = 50 – Non-uniform initialization
Laminar flow with vortex shedding (maybe on the limit of the onset of the Von Karman street.
http://www.wolfdynamics.com/training/turbulence/unscyl2.gif

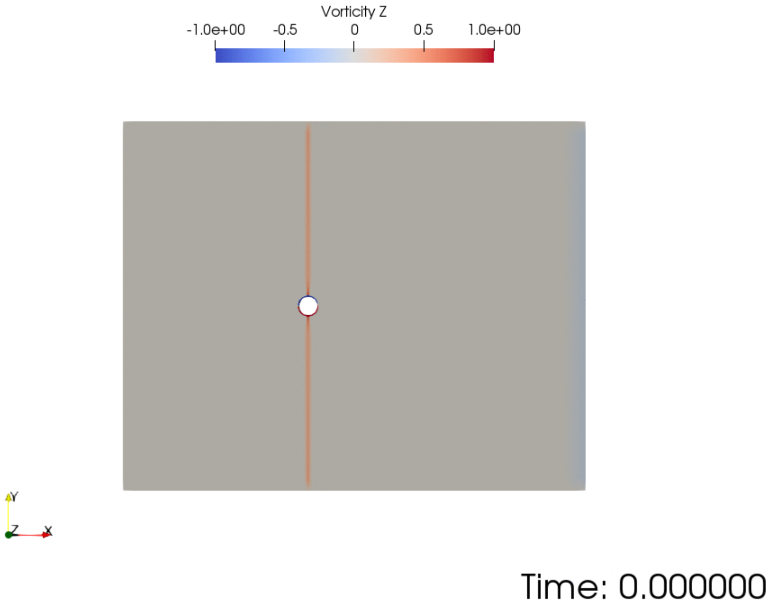


Reynolds = 20000 – Non-uniform initialization
Turbulent flow with vortex shedding (turbulence model enable)
http://www.wolfdynamics.com/training/turbulence/unscyl4.gif

What happens when we increase the Reynolds number?



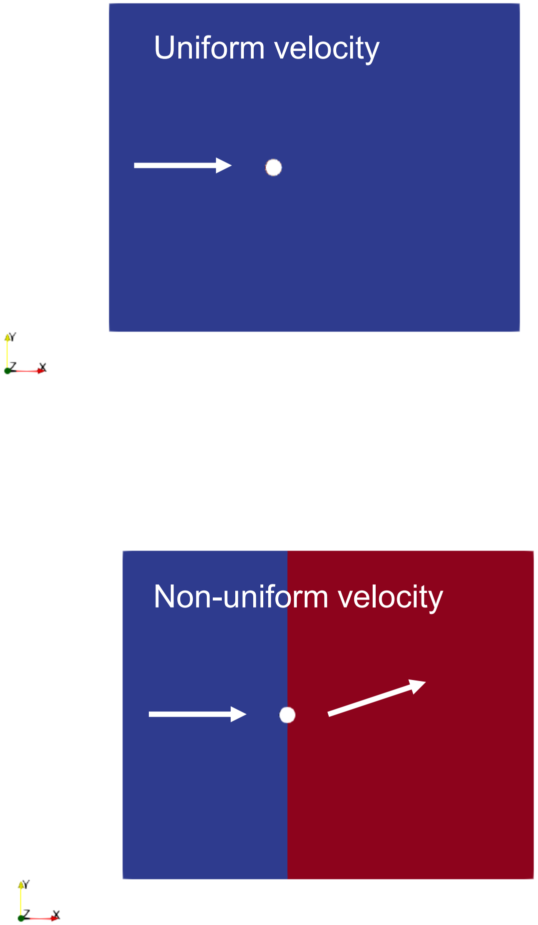
Reynolds = 20000 – Non-uniform initialization
No turbulence model enable
http://www.wolfdynamics.com/training/turbulence/unscyl5.gif



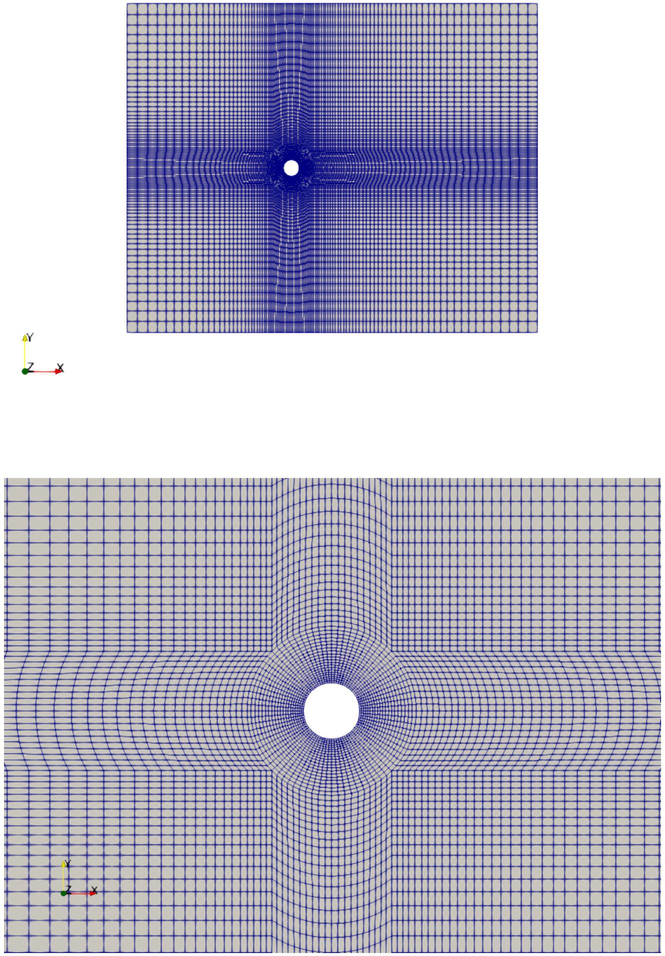
Reynolds = 20000 – Non-uniform initialization
Turbulence model enable (k-omega SST)
http://www.wolfdynamics.com/training/turbulence/unscyl4.gif

Introduction to turbulence modeling – Basic concepts

What happens when we increase the Reynolds number?

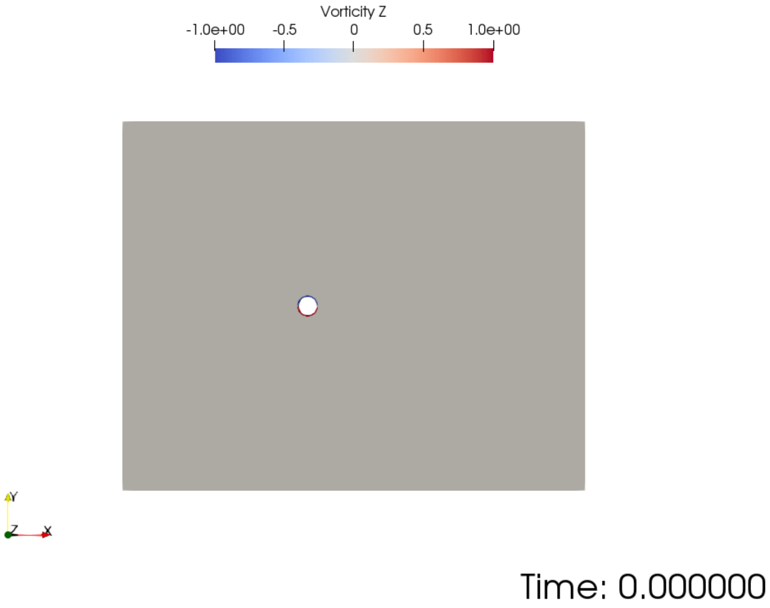


Field initialization - Velocity

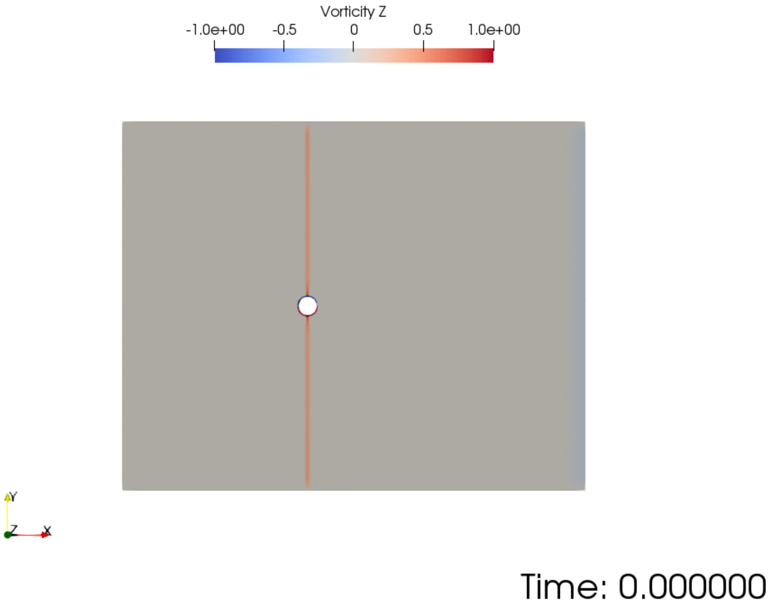


Mesh

What happens when we increase the Reynolds number?



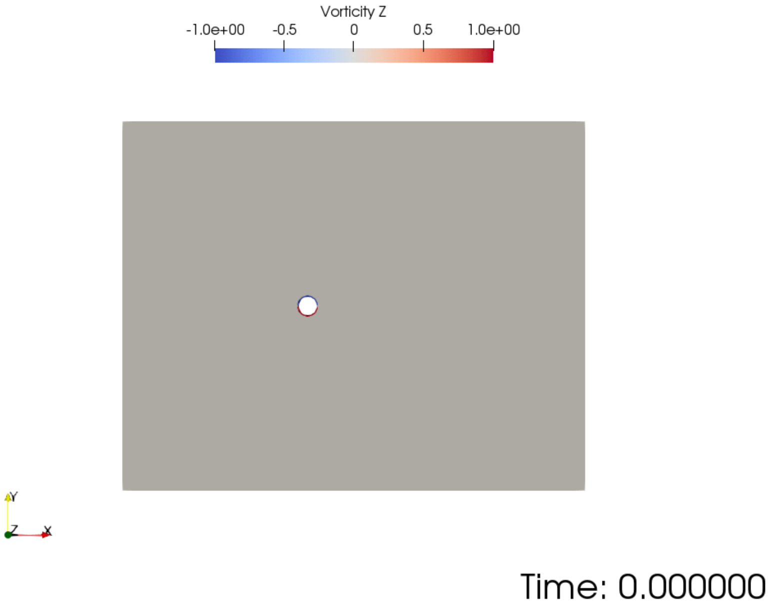
Reynolds = 20000 – Uniform initialization
Turbulence model enable (k-omega SST)
http://www.wolfdynamics.com/training/turbulence/unscyl6.gif



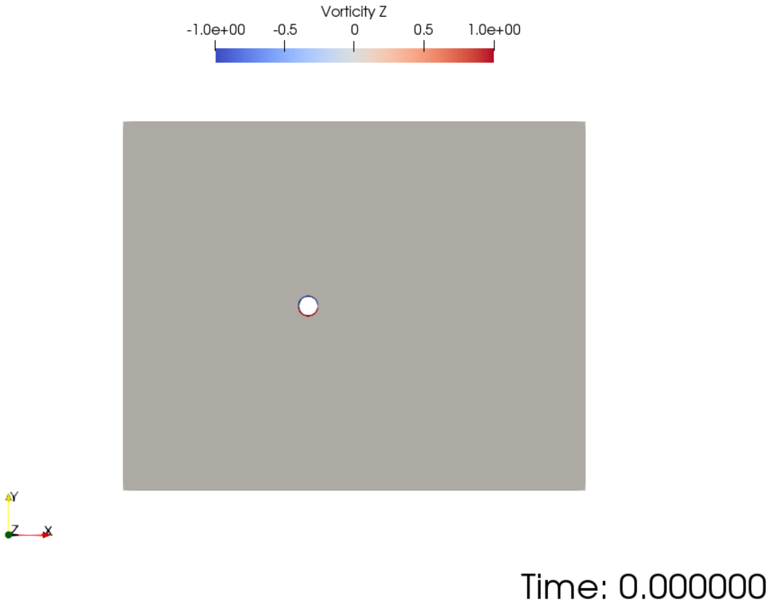
Reynolds = 20000 – Non-uniform initialization
Turbulence model enable (k-omega SST)
http://www.wolfdynamics.com/training/turbulence/unscyl4.gif

Introduction to turbulence modeling – Basic concepts

What happens when we increase the Reynolds number?

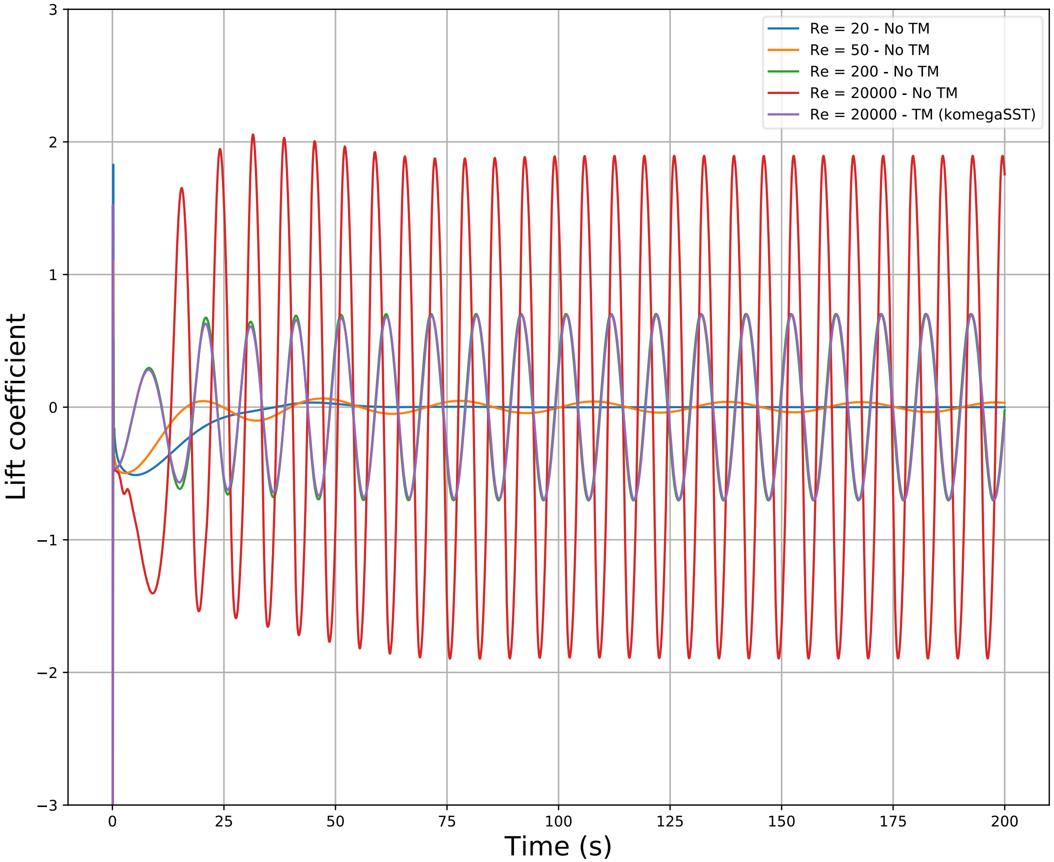


Reynolds = 20000 – Uniform initialization
Turbulence model enable (k-omega SST)
http://www.wolfdynamics.com/training/turbulence/unscyl6.gif



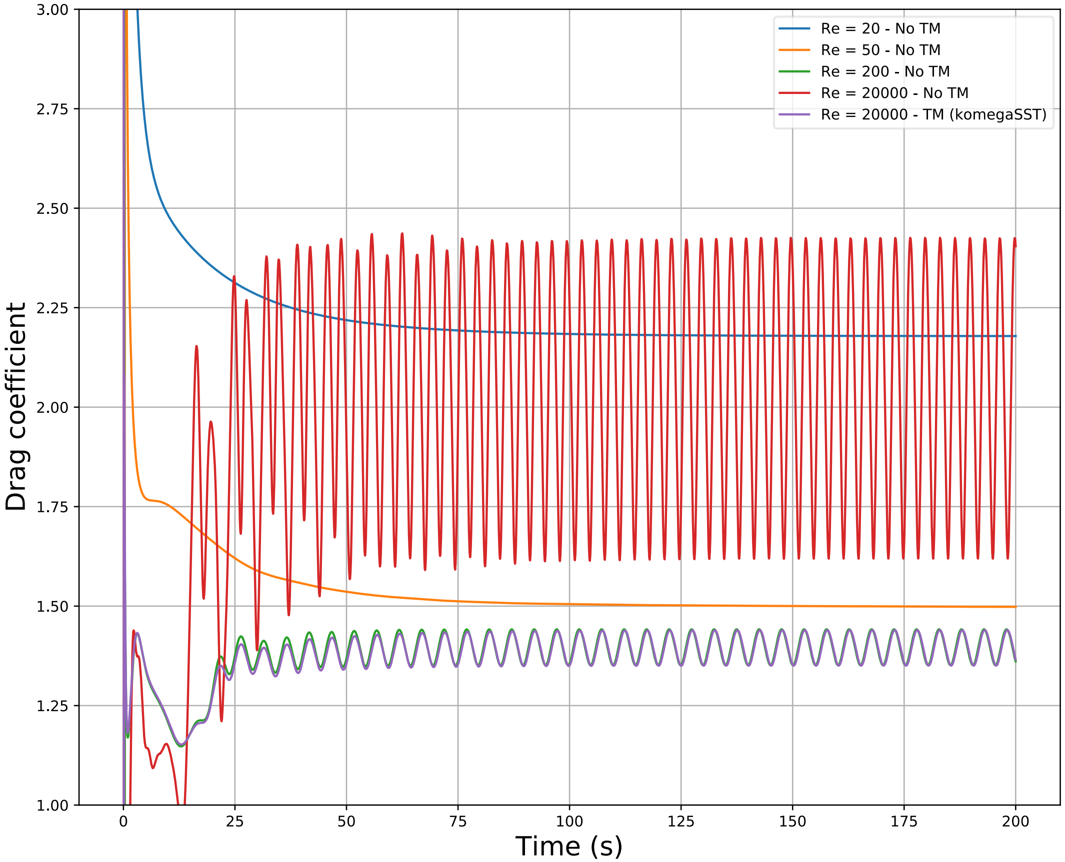
Reynolds = 20000 – Uniform initialization
Turbulence model enable (LES Smagorisky)
http://www.wolfdynamics.com/training/turbulence/unscyl7.gif

What happens when we increase the Reynolds number?



