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# **Turbulence and CFD models: Theory and applications**

# Roadmap to Lecture 4

## **1. Practical turbulence estimates**

# Practical turbulence estimates

## Introduction

- In Lecture 3, Kolmogorov scales, Taylor scales, and integral scales were introduced.
- We then explored the concepts of energy spectrum, energy cascade, integral length scale, and grid length scale.
- We also studied the basic concepts of turbulence near the wall, we introduced the Law of the Wall, and the non-dimensional quantity  $y^+$ .
- Finally, we took a glimpse to a turbulence model.
- At this point, the question is,
  - How can we use this information?
  - How can we get an initial estimate of the variables related to the turbulence model?
  - How can we estimate the meshing requirements?
- Hereafter, we will give some standard practices on how to get turbulence estimates.
- Have in mind that many commercial applications have these relationships already implemented, so you do not need to do these computations (which are trivial).

# Practical turbulence estimates

## Introduction

- Using everything we have learned so far, we can get global estimates for the following variables:
  - Eddies velocity, eddies size, and time scales (integral, Taylor, and Kolmogorov).
  - Rough estimates of the number of grid points or cells needed.
  - Energy dissipation rate  $\epsilon$ .
  - Turbulent kinetic energy  $k$ .
  - Turbulent kinematic viscosity  $\mu_t$ .
  - Turbulent intensity  $I_t$ .
  - Wall distance units  $y^+$ .
- Remember, we will compute initial estimates for global quantities.
- If you want to get the local values, you will need to run a simulation where you will resolve a transport equation for these variables.
- I cannot stress this enough; we will compute rough estimates which are fine for initial conditions or generating an initial mesh.

# Practical turbulence estimates

## Introduction

- Let us first compute the integral eddy length scale, turbulence intensity, turbulent kinetic energy, and turbulent dissipation.
- We will use the **LIKE** acronym [1] to describe the workflow that we will use to compute these practical estimates.

• <b>L</b>	=	$l$	=	integral eddy length scale
• <b>I</b>	=	$I_t$	=	turbulence intensity
• <b>K</b>	=	$k$	=	turbulent kinetic energy
• <b>E</b>	=	$\epsilon$	=	turbulent dissipation

- We already know many relations from Lecture 3.
- We will introduce a few new equations.
- Many of these relationships can be derived from dimensional analysis.
- Remember to always check the dimensional groups.

# Practical turbulence estimates

## A reminder about the units

Derived quantity	Symbol	Dimensional units	SI units
Velocity	$u$	$LT^{-1}$	m/s
Density	$\rho$	$ML^{-3}$	kg/m <sup>3</sup>
Kinematic viscosity	$\nu$	$L^2T^{-1}$	m <sup>2</sup> /s
Dynamic viscosity	$\mu$	$ML^{-1}T^{-1}$	kg/m-s
Energy dissipation rate per unit mass	$\epsilon$	$L^2T^{-3}$	m <sup>2</sup> /s <sup>3</sup>
Turbulent kinetic energy per unit mass	$k$	$L^2T^{-2}$	m <sup>2</sup> /s <sup>2</sup>
Length scales	$l$	L	m
Wavelength	$\kappa$	$L^{-1}$	1/m
Intensity	$I_t$	-	-
Stress	$\tau$	$ML^{-1}T^{-2}$	kg/(m-s <sup>2</sup> ) or N/m <sup>2</sup> or Pa

# Practical turbulence estimates

## Integral eddy length scale – LIKE

- Usually, the integral length scales  $l$  are represented by a characteristic dimension of the domain,

$$l \sim x_{char}$$

- That is, the system characteristic length places a limit on the maximum integral eddy length.
- In practice, this limit is not reached nor there is a typical value.
- Therefore, conservative approximations are often used based on a percentage of the system characteristic length.
- For example,
  - If you are simulating the flow about a cylinder, you can say that the largest eddies are about 70% of the cylinder diameter.
  - If you are simulating the flow in a pipe, you can say that the largest eddies are about 90% the diameter of the pipe.
  - If you are simulating the flow about an airfoil (with no large flow separation), you can say that the largest eddies are about the airfoil thickness.