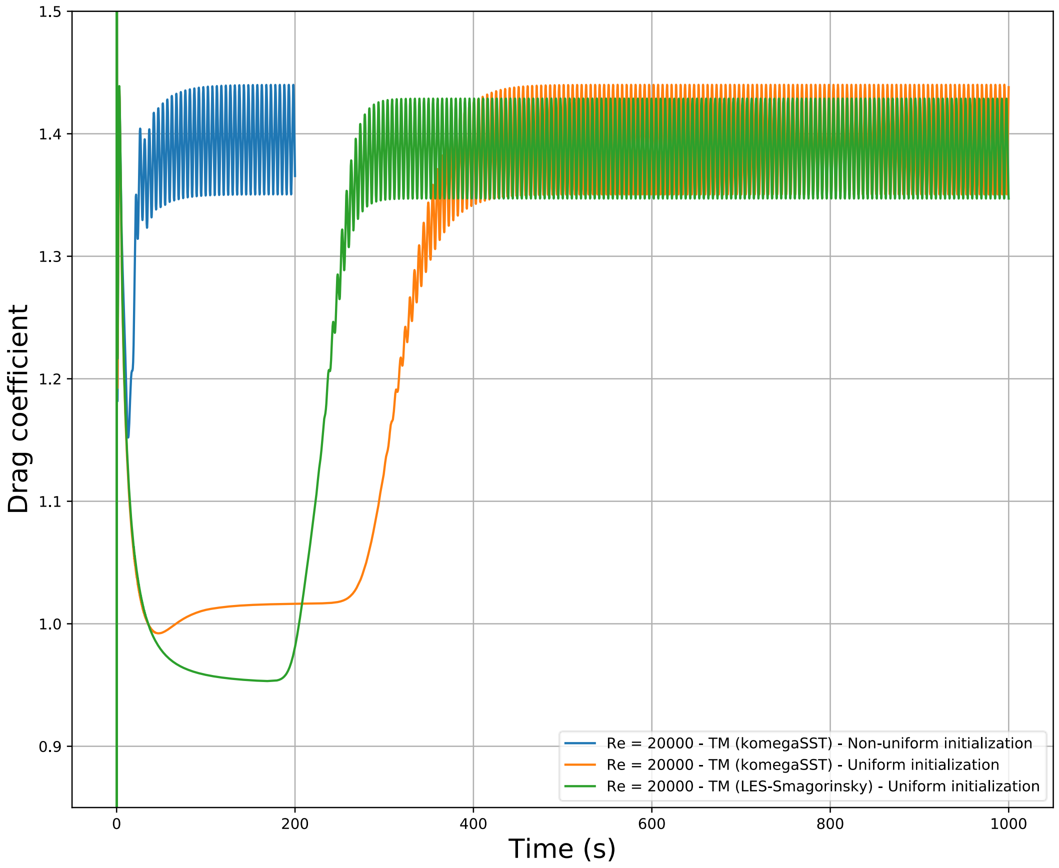
Reynolds	Turbulence model	Mean c_l
20	No	-0.00012
50	No	0.00274
200	No	-0.00149
20000	No	0.02176
20000	K-Omega SST	-0.00214

What happens when we increase the Reynolds number?



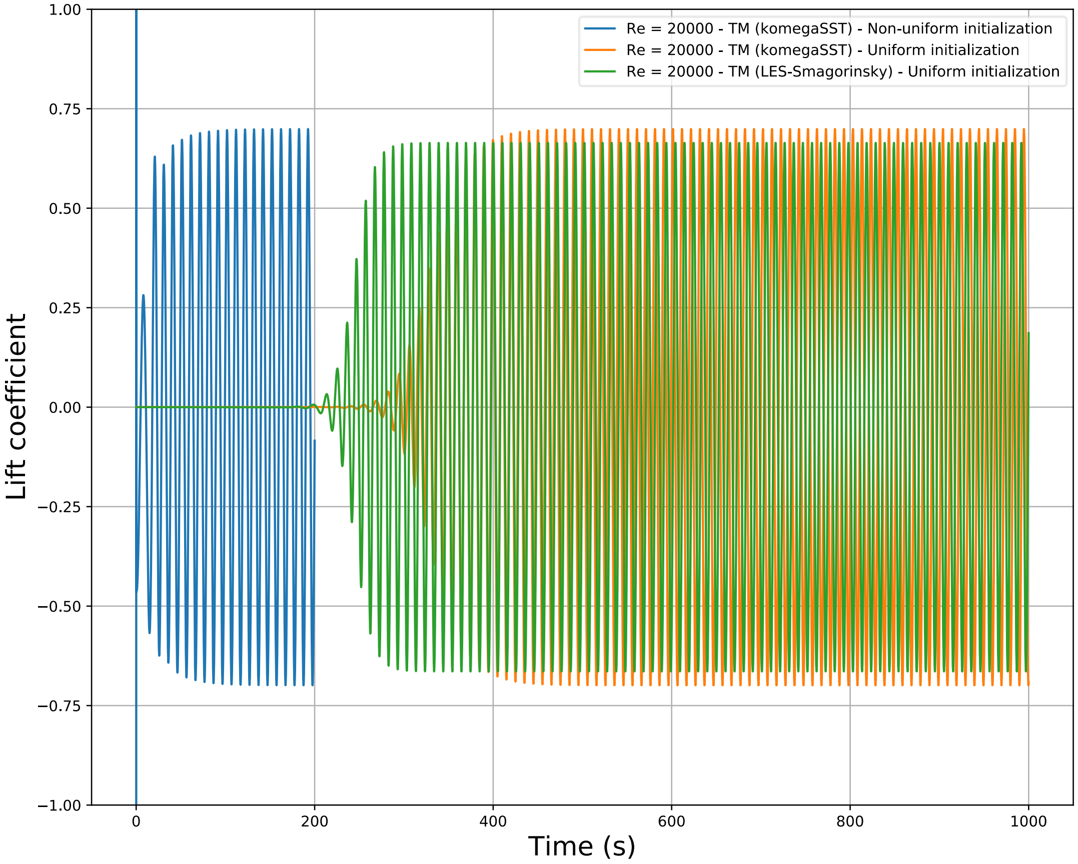
Reynolds	Turbulence model	Mean c_d
20	No	2.17987
50	No	1.50056
200	No	1.39786
20000	No	2.05043
20000	K-Omega SST	1.39459

What happens when we increase the Reynolds number?



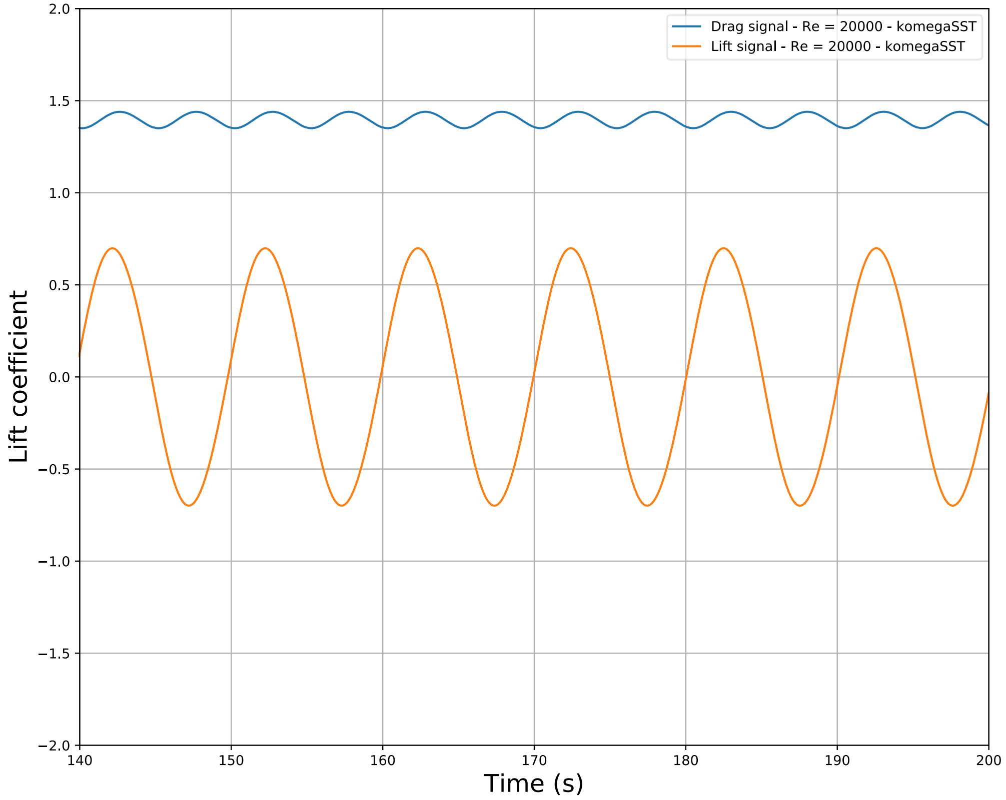
Reynolds	Turbulence model	Mean c_d
20000	K-Omega SST (NUI)	1.39786
20000	K-Omega SST (UI)	1.39617
20000	LES-Smagorinsky (UI)	1.38865

What happens when we increase the Reynolds number?

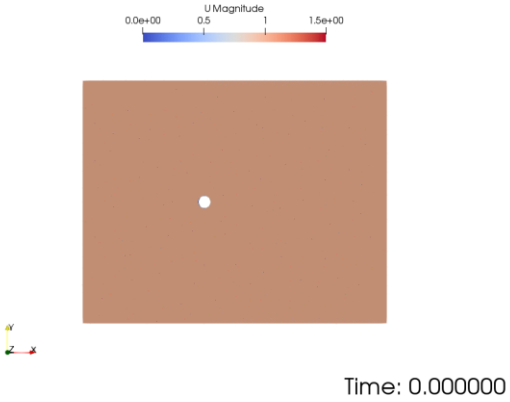


Reynolds	Turbulence model	Mean c_l
20000	K-Omega SST (NUI)	-0.00214
20000	K-Omega SST (UI)	0.00190
20000	LES-Smagorinsky (UI)	-0.00118

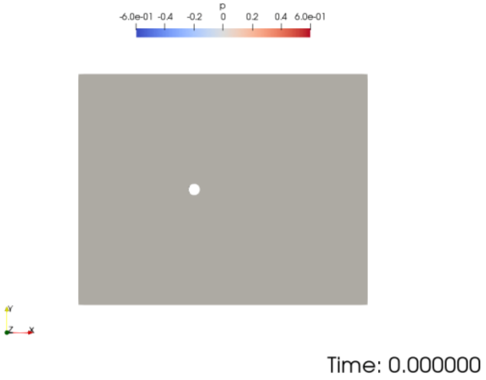
What happens when we increase the Reynolds number?



http://www.wolfdynamics.com/training/turbulence/unscyl8.gif

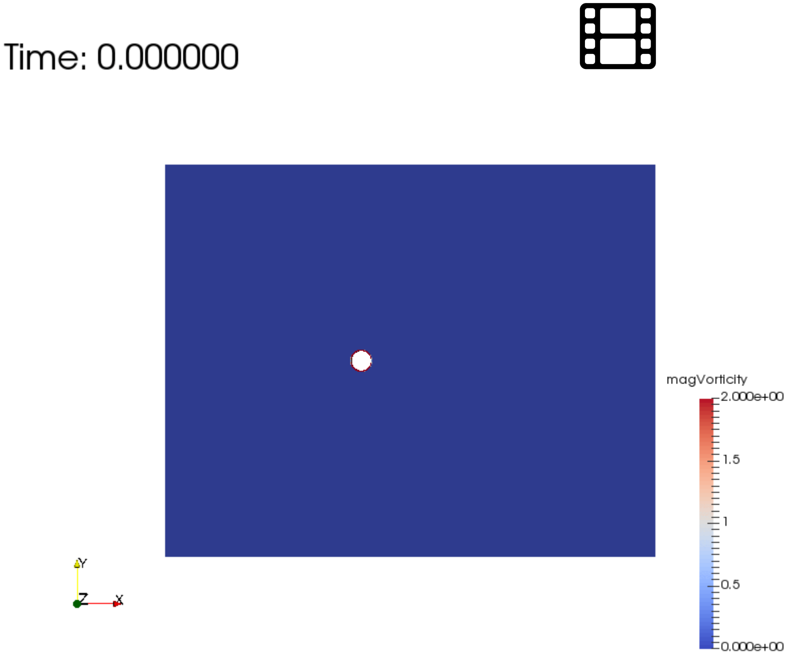


http://www.wolfdynamics.com/training/turbulence/unscyl9.gif



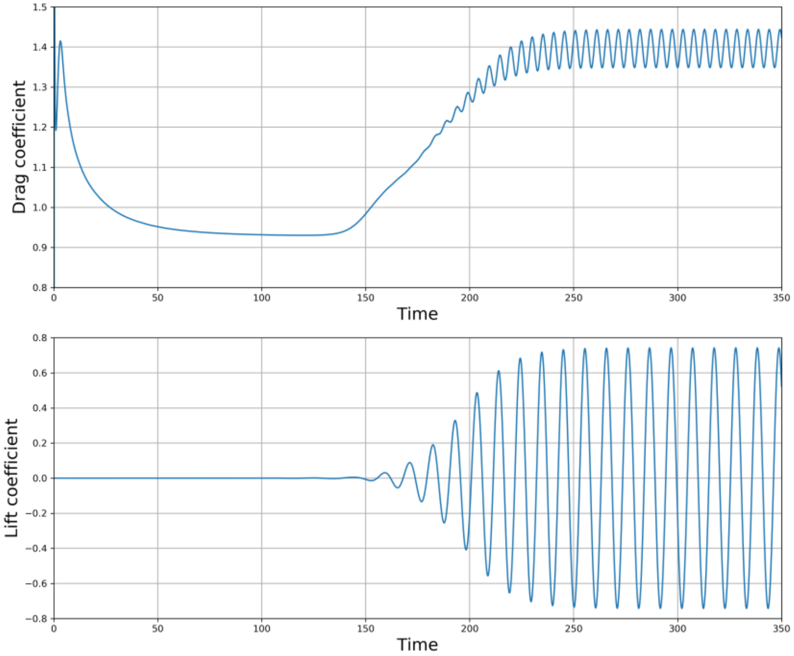
- Do you notice anything peculiar in the force coefficient signals?
- Look at the frequencies?
- Try different Reynolds number, do you see the same behavior?

Vorticity does not always mean turbulence



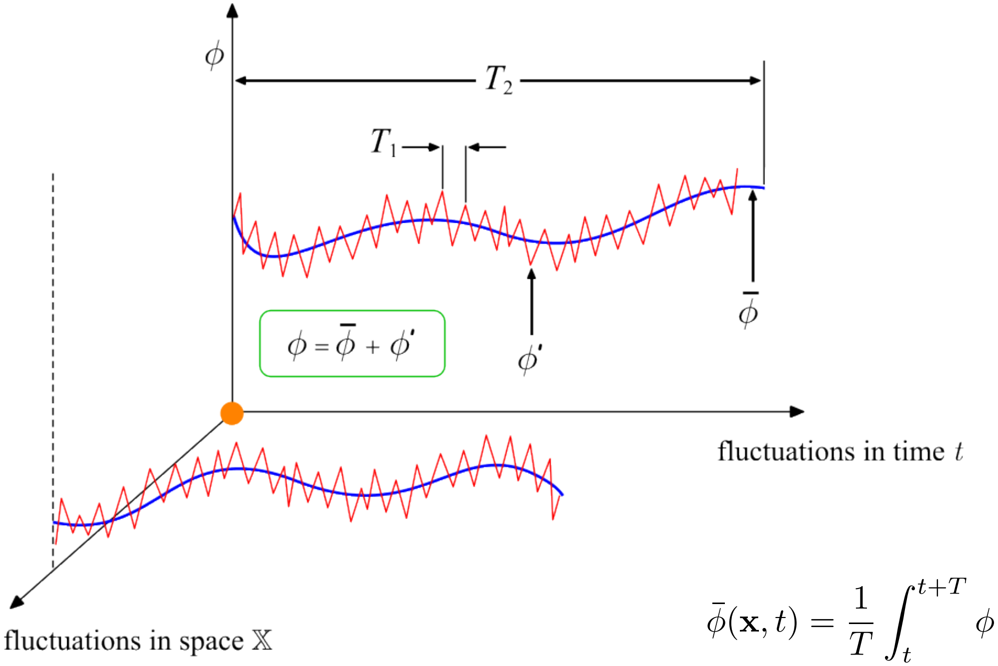
Instantaneous vorticity magnitude field

http://www.wolfdynamics.com/training/turbulence/image6.gif



- The Reynolds number in this case is 200, for these conditions, the flow still is laminar.
- We are in the presence of the Von Karman vortex street, which is the periodic shedding of vortices caused by the unsteady separation of the fluid around blunt bodies.
- Vorticity is not a direct indication of turbulence.
- However, turbulent flows are rotational, they exhibit vortical structures.

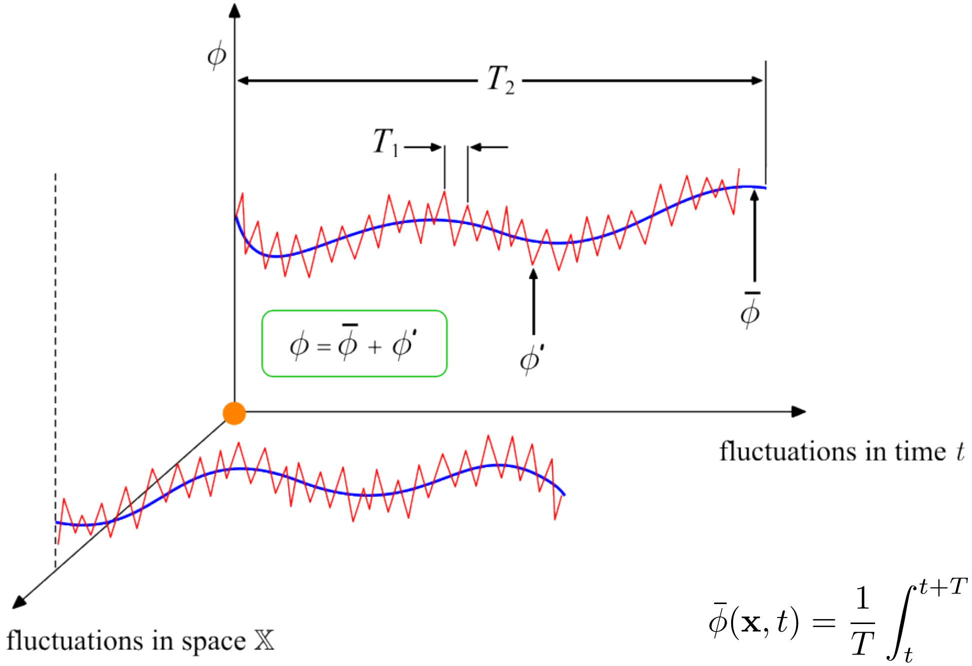
Turbulence modeling – Fluctuations of transported quantities



$$\bar{\phi}(\mathbf{x},t) = \frac{1}{T}\int_{t}^{t+T} \phi(\mathbf{x},t)\mathrm{d}t, \quad T_1 << T << T_2$$

- We have defined turbulence as an unsteady, aperiodic motion in which velocity components and every transported quantity fluctuate in space and time.
- For most engineering applications it is impractical to account for all these instantaneous fluctuations.
- Therefore, we need to somehow remove, avoid, or filter those small scales by using models.

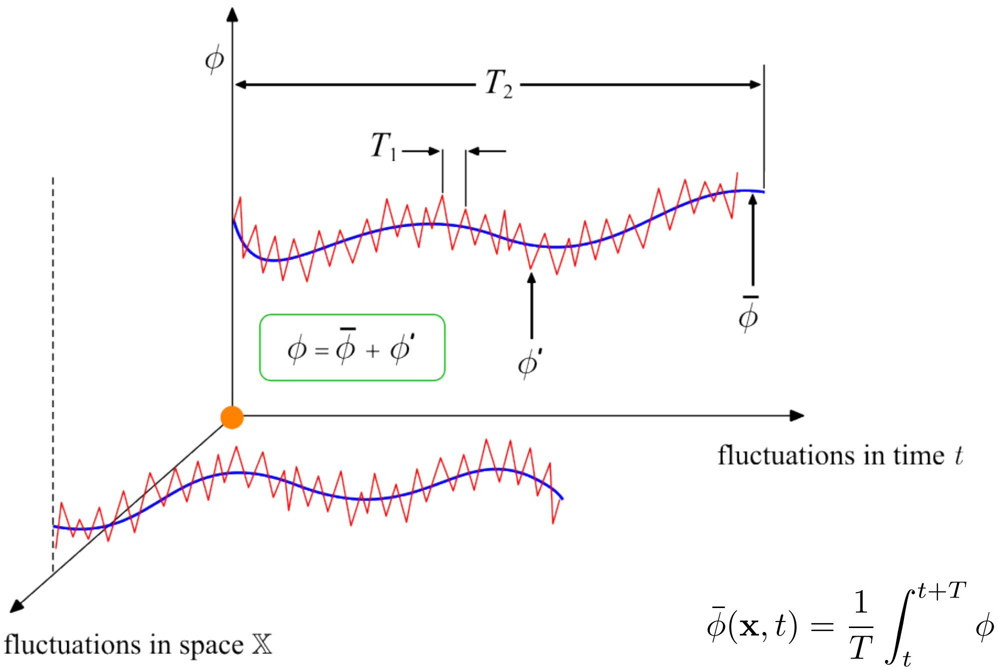
Turbulence modeling – Fluctuations of transported quantities



$$\bar{\phi}(\mathbf{x},t) = \frac{1}{T}\int_{t}^{t+T} \phi(\mathbf{x},t)\mathrm{d}t, \quad T_1 << T << T_2$$

- To remove, avoid, or filter the instantaneous fluctuations (or small scales), two methods can be used: Reynolds averaging and filtering the governing equations.
- Both methods introduce additional terms that must be modeled for closure, Turbulence Modeling.
- We are going to talk about closure methods later.

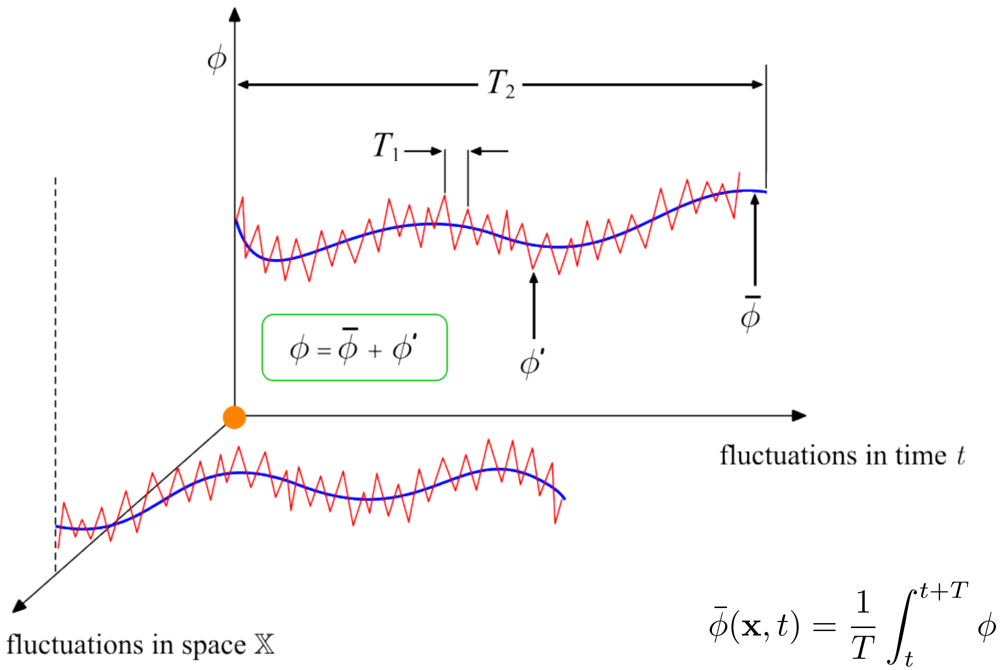
Turbulence modeling – Fluctuations of transported quantities



$$\bar{\phi}(\mathbf{x},t) = \frac{1}{T}\int_{t}^{t+T} \phi(\mathbf{x},t)\mathrm{d}t, \quad T_1 << T << T_2$$

- The objective of turbulence modeling is to develop equations that will predict the time averaged primitive fields (velocity, pressure, temperature, concentration, and so on), without calculating the complete turbulent flow pattern as a function of time.

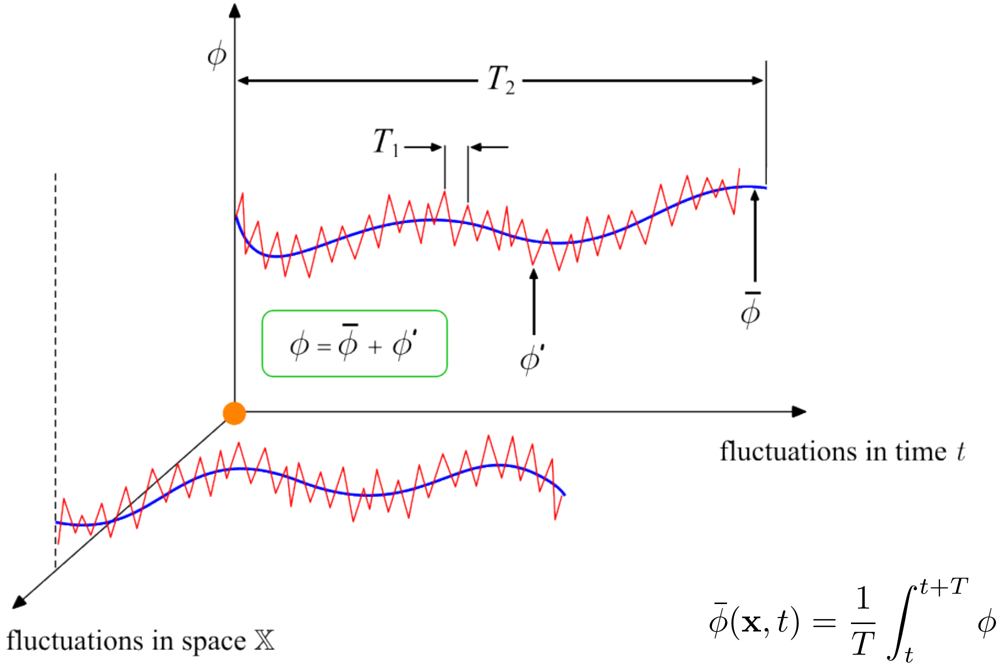
Turbulence modeling – Fluctuations of transported quantities



$$\bar{\phi}(\mathbf{x},t) = \frac{1}{T}\int_{t}^{t+T} \phi(\mathbf{x},t)\mathrm{d}t, \quad T_1 << T << T_2$$

- In other words, we do not want to resolve all the space and time scales.
  - This reduces the computational time and resources.
  - However, you can also compute instantaneous values and calculate other statistical properties (*e.g.*, RMS, two point-correlations, PDF, and so on).
  - Turbulence modeling is very accurate, if you know what are you doing.

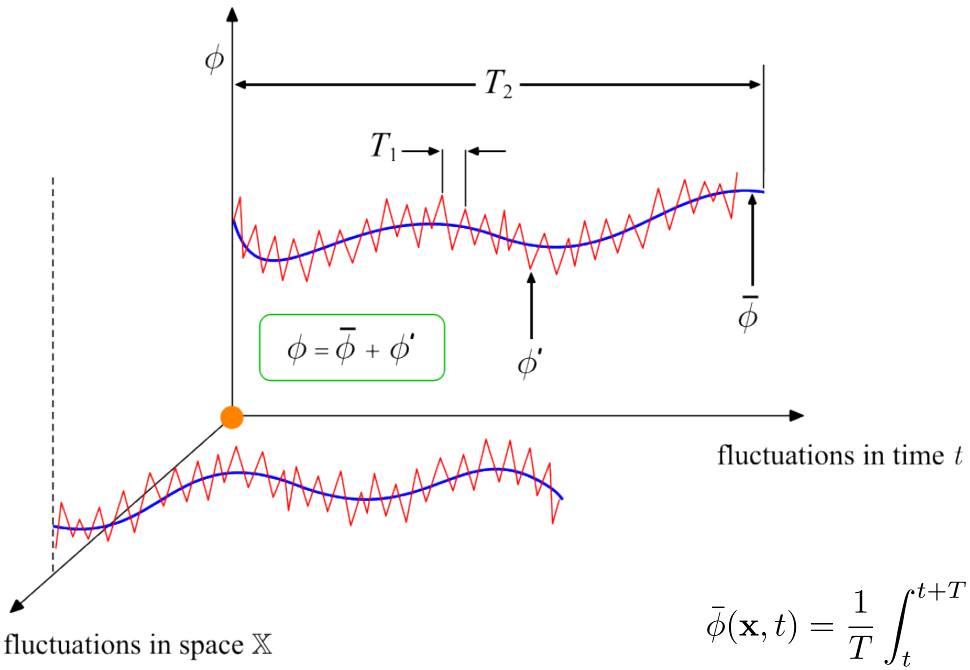
Turbulence modeling – Fluctuations of transported quantities



$$\bar{\phi}(\mathbf{x},t) = \frac{1}{T}\int_t^{t+T} \phi(\mathbf{x},t)\mathrm{d}t, \quad T_1 << T << T_2$$

- Important to understand:
    - The time averaged flow pattern is a statistical property of the flow.
    - It is not the actual flow pattern.
    - The flow pattern changes from instant to instant.
    - In engineering applications, most of the time it is enough to know the average value.

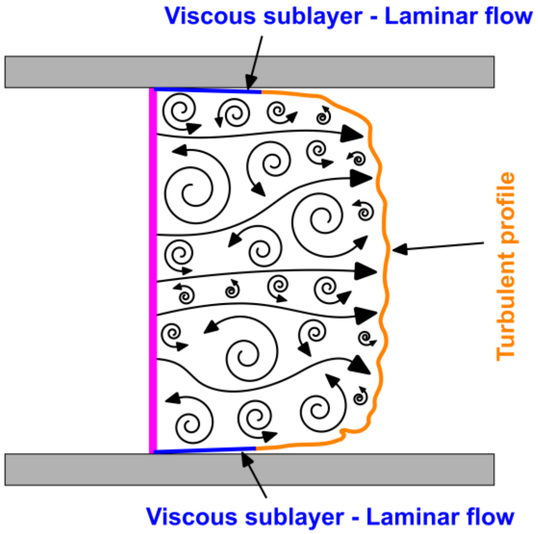
Turbulence modeling – Fluctuations of transported quantities



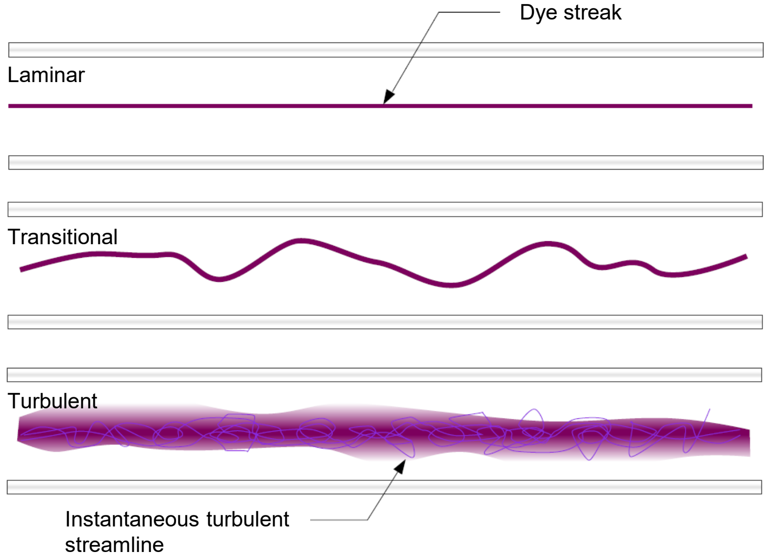
$$\bar{\phi}(\mathbf{x},t) = \frac{1}{T}\int_{t}^{t+T} \phi(\mathbf{x},t)\mathrm{d}t, \quad T_1 << T << T_2$$

- Later we are going to made a distinction between averaging stationary turbulence, averaging nonstationary turbulence, and statistically steady turbulence.

Turbulence modeling – Fluctuations of transported quantities



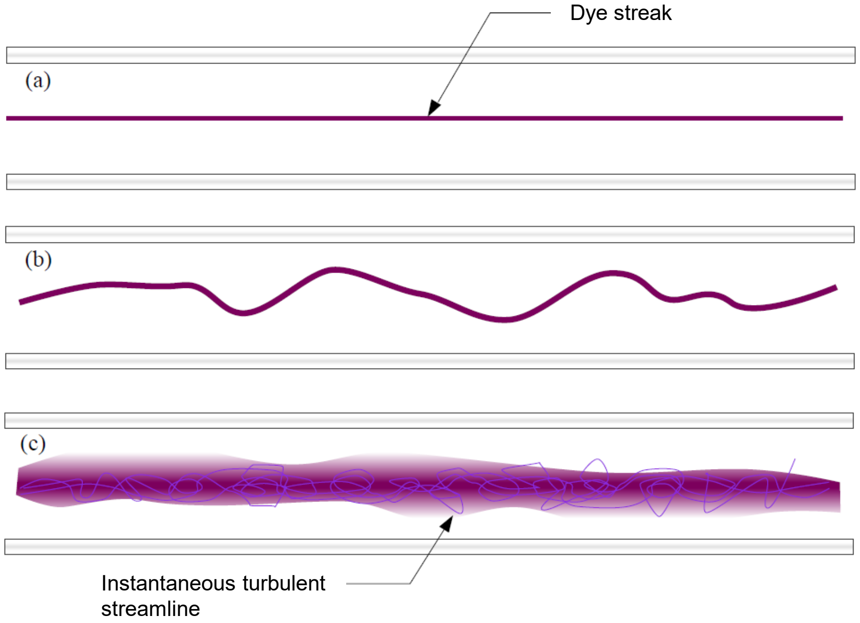
Wall bounded turbulence



Shear free turbulence

- **Turbulent flows can originate at the walls.** When this is the case, we talk about wall bounded turbulence.
- **Turbulent flows can also originate in the absence of walls (or far from walls).** When this is the case, we talk about shear free turbulence (usually jets, heated walls, atmospheric flows).
- But indifferently of the type of turbulence, that is, wall bounded or shear free, turbulent motion has a direct effect on the velocity profiles and mixing of transported quantities (heat transfer rate, species concentration, mass transfer, and so on).

Turbulence modeling – Fluctuations of transported quantities

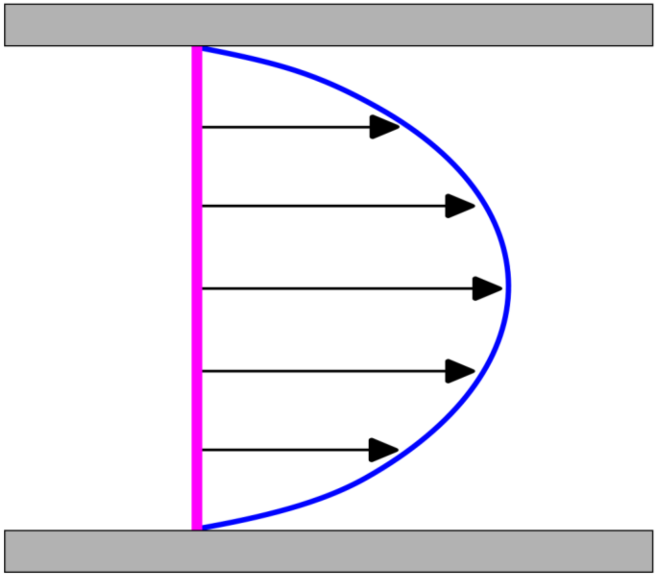


Flow in a pipe. (a) Laminar, (b) Transitional, (c) Turbulent

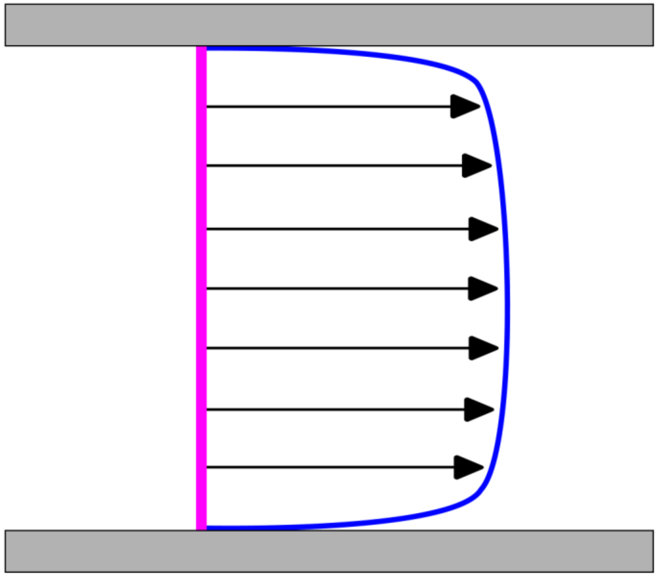
- Turbulence has a direct effect on the velocity profiles and mixing of transported quantities.
    - **Case (a)** correspond to a laminar flow, where the dye can mix with the main flow only via molecular diffusion, this kind of mixing can take very long times.
    - **Case (b)** shows a transitional state where the dye streak becomes wavy, but the main flow still is laminar.
    - **Case (c)** shows the turbulent state, where the dye streak changes direction erratically, and the dye has mixed significantly with the main flow due to the velocity fluctuations.
- This image give us an idea of what happens at the core if the flow, but what about the walls?

Introduction to turbulence modeling – Basic concepts

Turbulence modeling – Fluctuations of transported quantities



Laminar flow in a pipe

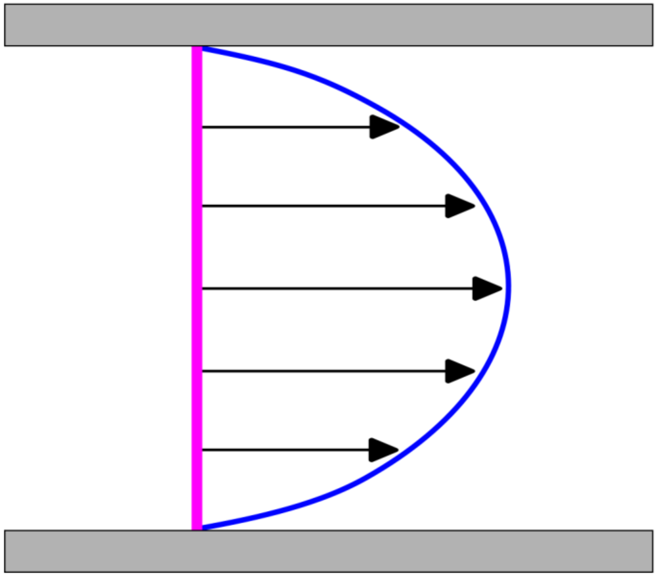


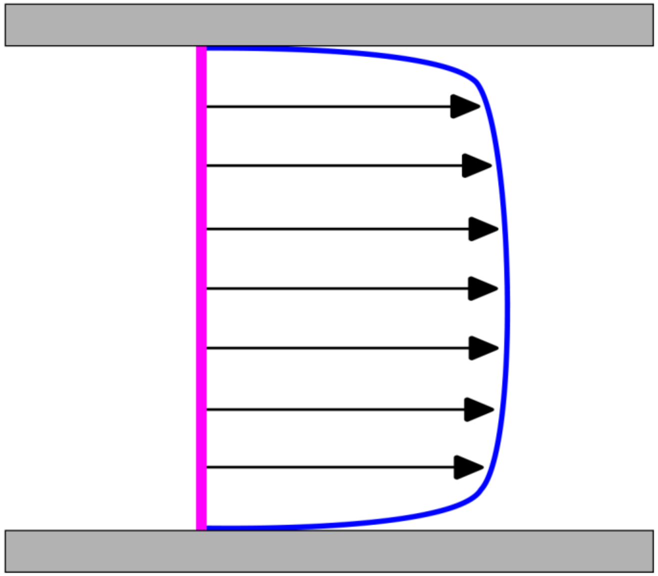
Turbulent flow in a pipe

- Turbulence has a direct effect on the velocity profiles and mixing of transported quantities.
  - In the **laminar case**, the velocity gradient close to the walls is small (therefore the shear stresses are lower).
  - The **turbulent case** shows two regions. One thin region close to the walls with very large velocity gradients (hence large shear stresses), and a region far from the wall where the velocity profile is nearly uniform.
  - In the illustration, the velocity profile of the turbulent case has been averaged.
  - In reality, random fluctuations of the velocity field and transported quantities are present.

Introduction to turbulence modeling – Basic concepts

Turbulence modeling – Fluctuations of transported quantities


Laminar flow in a pipe


Turbulent flow in a pipe

- Turbulence has a direct effect on the velocity profiles and mixing of transported quantities.
  - The larger the velocity gradient close to the walls, the larger the wall shear stresses will be.
  - Recall that the wall shear stress τ_{wall} for laminar flows or in the viscous sub-layer* is equal to:

$$\tau_{wall} = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

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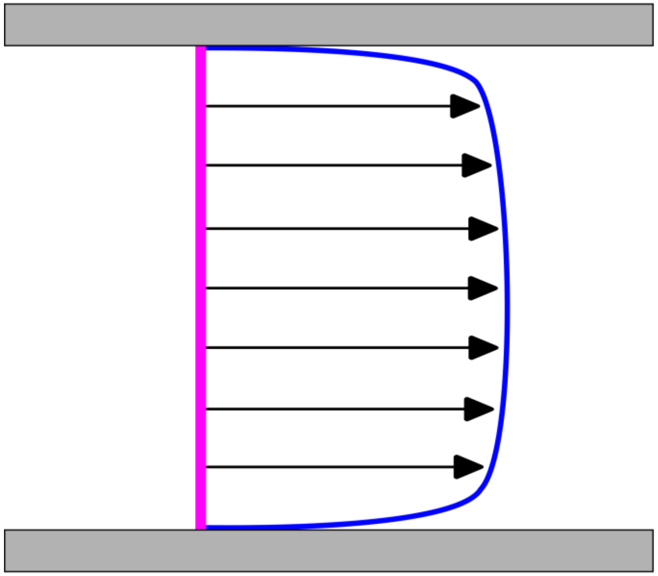
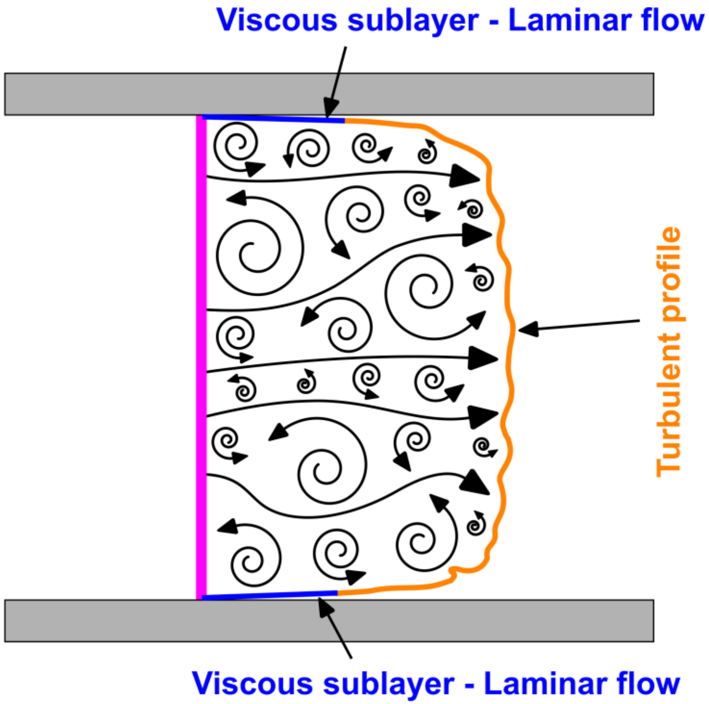
* The viscous sub-layer is region of the turbulent boundary layer very close to the wall where the flow is laminar.

Turbulence modeling – Fluctuations of transported quantities



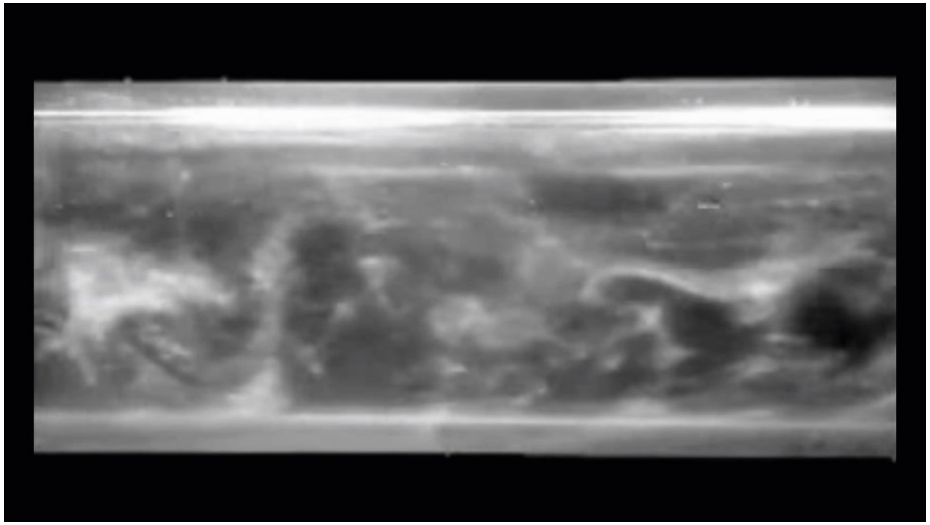
Averaged turbulent flow



Instantaneous turbulent flow

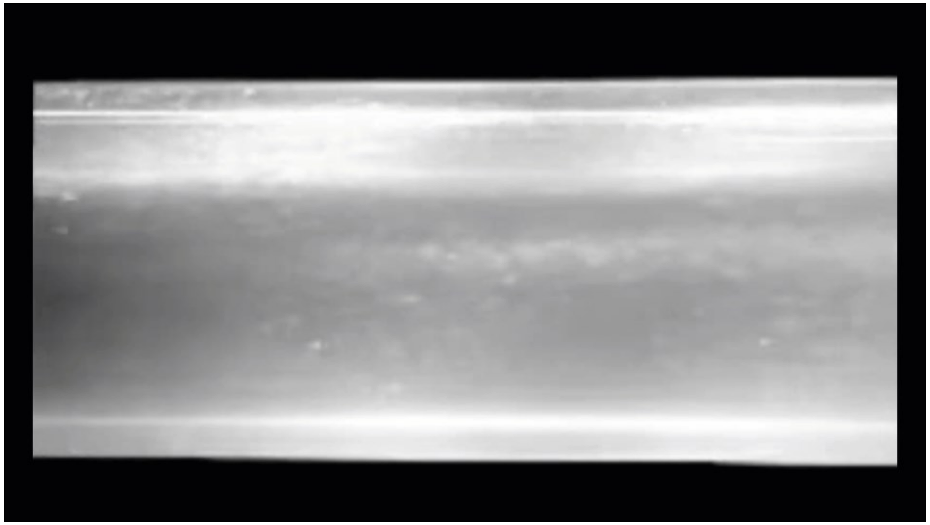
- In the left figure, the velocity profile has been averaged.
- In reality, the velocity profile fluctuates in time (right figure).
- The thin region close to the walls has very large velocity gradients and is laminar.
- Far from the flows, the flow becomes turbulent.
- Turbulence increases the wall shear stresses and enhances mixing.

Turbulence modeling – Fluctuations of transported quantities



Turbulent flow in a pipe

www.wolfdynamics.com/training/turbulence/video-pipe1.gif



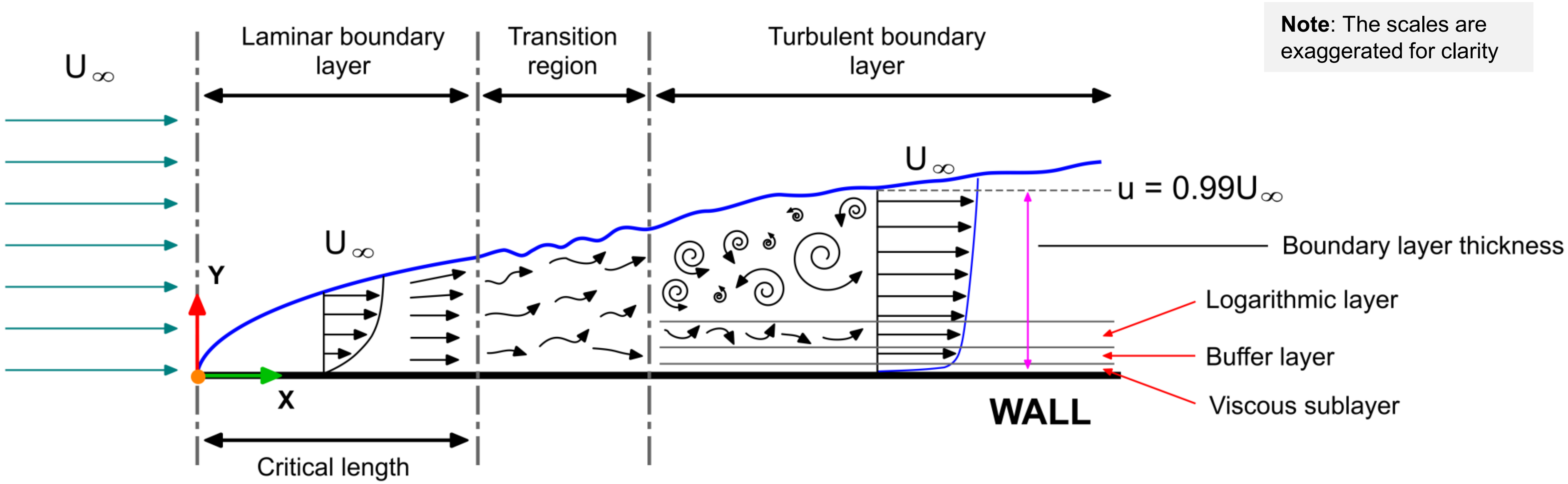
Laminar flow in a pipe

www.wolfdynamics.com/training/turbulence/video-pipe2.gif

Video credit: https://gfm.aps.org/meetings/dfd-2017/59997842b8ac316d3884197e
DOI: https://doi.org/10.1103/APS.DFD.2017.GFM.V0013

This video has been edited to fit this presentation.

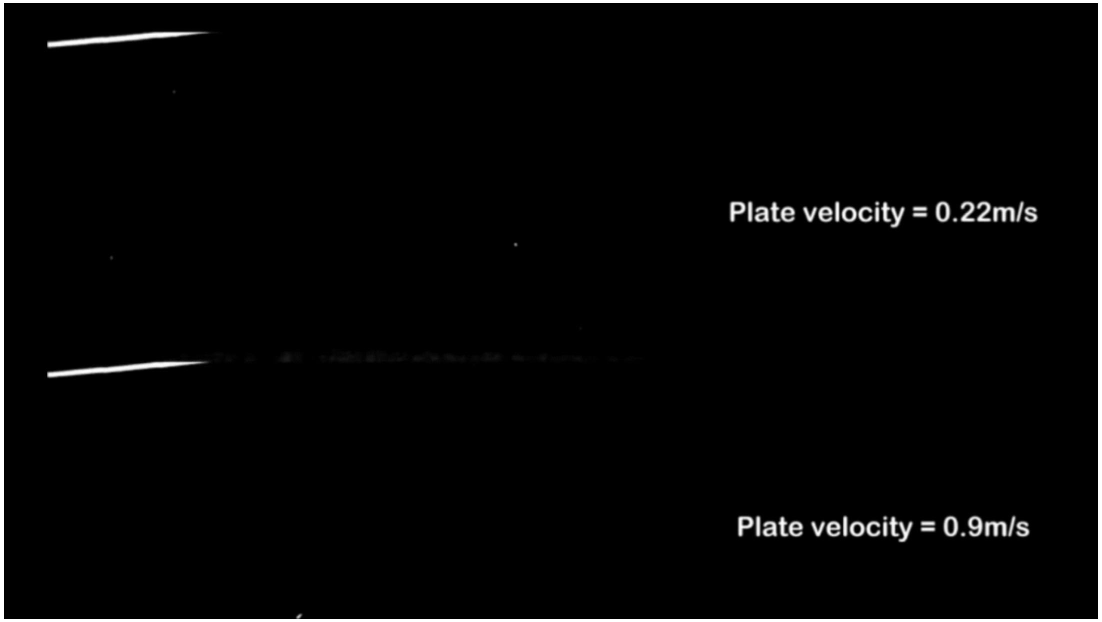
Turbulence modeling – Boundary layer



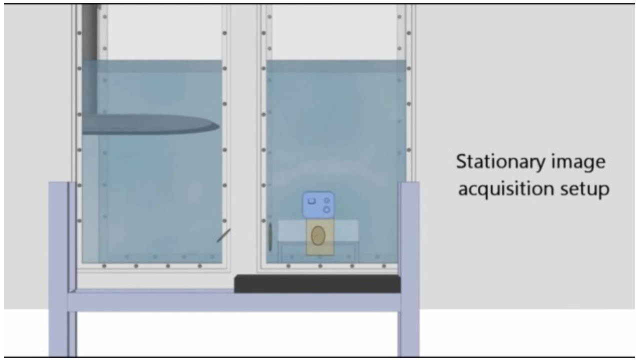
Boundary layer (Laminar-Transitional-Turbulent flow)

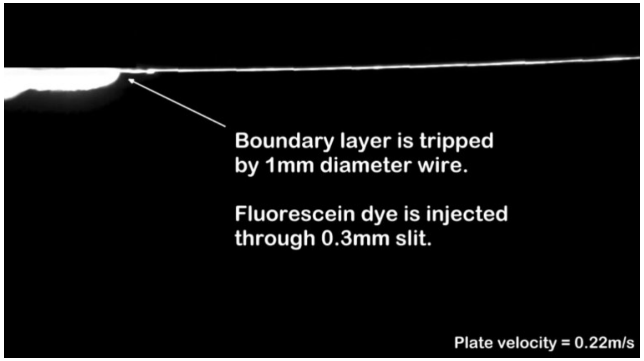
- In this case, a laminar boundary layer starts to form at the leading edge.
- As the flow proceeds further downstream, large shear stresses and velocity gradient develop within the boundary layer. At one point the flow becomes turbulent.
- The turbulent motion increases the mixing and the boundary layer mixing.
- What is happening in the transition region is not well understood. The flow can become laminar again or can become turbulent.
- As for the pipe flow, the velocity profiles in the laminar and turbulent regions are different.

Turbulence modeling – Fluctuations of transported quantities



Turbulent boundary layer on a flat plate
www.wolfdynamics.com/training/turbulence/video-bl1.mp4





Video credit: https://www.youtube.com/watch?v=e1TbkLIDWys
arXiv: https://arxiv.org/abs/1210.3881

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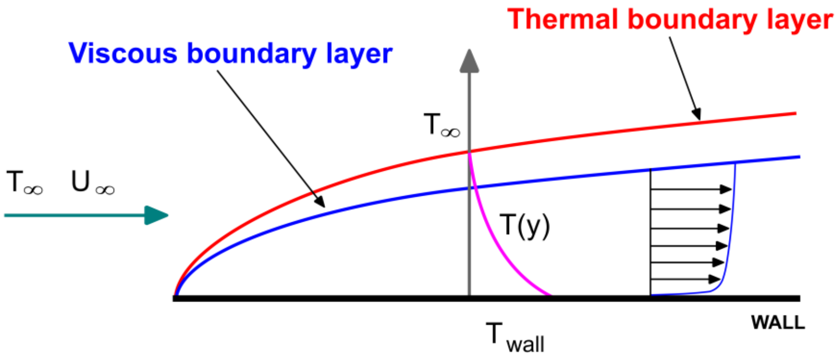
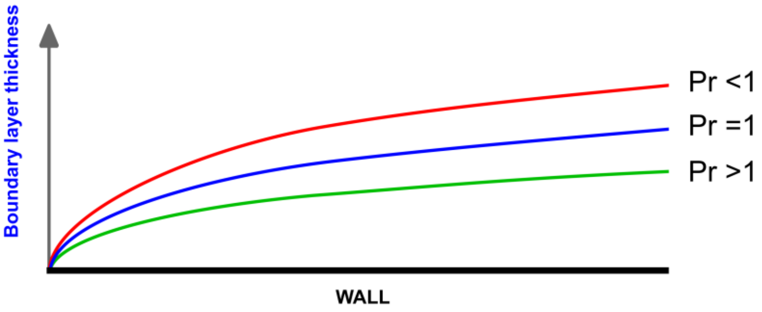
Turbulence modeling – Thermal boundary layer



Thermal boundary layer vs. Viscous boundary layer
Forced convection

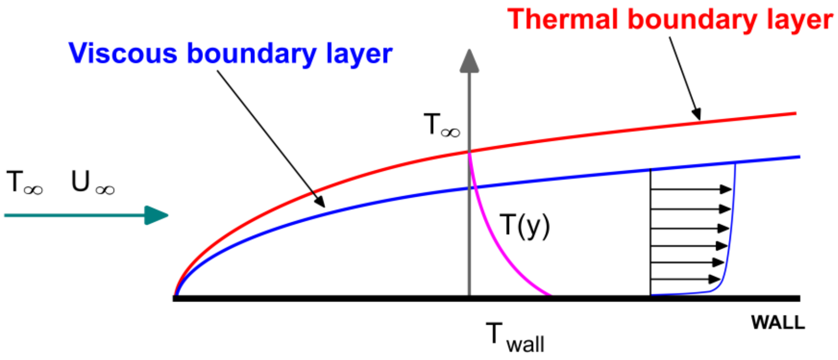


Thermal boundary layer in function of Prandtl number (Pr)

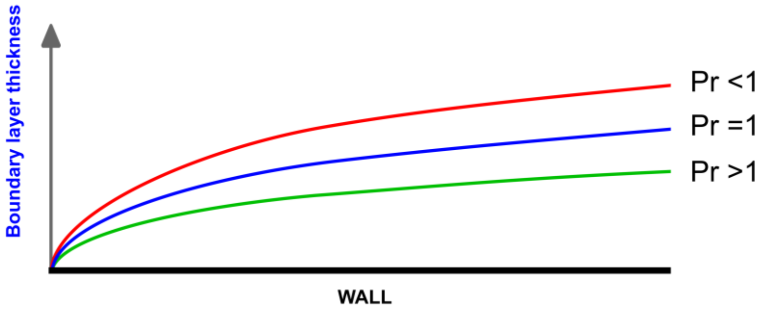
Momentum and thermal boundary layer

- Just as there is a viscous boundary layer in the velocity distribution (or momentum), there is also a thermal boundary layer.
- Thermal boundary layer thickness is different from the thickness of the viscous sublayer (momentum), and is fluid dependent.
- The thickness of the thermal sublayer for a high Prandtl number fluid (e.g., water) is much less than the momentum sublayer thickness.

Turbulence modeling – Thermal boundary layer



Thermal boundary layer vs. Viscous boundary layer
Forced convection

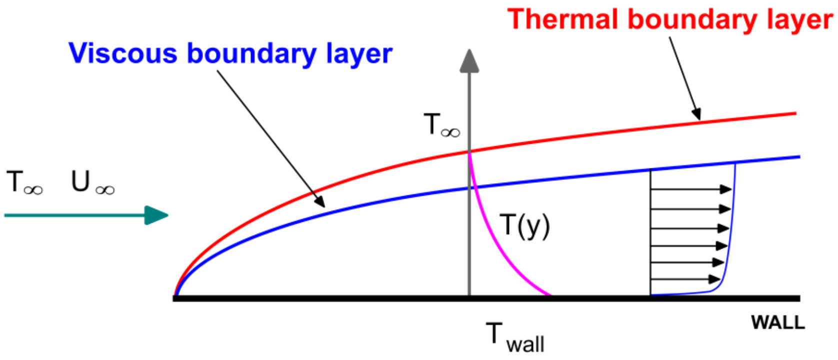


Thermal boundary layer in function of Prandtl number (Pr)

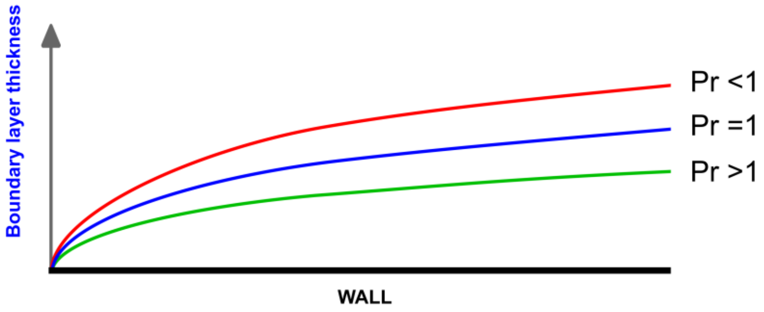
Momentum and thermal boundary layer

- For fluids of low Prandtl numbers (*e.g.*, air), it is much larger than the momentum sublayer thickness.
- For Prandtl number equal 1, the thermal boundary layer is equal to the momentum boundary layer.
- Note that the thickness of the thermal sublayer for a high Prandtl number fluid (e.g., water) is much less than the momentum sublayer thickness.
- Therefore, the mesh requirements close to the walls need to be corrected for the thinner thermal boundary layer.

Turbulence modeling – Thermal boundary layer



Thermal boundary layer vs. Viscous boundary layer
Forced convection



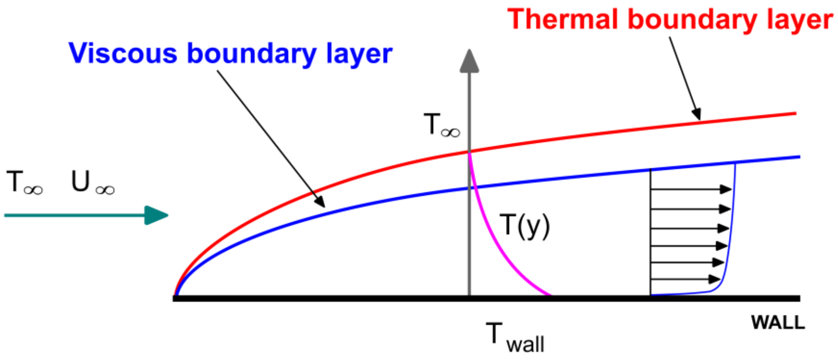
Thermal boundary layer in function of Prandtl number (Pr)

Momentum and thermal boundary layer

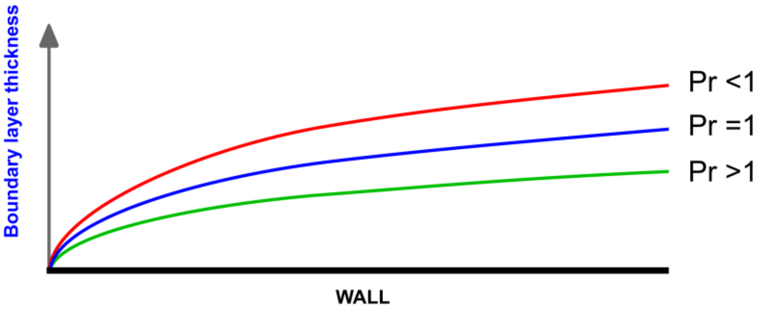
- Using the following relationships, the y^+ value, and therefore the distance from to wall to the first mesh node normal to the wall, can be corrected as follows,

$$\frac{\delta_{\text{hydrodynamic b.l.}}}{\delta_{\text{thermal b.l.}}} \approx \frac{y_{\text{viscous s.l.}}}{y_{\text{thermal s.l.}}} \approx \sqrt{Pr} \qquad \text{therefore} \qquad y^+ \approx \frac{1}{\sqrt{Pr}}$$

Turbulence modeling – Thermal boundary layer



Thermal boundary layer vs. Viscous boundary layer
Forced convection



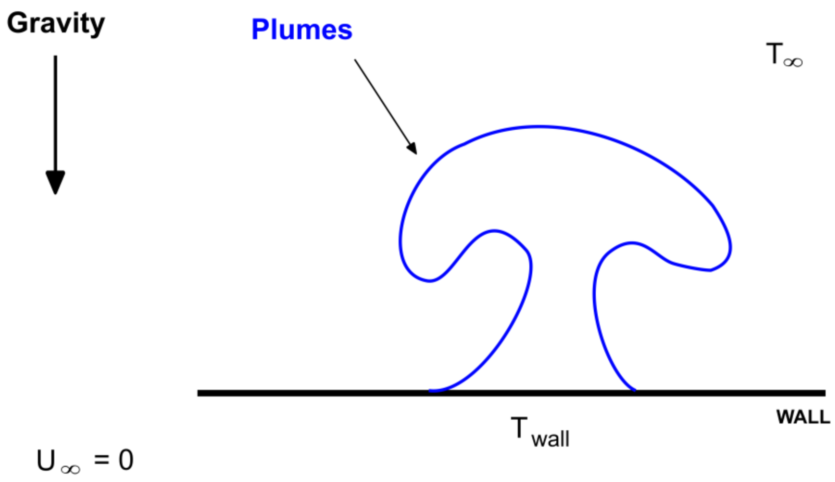
Thermal boundary layer in function of Prandtl number (Pr)

Momentum and thermal boundary layer

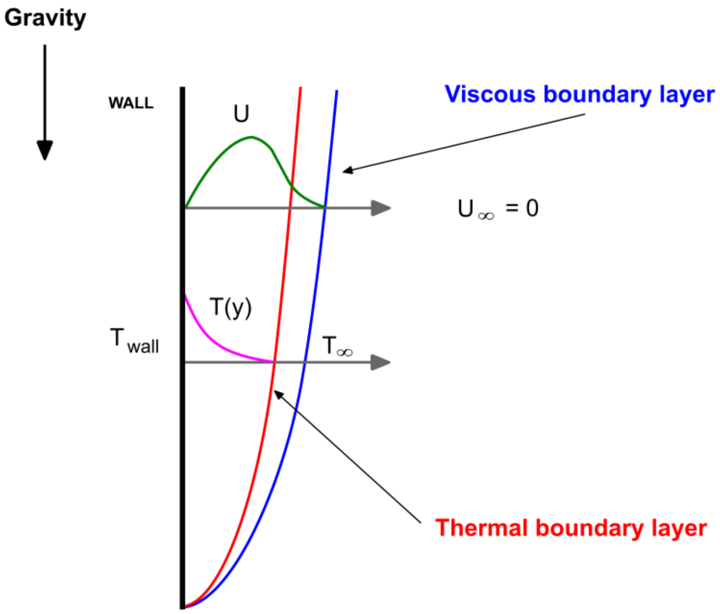
- A similar situation exists when working with multispecies transport and using the Schmidt number (Sc).
- If the Schmidt number is considerably larger than unity, then the thickness of the thermal diffusion sublayer is much less than the momentum sublayer thickness.
- Using the Schmidt number, the y^+ value, and therefore the distance from to wall to the first mesh node normal to the wall, can be corrected as follows,

$$y^+ \approx \frac{1}{\sqrt[3]{Sc}}$$

Turbulence modeling – Thermal boundary layer



Horizontal heated plate immersed in a quiescent fluid.
Natural convection



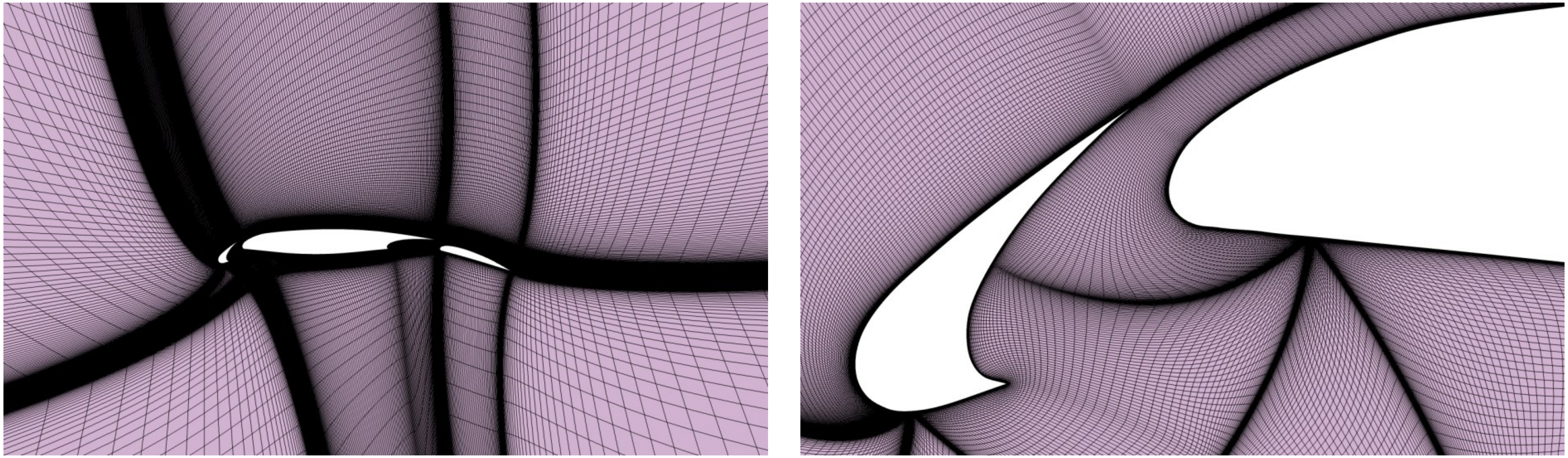
Vertical heated plate immersed in a quiescent fluid.
Natural convection.

Natural convection in a heated plate

- As the fluid is warmed by the plate, its density decreases, and a buoyant force arises which induces flow motion in the vertical or horizontal direction.
- The force is proportional to $(\rho - \rho_\infty) \times g$, therefore gravity must be considered.

Introduction to turbulence modeling – Basic concepts

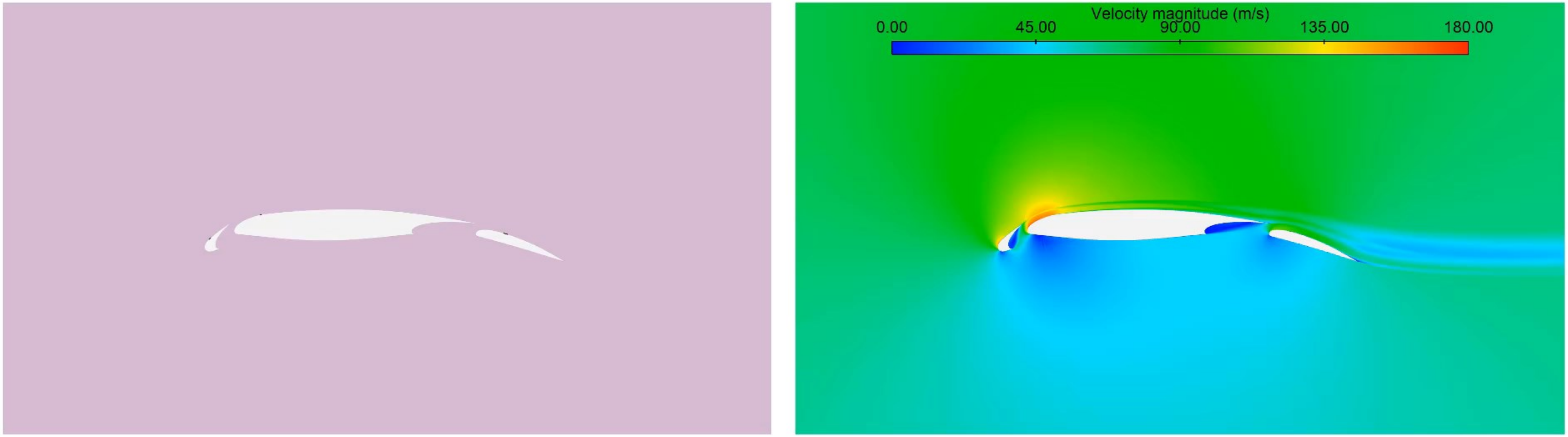
Turbulence modeling – Laminar separation bubbles and transition to turbulence



- Maybe, the most challenging topic of turbulence modeling is the prediction of transition to turbulence.
- Trying to predict transition to turbulence in CFD requires very fine meshes and well calibrated models.
- Many traditional turbulence models assume that the boundary layer is turbulent in all its extension.
- But assuming that the boundary layer is entirely turbulent might not be a good assumption, as in some regions the boundary layer might still be laminar, so we may be overpredicting drag forces or predicting wrong separation points.
- In many applications, transition to turbulence is preceded by laminar separation bubbles (LSB), which are laminar recirculation areas that separate from the wall and reattach in a very short distance and are very sensitive to disturbances.
- After the LSB, the flow becomes turbulent.

Introduction to turbulence modeling – Basic concepts

Turbulence modeling – Laminar separation bubbles and transition to turbulence

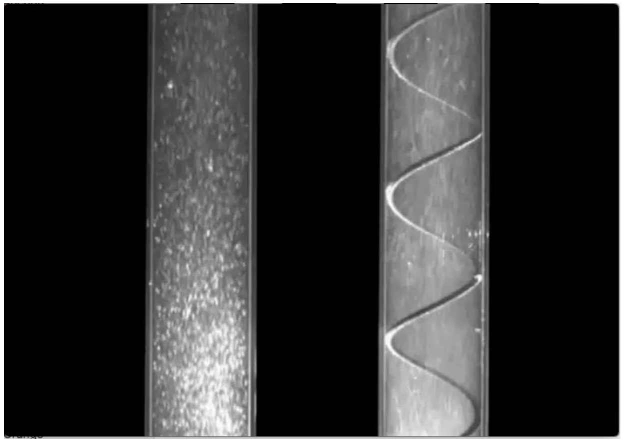


Laminar separation bubbles

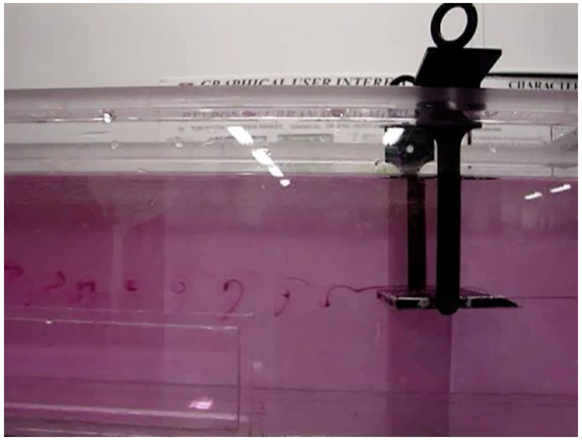
http://www.wolfdynamics.com/images/airfoil.mp4

Introduction to turbulence modeling – Basic concepts

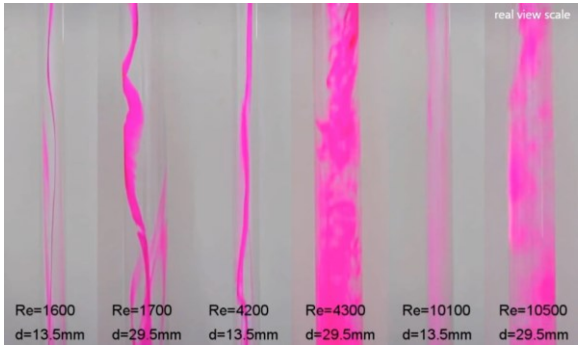
Turbulence flows videos



Laminar-Turbulent flow in a pipe
Smooth vs corrugated tube
https://www.youtube.com/watch?v=WG-YCpAGgQQ



Flow over a flat plate
Attached and separated boundary layer
https://www.youtube.com/watch?v=zsO5BQA_CZk



Reynolds' dye experiment
Various types of flow - Laminar-Transitional-Turbulent
https://www.youtube.com/watch?v=ontHCuI6eB4

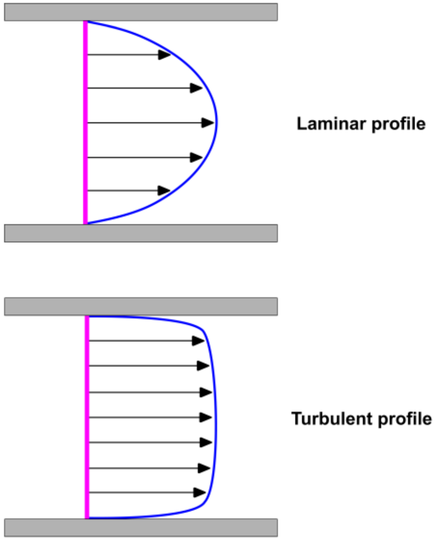
Laminar-Turbulent vortex shedding
https://www.youtube.com/watch?v=JI0M1gVNhbw

Roadmap to Lecture 2

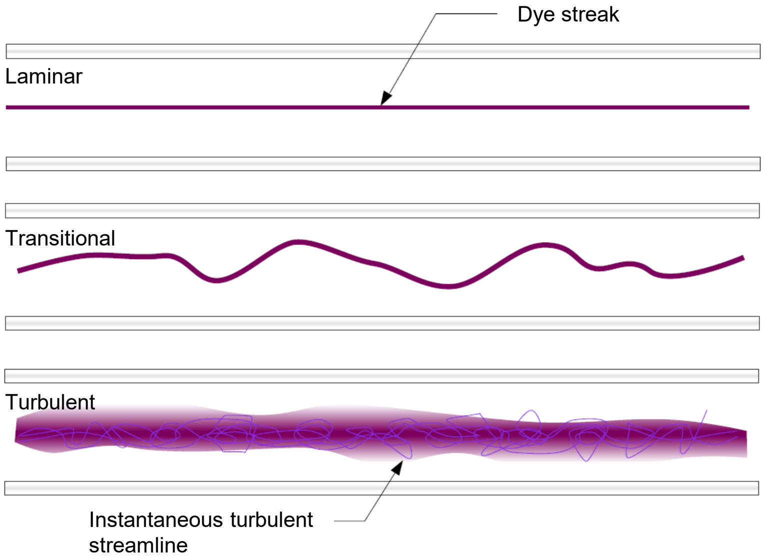
1. ~~The turbulent world around us~~
2. ~~Turbulence, does it matter?~~
3. ~~Introduction to turbulence modeling – Basic concepts~~
4. **Wall bounded flows and shear flows**
5. A peek to the turbulence closure problem, some correlations in turbulence modeling, and the energy cascade

Wall bounded flows and shear flows

Wall bounded and shear free flows



Wall bounded turbulence

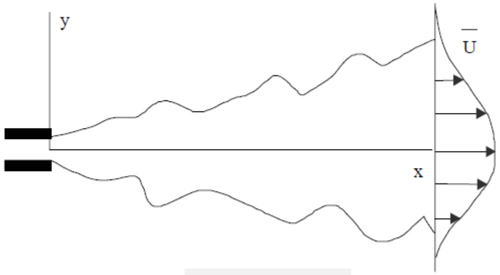


Shear free turbulence

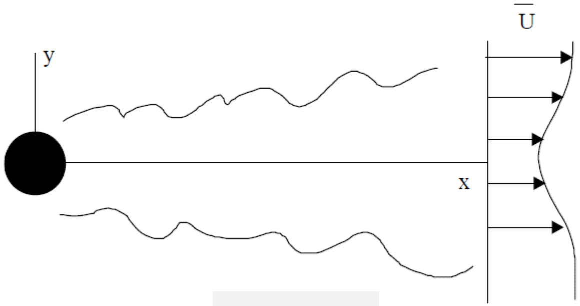
- Flows (laminar or turbulent), can originate at the walls. When this is the case, we talk about wall bounded turbulence (boundary layers).
- They can also originate in the absence of walls (not bounded to walls), or far from walls. When this is the case, we talk about shear free turbulence (jets, har wakes, mixing layers, thermal plumes, atmospheric flows).

Wall bounded flows and shear flows

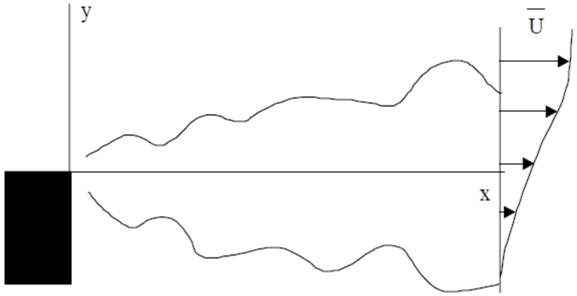
Shear free flows samples



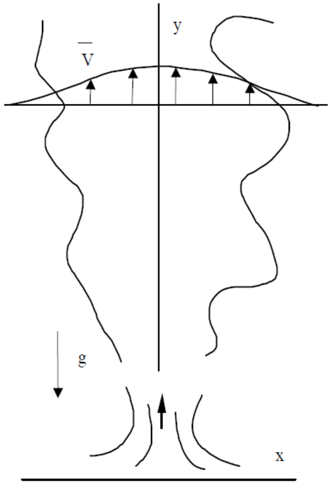
Jet



Far wake



Mixing layer



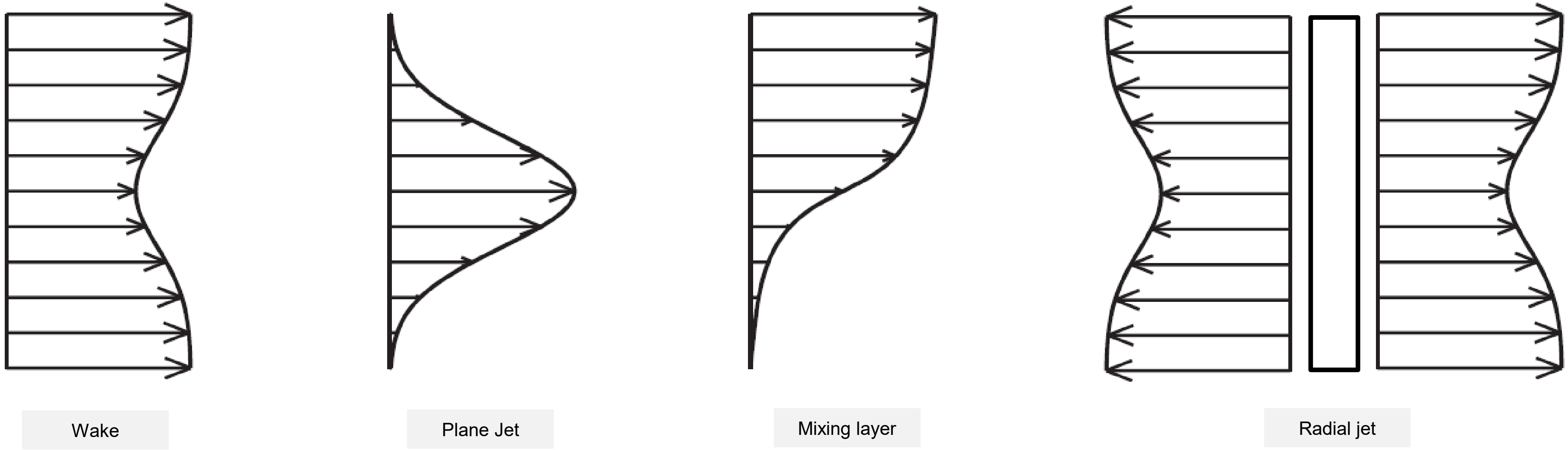
Thermal plume

Photo credit: Modeling and Simulation of Turbulent Flows. Roland Schiestel. Wiley, 2007.

Wall bounded flows and shear flows

Shear free flows samples

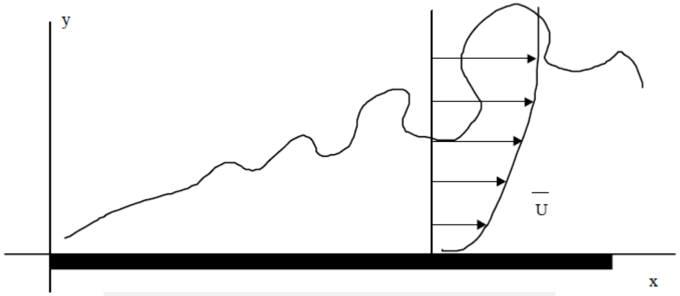
Different velocity profiles of shear free flows



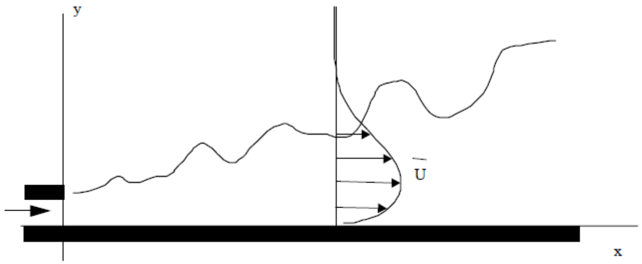
- Under certain conditions, many of these flows have analytical solutions.
- Therefore, turbulence models can be calibrated or validated using these solutions.
- Sample of analytical solutions (self-similarity solutions):
  - Far wake (Schlichting-Gersten, 2017)
  - Mixing layer (Liepmann and Laufer, 1947)
  - Plane jet (Witze and Dwyer, 1976).

Wall bounded flows and shear flows

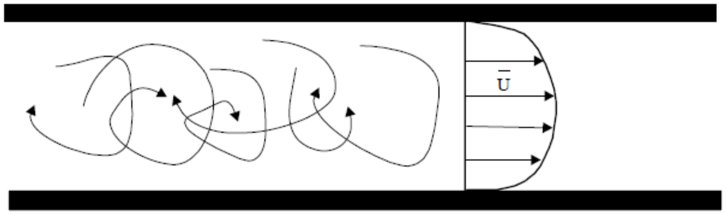
Wall bounded flows samples



Turbulent boundary layer on a flat plate



Turbulent wall jet



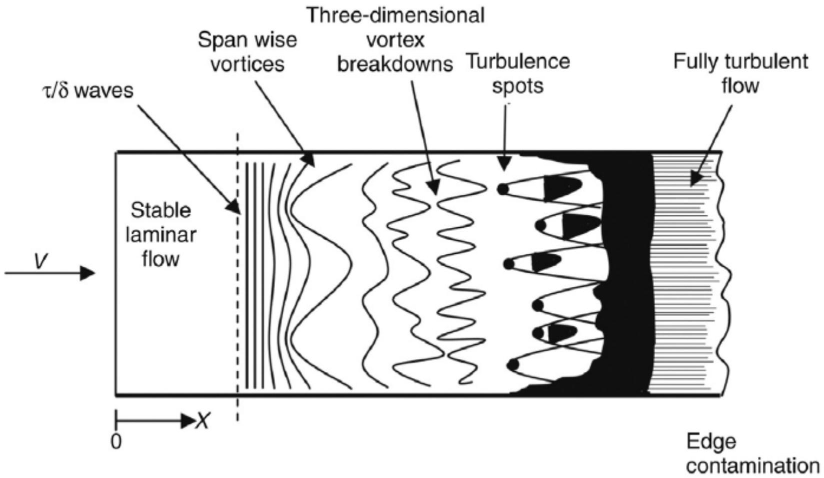
Turbulent flow in a channel or pipe

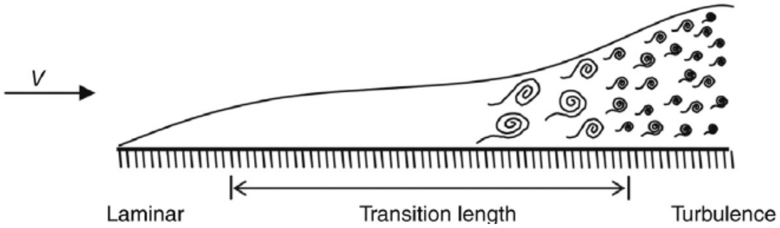
- Walls can be heated, which adds buoyancy and thermal diffusivity to the physics.
- Walls can also be rough, which will affect the boundary layer.
- Do you think in something else?

Photo credit: Modeling and Simulation of Turbulent Flows. Roland Schiestel. Wiley, 2007.

Wall bounded flows and shear flows

Wall bounded flows samples





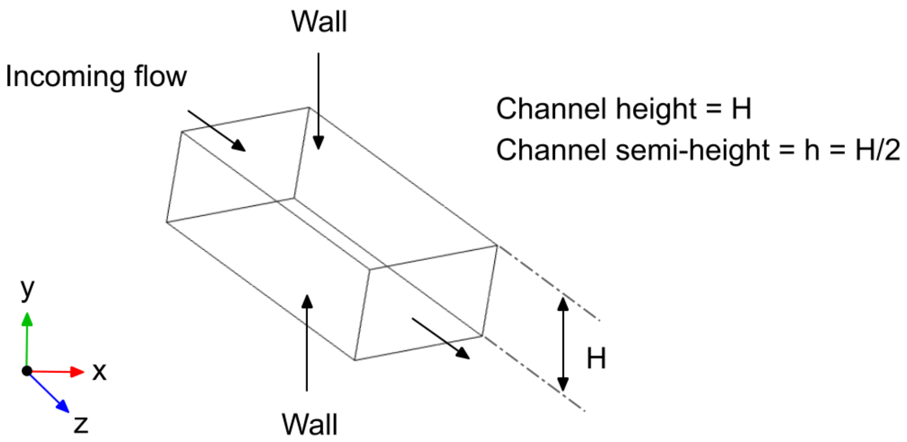
- In these cases, the no-slip boundary condition must be enforced at the walls, and we expect to find a boundary layer similar to the one depicted in the figure.
- The transition to turbulence can follow three different paths:
  - Natural.
  - Bypass.
  - Forced.
- Under certain conditions, these flows have analytical solutions.
- Therefore, turbulence models can be calibrated or validated using these solutions.
- Sample of analytical solutions:
  - Channel flows (Pope, 2000).
  - Boundary layers (Prandtl, 1925; Bradshaw et al. 1995; Pope, 2000).
  - Coutte flow (Schlichting-Gersten, 2017)

Photo credit: Basics of Engineering Turbulence. David Ting. Academic Press, 2016.

Wall bounded flows and shear flows

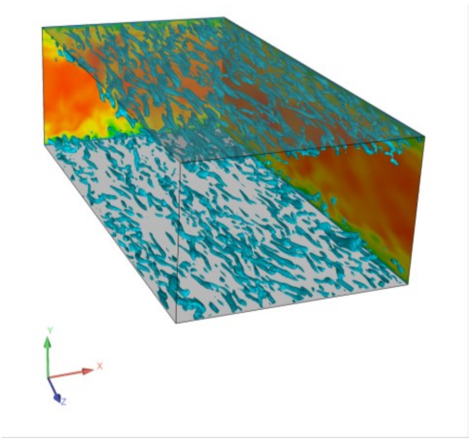
Wall bounded flow – Channel flow

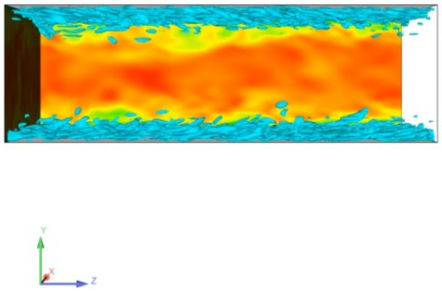
- You will find many research on the channel flow as it has an analytical solution under very specific conditions.



Channel height = H
Channel semi-height = h = H/2

Periodic boundary conditions in the streamwise direction (x) and spanwise direction (z)





Wall bounded flows and shear flows

Wall bounded flow – Channel flow

- The channel flow is usually characterize using the Re_τ, which is defined as follows,

$$Re_\tau = \frac{U_\tau \times h}{\nu} \qquad Re \approx 20 \times Re_\tau$$

- Where *h* is the channel semi-height, ν is the kinematic viscosity, and U_τ is the shear velocity and is defined as follows,

$$U_\tau = \left(\frac{\tau_{wall}}{\rho} \right)^{0.5}$$

- In the previous equation, τ_{wall} is the shear stress at the wall ($\tau_{wall} = \mu \times \frac{\partial u}{\partial y}$).

Streamwise velocity component

Component normal to the wall

- With these conditions and according to the theory of equilibrium for channels, the equilibrium between the imposed pressure drop and the wall shear stresses is given by,

$$\frac{\partial P}{\partial x} h = -\tau_{wall}$$

- At this point, we only need to set a pressure drop.

For a complete derivation of these results, refer to: S. Pope. Turbulent Flows, Cambridge University Press, 2000.

Wall bounded flows and shear flows

Wall bounded flow – Channel flow

- The following DNS simulation was conducted at a Re_τ equal to,

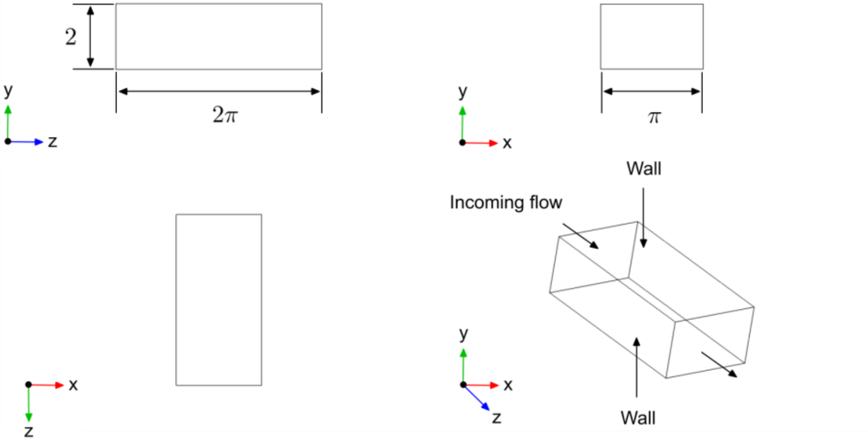
$$Re_\tau = \frac{U_\tau \times h}{\nu} = 590$$

- With the following parameters,

$$\rho = 1 \frac{kg}{m^3}$$

$$\mu = 0.001695 \frac{kg}{m\,s}$$

$$h = 1\,m$$



Periodic boundary conditions in the streamwise direction (x) and spanwise direction (z)

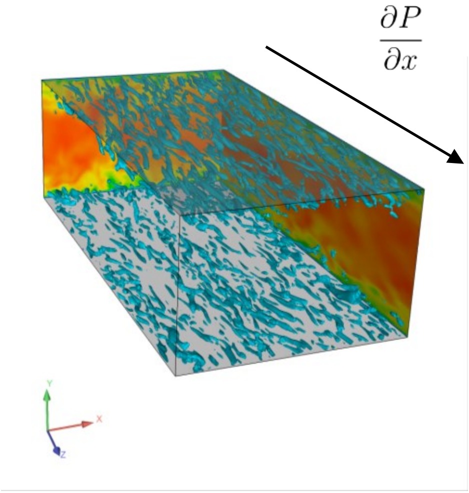
- Where *h* is the channel semi-height.
- Notice that this is an incompressible flow.

Wall bounded flows and shear flows

Wall bounded flow – Channel flow

- Periodic boundary conditions in the streamwise (z) and spanwise (x) directions were used.
- The top and bottom walls are no-slip walls.
- To onset the fluid flow we imposed a pressure drop equal to 1 Pa, such as,

$$\frac{\partial P}{\partial x} = -1\frac{Pa}{m}$$



- Therefore, the shear stresses at the wall are equal to,

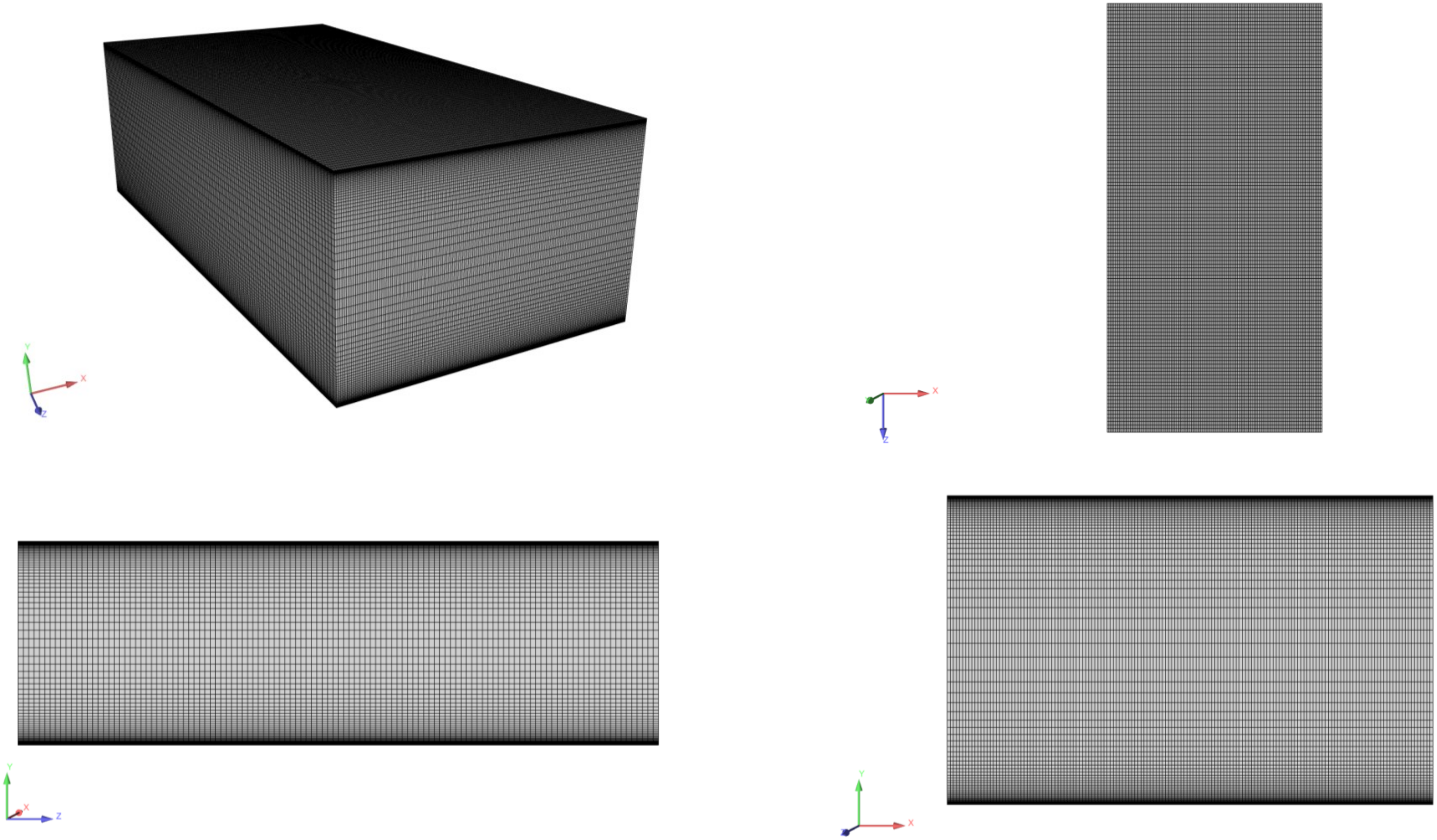
$$\tau_{wall} = 1\,Pa$$

- Finally, the shear velocity is equal to,

$$U_\tau = \left(\frac{\tau_{wall}}{\rho}\right)^{0.5} = 1\frac{m}{s}$$

Wall bounded flows and shear flows

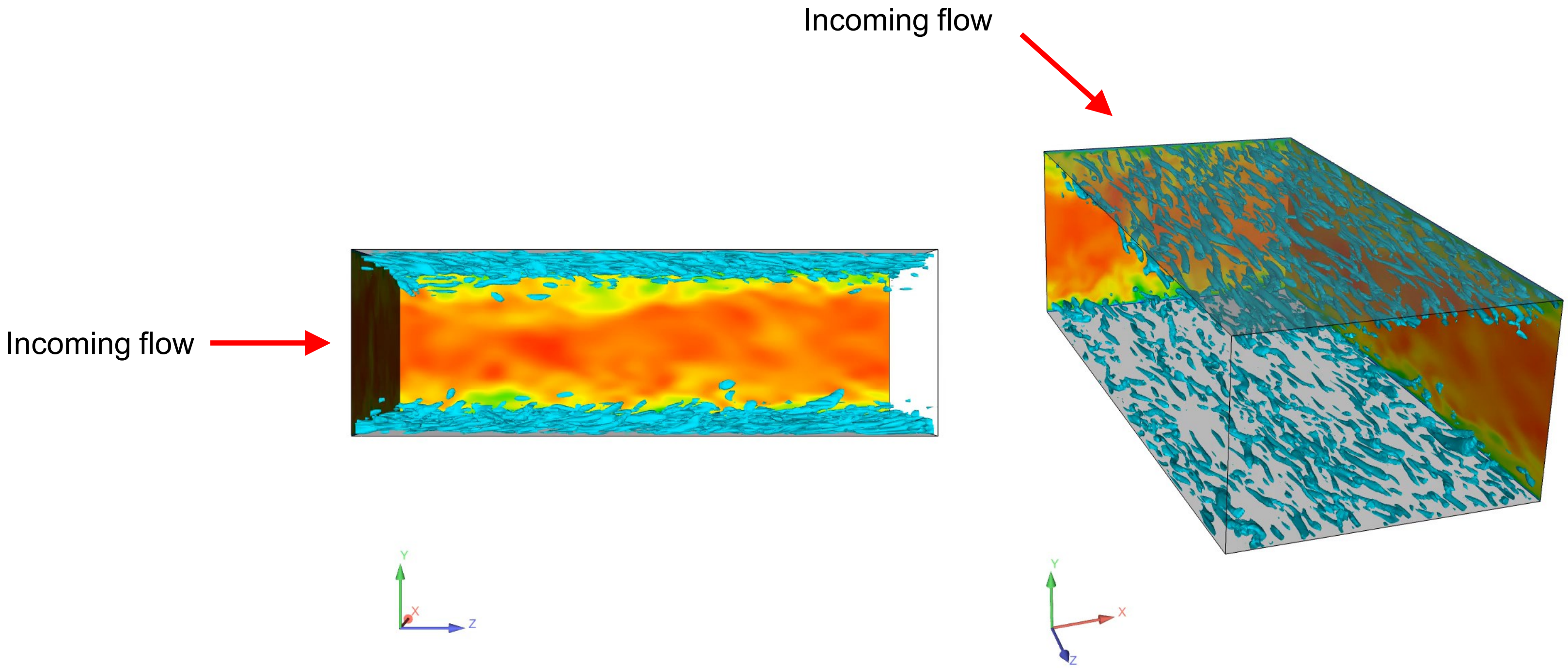
Wall bounded flow – Channel flow



Mesh

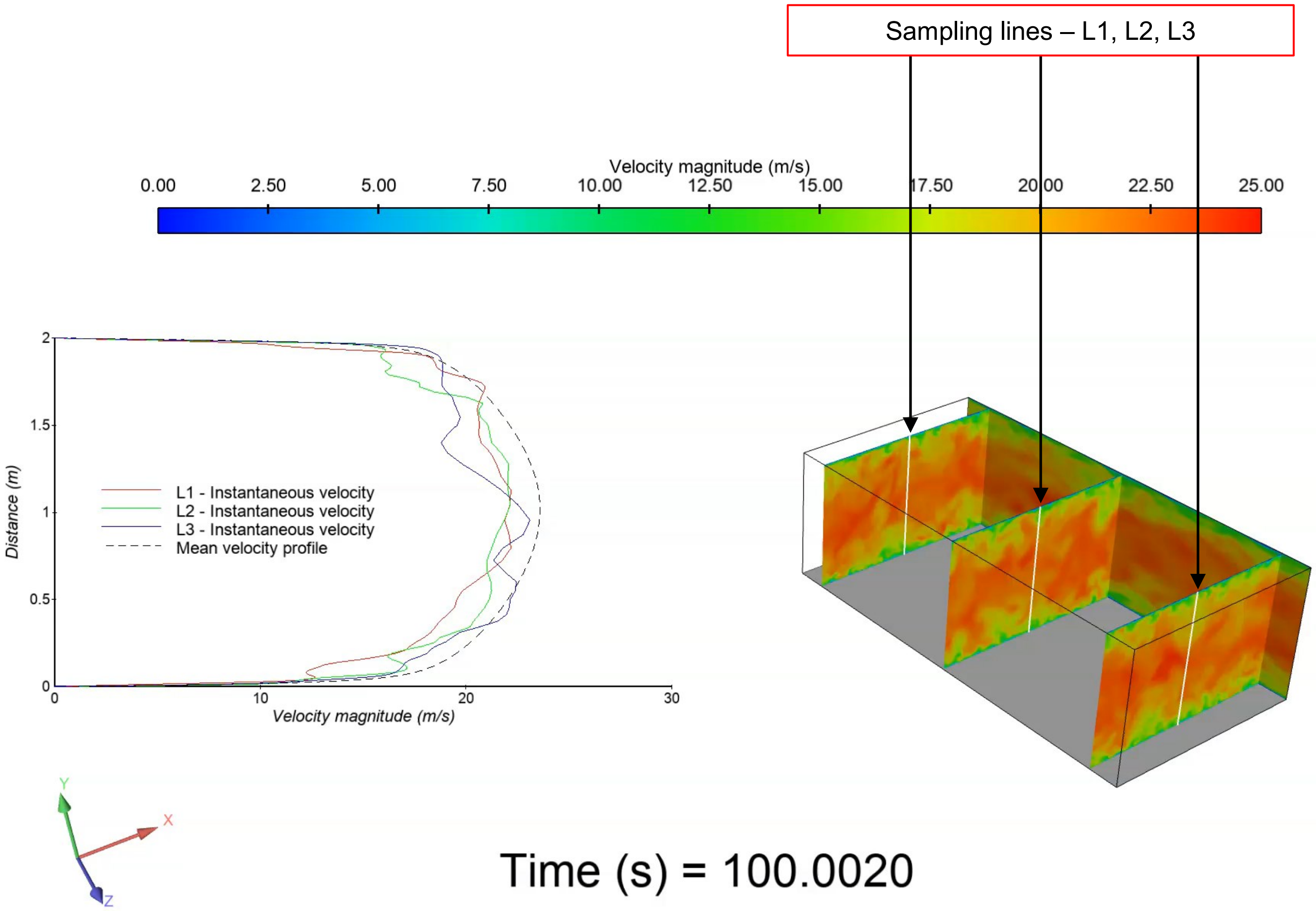
Orthogonal hexahedral mesh – Approximately 1.8 million elements

Wall bounded flow – Channel flow



Wall bounded flow – Channel flow
http://www.wolfdynamics.com/training/turbulence/channel1.mp4

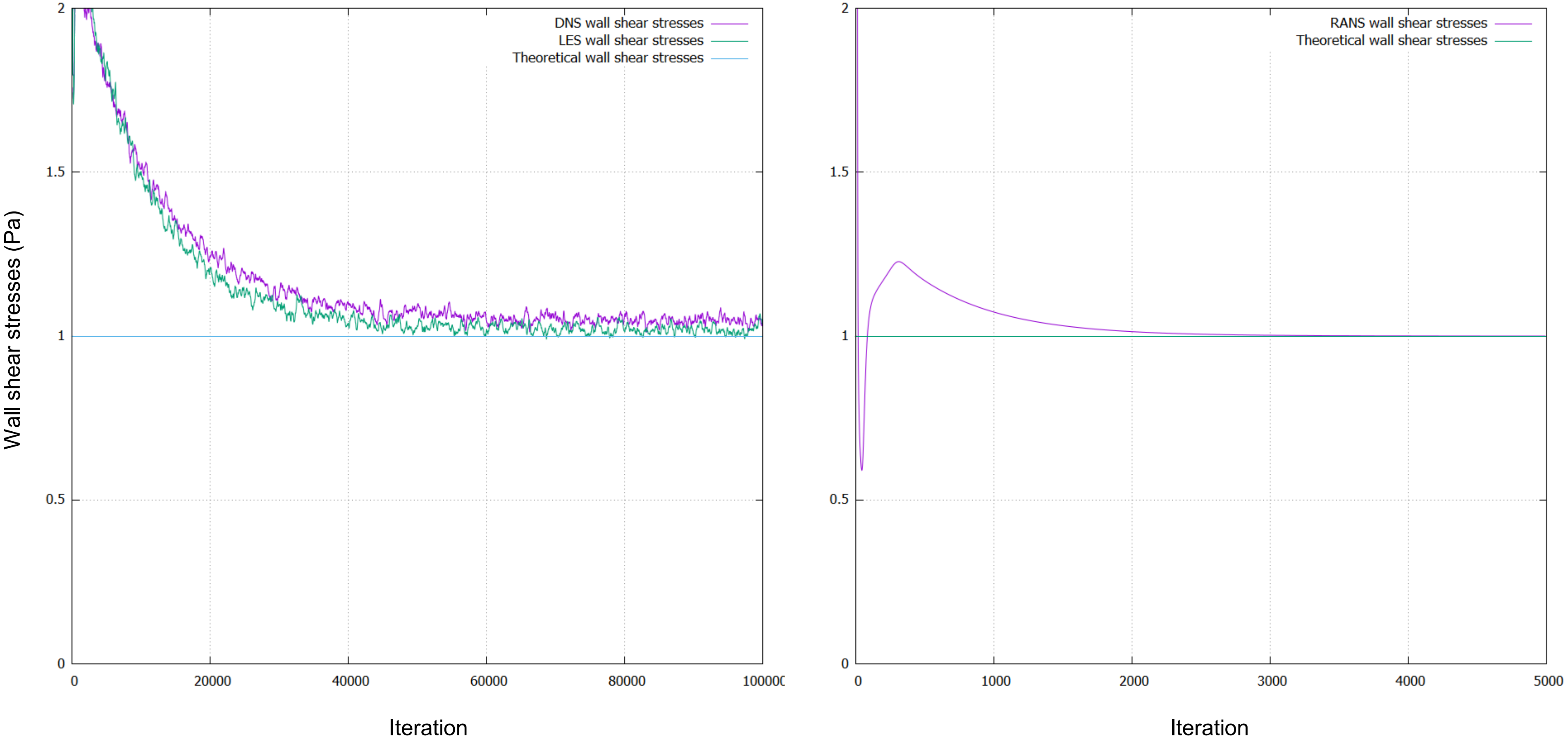
Wall bounded flow – Channel flow


Sampling lines – L1, L2, L3

Wall bounded flow – Channel flow
http://www.wolfdynamics.com/training/turbulence/channel2.mp4

Wall bounded flows and shear flows

Wall bounded flow – Channel flow



- The DNS both took approximately 150 hours on 8 cores (CFL < 1).
- The RANS simulation took approximately 1 hour on 1 core.

Wall bounded flows and shear flows

Shear free flow – Kelvin-Helmholtz instability

Kelvin Helmholtz Clouds
Photo credit: **https://www.deviantart.com/yenom/art/Kelvin-Helmholtz-Clouds-5368556**
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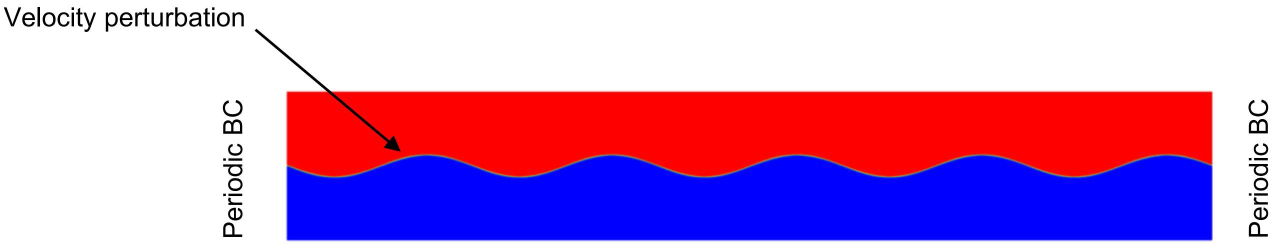
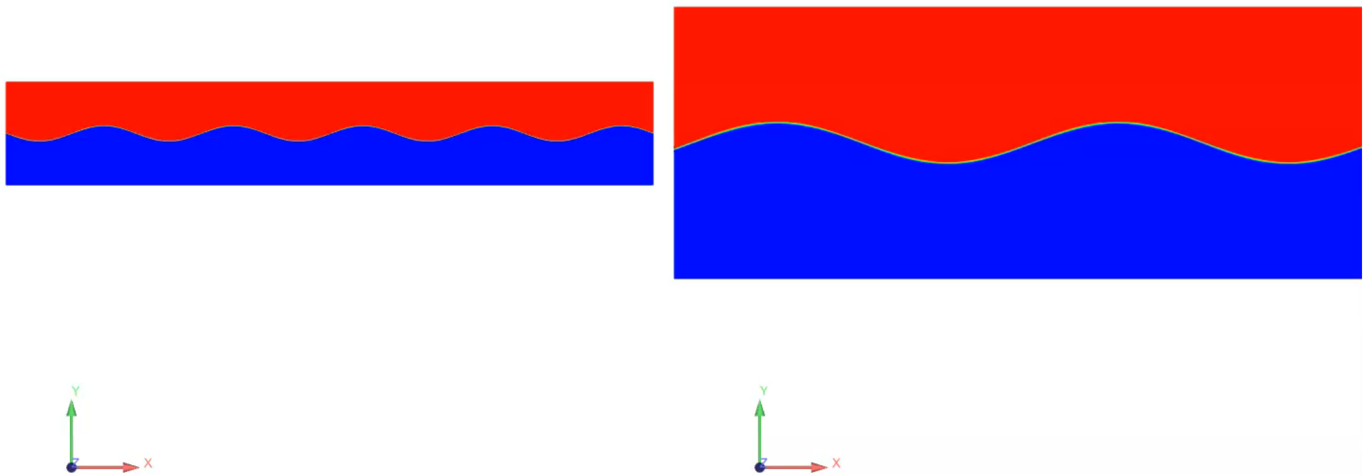


Initialization for the numerical simulation

Wall bounded flows and shear flows

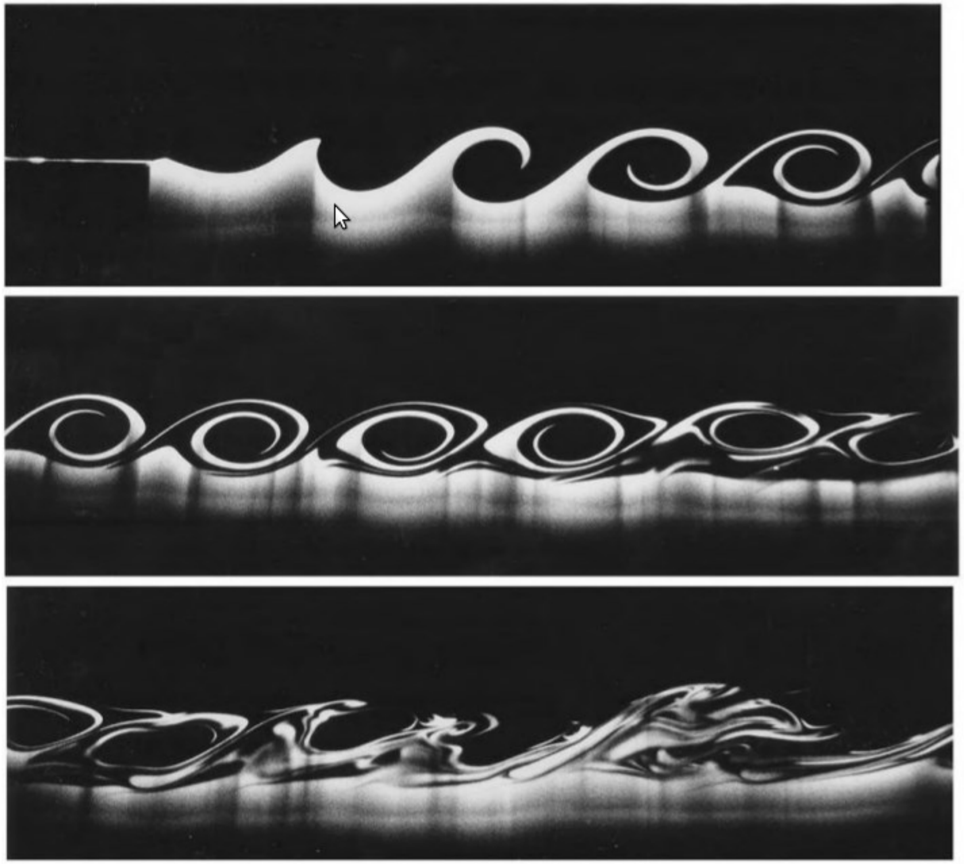
Shear free flow – Kelvin-Helmholtz instability



Numerical simulation of the Kelvin-Helmholtz instability

www.wolfdynamics.com/training/turbulence/KH.mp4

Shear free flow – Kelvin-Helmholtz instability



Development of a Kelvin-Helmholtz instability in the laboratory. The upper and faster moving layer is slightly less dense than the lower layer. At first, waves form and overturn in a two-dimensional fashion but, eventually, three-dimensional motions appear that lead to turbulence and complete mixing.

B. Cushman-Roisin, J. Beckers, Introduction to Geophysical Fluid Dynamics - Physical and Numerical Aspects (Academic Press, 2009), 393-401.

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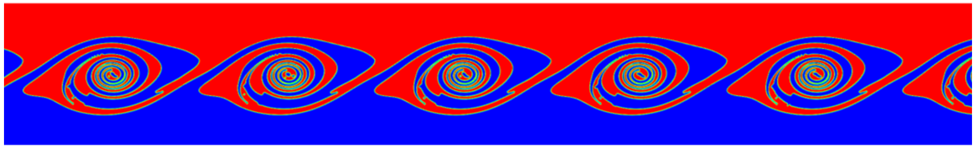


Outcome of the numerical simulation for similar conditions

Roadmap to Lecture 2

1. ~~The turbulent world around us~~
2. ~~Turbulence, does it matter?~~
3. ~~Introduction to turbulence modeling – Basic concepts~~
4. ~~Wall bounded flows and shear flows~~
5. **A peek to the turbulence closure problem, some correlations in turbulence modeling, and the energy cascade**

A peek to the turbulence closure problem and some correlations

Turbulence modeling – Starting equations

- For an incompressible flow, the exact Navier-Stokes equations (NSE) are written as follows,

$$\nabla \cdot (\mathbf{u}) = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = \frac{-\nabla p}{\rho} + \nu \nabla^2 \mathbf{u}$$

- These equations are valid for laminar and turbulent flows.
- However, as we will see later, using the exact NSE to tackle a turbulent flow, is too expensive from the computational point of view.
- Therefore, the need of using turbulence model to reduce the computational cost and at the same time getting physically realistic results.
- By the way, when we solve the exact NSE we are doing what is called direct numerical simulations (DNS).

A peek to the turbulence closure problem and some correlations

Turbulence modeling – Governing equations

- Let us introduce the incompressible Reynolds-averaged Navier-stokes equations (RANS) used in turbulence modeling.

If we retain this term we talk about URANS equations
and if we drop it we talk about RANS equations

Reynolds stress tensor
This term requires modeling

$$\nabla \cdot (\bar{\mathbf{u}}) = 0$$

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + \nabla \cdot (\bar{\mathbf{u}}\bar{\mathbf{u}}) = \frac{-\nabla \bar{p}}{\rho} + \nu \nabla^2 \bar{\mathbf{u}} - \frac{1}{\rho} \nabla \cdot \tau^R$$

- These equations are known as the exact RANS equations.
- The differences between the exact NSE and the exact RANS equations are the overbar on top of the primitive variables and the extra term τ^R.
- The overbar over the primitive variables in the RANS equations means that the quantities have been averaged to remove the fluctuations.
- We will talk more about these equations later.

A peek to the turbulence closure problem and some correlations

Turbulence modeling – Governing equations

- Our goal, at this point, is to model somehow the Reynolds stress tensor,

$$\tau^R = -\rho\left(\overline{\mathbf{u}'\mathbf{u}'}\right) = -\begin{pmatrix} \rho\overline{u'u'} & \rho\overline{u'v'} & \rho\overline{u'w'} \\ \rho\overline{v'u'} & \rho\overline{v'v'} & \rho\overline{v'w'} \\ \rho\overline{w'u'} & \rho\overline{w'v'} & \rho\overline{w'w'} \end{pmatrix}$$

- The Reynolds stress tensor basically correlates the velocity fluctuations.
- In CFD we do not want to resolve these velocity fluctuations, as it requires very fine meshes and very small time-steps.
- The RANS/URANS approach to turbulence modeling requires the Reynolds stresses to be appropriately modeled.
- The rest of the terms appearing in the governing equations, can be computed from the mean flow.

A peek to the turbulence closure problem and some correlations

Turbulence modeling – Governing equations

- By using a few hypotheses (we will talk about this later), we can write down the solvable RANS equations starting from the exact RANS equations,

$$\nabla \cdot (\bar{\mathbf{u}}) = 0$$

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + \nabla \cdot (\bar{\mathbf{u}}\bar{\mathbf{u}}) = -\frac{1}{\rho}\left(\nabla \bar{p} + \underbrace{\frac{2}{3}\rho \nabla k}_{\text{Normal stresses arising from the Boussinesq approximation}}\right) + \nabla \cdot \left[\frac{1}{\rho}\overbrace{\left(\mu + \underset{\text{Turbulent viscosity}}{\mu_t}\right)}^{\text{Effective viscosity}} \nabla \bar{\mathbf{u}}\right]$$

- The problem now reduces to computing the turbulent eddy viscosity μ_T in the momentum equation.
- This is the closure problem in turbulence modeling.

A peek to the turbulence closure problem and some correlations

Turbulence modeling – Governing equations

- The turbulent eddy viscosity can be computed using models.
- There are many models, each having different requirements and limitations. We will talk more about models later.
- For example, we can compute the turbulent eddy viscosity using the $k-\omega$ turbulence model, which reads as follows,

$$\rho\frac{\partial k}{\partial t}+\rho\nabla.\left(\bar{\mathbf{u}}k\right)=\tau^R:\nabla\bar{\mathbf{u}}-\overset{\overset{\text{Constant}}{\downarrow}}{\beta^*}\rho k\omega+\nabla.\left[\left(\mu+\overset{\overset{\text{Constant}}{\downarrow}}{\sigma^*}\mu_T\right)\nabla k\right]$$

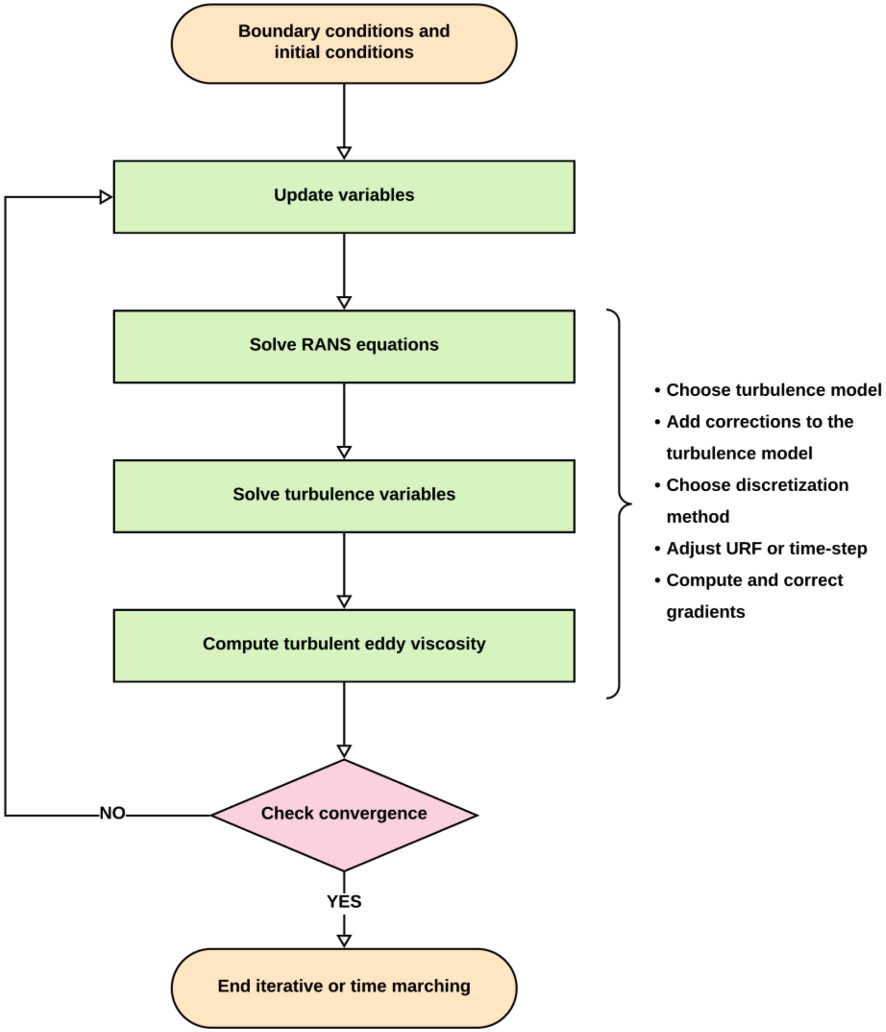
$$\rho\frac{\partial \omega}{\partial t}+\rho\nabla.\left(\bar{\mathbf{u}}\omega\right)=\underset{\underset{\text{Constant}}{\uparrow}}{\alpha}\frac{\omega}{k}\tau^R:\nabla\bar{\mathbf{u}}-\underset{\underset{\text{Constant}}{\uparrow}}{\beta}\rho\omega^2+\nabla.\left[\left(\mu+\underset{\underset{\text{Constant}}{\uparrow}}{\sigma}\mu_T\right)\nabla\omega\right]$$

$$\mu_t=\frac{\rho k}{\omega}$$

- The turbulent eddy viscosity is not a physical property.
- It has been introduced to take into account the increased mixing and shear stresses due to the turbulence

A naïve CFD loop for turbulence modeling

- The first step consist in defining the boundary and initial conditions of all variables.
- Including the variables related to the turbulence model.
- The loop will solve first the RANS equations.
- Then it will move to the next step, where it solves the equations of the turbulence model using the mean values of the RANS equations.
- It will then compute the turbulent eddy viscosity.
- At this point, it will check the convergence.
- If it is necessary to keep iterating, it will update the variables and it will do another sweep.



- Choose turbulence model
- Add corrections to the turbulence model
- Choose discretization method
- Adjust URF or time-step
- Compute and correct gradients

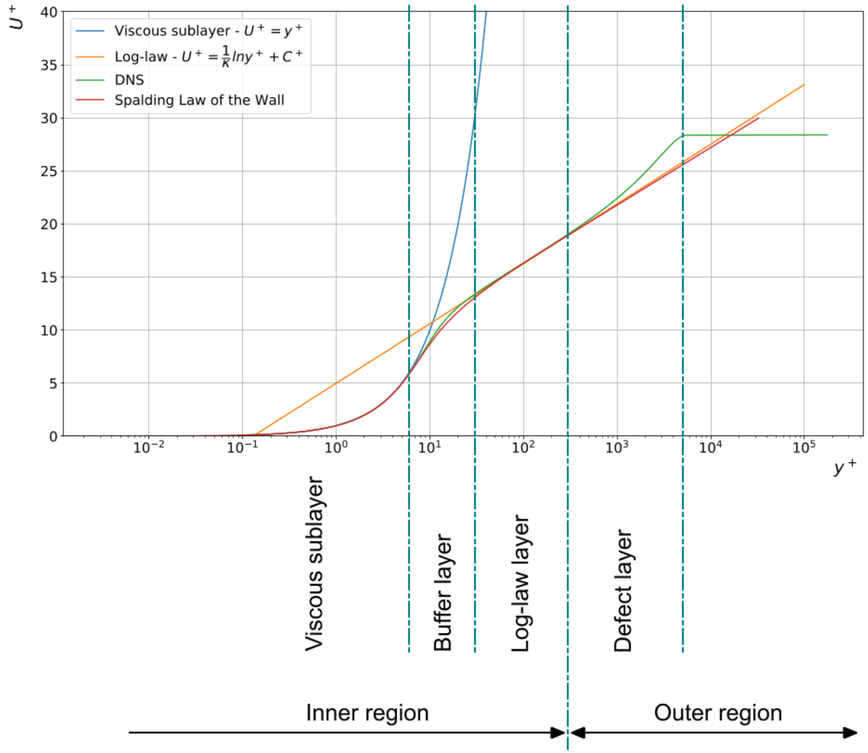
A peek to the turbulence closure problem and some correlations

Turbulence near the wall – Law of the wall

- Let us briefly explore the following plot.
- We will talk **a lot** about this plot later.
- In the plot, U^+ and y^+ are non-dimensional quantities that we will define later.
- The law of the wall, is one of the most famous empirically determined relationships in turbulent flows near solid boundaries.

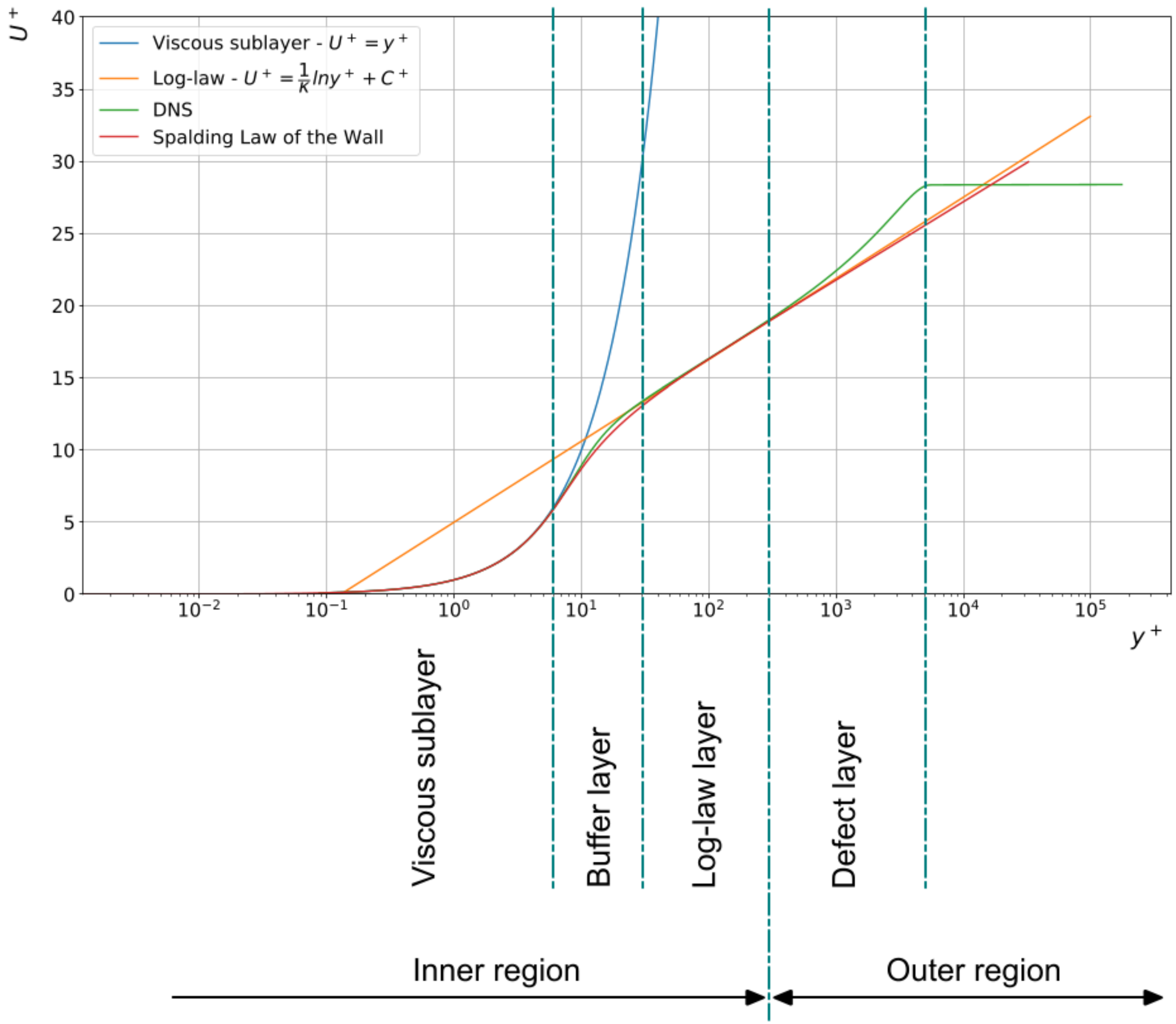
$$U^+ = f_w\left(y^+\right)$$

- In the viscous sublayer, the streamwise velocity varies linearly.
- And in the log-law layer, the streamwise velocity varies logarithmically
- This behavior has been measured in experiments and numerical simulations, for both, internal and external flows.



A peek to the turbulence closure problem and some correlations

Turbulence near the wall – Law of the wall



Viscous sublayer

$$y^+ < 5$$

$$u^+ = y^+$$

Buffer layer

$$5 < y^+ < 30$$

$$u^+ \neq y^+$$

$$u^+ \neq \frac{1}{\kappa} \ln y^+ + C^+$$

Log-law layer

$$30 < y^+ < 300$$

$$u^+ = \frac{1}{\kappa} \ln y^+ + C^+$$

k and C⁺ are empirical constants

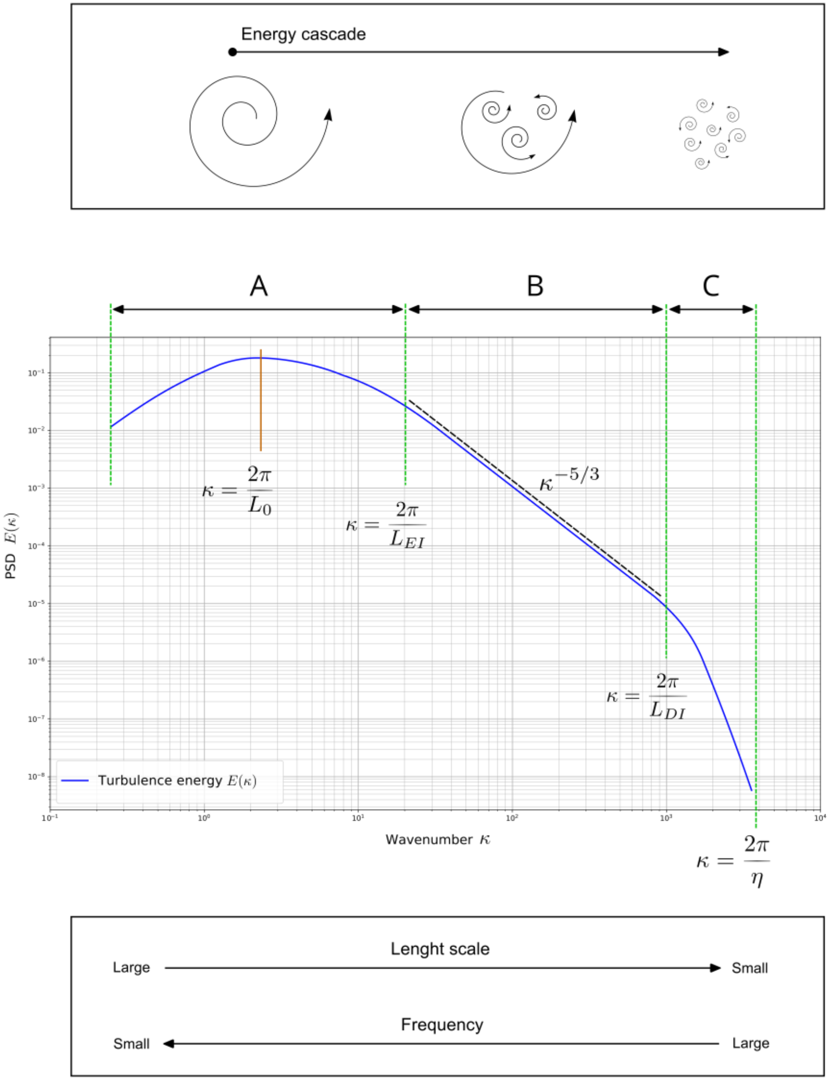
Note: the range of y⁺ values might change from reference to reference but roughly speaking they are all close to these values.

A peek to the turbulence closure problem and some correlations

The turbulence kinetic energy spectrum

- Let us briefly explore the energy cascade and the universal equilibrium theory of turbulence (K41).
- The energy cascade process consist in the energy transfer from the largest eddies (A) down to the smallest eddies (C), where turbulence kinetic energy is dissipated.
- According to the K41 theory, there is a range of eddy sizes between the largest and smallest for which the cascade process is independent of the statistics of the energy containing eddies and of the effect of viscosity.
- As a consequence, a range of wavenumbers exists in which the energy transferred by inertial effects (B), wherefore $E(\kappa)$ depends only upon ϵ (dissipation) and κ (wavenumber),

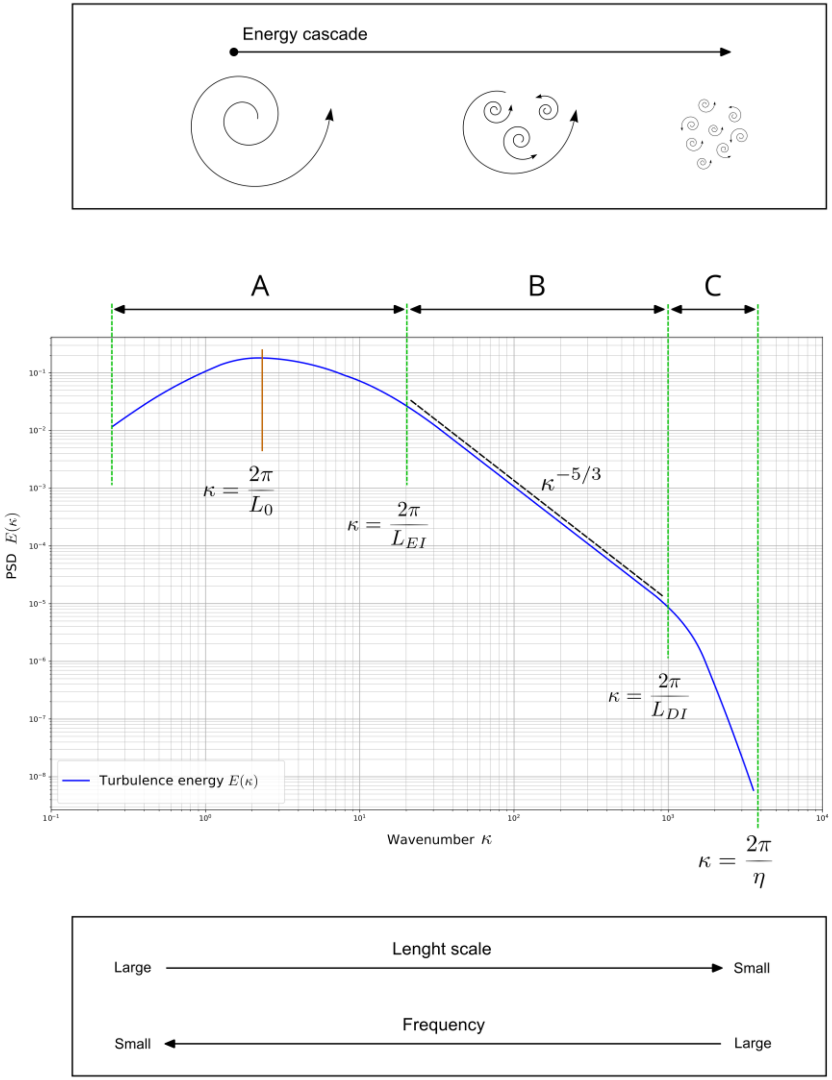
$$E(\kappa) = C_K \epsilon^{2/3} \kappa^{-5/3}$$



A peek to the turbulence closure problem and some correlations

The turbulence kinetic energy spectrum

- This relationship is sometimes known as the -5/3 law because the slope of the inertial sub-range (B) is equal to -5/3.
- The existence of this inertial sub-range has been verified by many experiments and numerical simulations.
- The energy spectrum equation $E(\kappa)$ is fundamental in turbulence modeling.
- The turbulent power spectrum represents the distribution of the turbulent kinetic energy across the various length scales.
- It is a direct indication of how energy is dissipated with eddies size.
- In CFD, the mesh resolution determines the fraction of the energy spectrum directly resolved.
- In scale-resolving simulations, the slope of the spectrum in the inertial range should be capture (-5/3).



Additional slides

Links to a few impressive turbulent simulations

Additional slides

Links to a few impressive turbulent simulations

- Supercomputer Simulation of NASA's Orion Launch Abort Vehicle
  - https://www.youtube.com/watch?time_continue=1&v=vFgxdD7_LPs&feature=emb_logo
- Aircraft landing gear air flow supercomputer simulation - NASA Ames Research Center
  - https://www.youtube.com/watch?v=-D5N_OnZ_Tg
- Turbulent Boundary Layer (DNS)
  - https://www.youtube.com/watch?v=Wr984EOmNaY
- DNS Re=400000 NACA4412
  - https://www.youtube.com/watch?v=aR-hehP1pTk
- Exploring Drone Aerodynamics With Computers
  - https://www.youtube.com/watch?v=hywBEaGiO4k
- Toward Urban Air Mobility: Air Taxis with Side-By-Side Rotors
  - https://www.youtube.com/watch?v=eA3SJIzWADQ
- A computational laboratory for the study of transitional and turbulent boundary layers
  - https://www.youtube.com/watch?v=wXsl4eyupUY
- Turning on a Dime – Asymmetric Vortex Formation in Hummingbird Maneuvering Flight
  - https://www.youtube.com/watch?v=PCj-82oYgUs