# Practical turbulence estimates

## Integral eddy length scale – LIKE

- You will find often the following relationships in the literature.
  - For internal flows (pipes and ducts), where  $D$  is the diameter or height,

$$l \approx 0.07D$$

- For boundary layers over surfaces, where  $\delta$  is the turbulent boundary layer thickness,

$$l \approx 0.4\delta$$



where the boundary layer thickness can be approximated using the following correlation (among many available in the literature),

$$\delta \approx \frac{0.37x}{Re_x^{1/5}}$$



# Practical turbulence estimates

## Integral eddy length scale – LIKE

- You will find often the following relationships in the literature.
  - For grid generated turbulence in wind tunnels, where  $S$  is the grid spacing,

$$l \approx 0.2S$$

- The following is a personal estimate that I often use,

$$l \approx 0.7h_b$$



- Where  $h_b$  is the blockage height in the direction of the incoming flow.
- For example:
  - Airfoil thickness, if it is aligned with the flow or at a low AOA.
  - If the airfoil is at a high AOA, the blockage height.
  - Frontal area of a body (cylinder, truck, and so on).
- This relationship can be use for internal and external flows

# Practical turbulence estimates

## Integral eddy length scale – LIKE

- If you have estimates for  $k$  and  $\epsilon$ , you can compute the integral length scales as follows.
  - Taylor suggests [1] that the integral length scales can be approximated as follows,

$$l \approx \frac{k^{3/2}}{\epsilon}$$

Use this estimate if you are interested in the largest integral length scale

- The previous estimate has been improved by using experimental data, as explained by Wilcox [2],

$$l = C_\mu \frac{k^{3/2}}{\epsilon} \quad \text{where} \quad C_\mu = 0.09$$

Use this estimate if you are interested in the average integral length scale

**This is the preferred correlation**

[1] G. I. Taylor. Statistical theory of turbulence. Proceedings of the Royal Society of London. 1935.

[2] D. Wilcox. Turbulence Modeling for CFD. DCW Industries Inc., 2010.

# Practical turbulence estimates

## Turbulence intensity – LIKE

- The turbulence intensity (also called turbulence level) is often abbreviated as follows,

$$I \qquad I_t \qquad T_u \qquad T' \qquad T_L$$

- The turbulence intensity can be computed as follows,

$$I = \frac{u_{\text{RMS}}}{\bar{u}}$$

← Intensity of velocity fluctuations

← Mean velocity – Freestream velocity

- The intensity of the velocity fluctuations (turbulence strength) is defined by the root mean square (RMS) of the velocity fluctuations,

$$u_{\text{RMS}} = \sqrt{\frac{1}{3} \left( \overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right)}$$

←

- It gives a measure of the dispersion of the velocity fluctuations squared ( or normal Reynolds stresses).
- It is nothing else but the standard deviation of the fluctuations.

# Practical turbulence estimates

## Turbulence intensity – LIKE

- The Reynolds stress components  $\overline{u'^2}$ ,  $\overline{v'^2}$ ,  $\overline{w'^2}$ , can also be regarded as the kinetic energy per unit mass of the fluctuating velocity in the three spatial directions.
- If we sum the normal Reynolds stresses and multiply by 0.5, we obtain the turbulent kinetic energy,

$$k = \frac{1}{2} \left( \overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right) = \frac{1}{2} \overline{u'_i u'_i}$$

- The normal Reynolds stresses can be normalized relative to the mean flow velocity, as follows,

$$\hat{u} \equiv \frac{\sqrt{\overline{u'^2}}}{\bar{u}} \quad \hat{v} \equiv \frac{\sqrt{\overline{v'^2}}}{\bar{u}} \quad \hat{w} \equiv \frac{\sqrt{\overline{w'^2}}}{\bar{u}}$$

These three quantities are known as relative intensities.

# Practical turbulence estimates

## Turbulence intensity – LIKE

- For anisotropic turbulence (*i.e.*, the normal components of the Reynolds stress tensor are unequal), a rough but useful estimate to the normal Reynolds stress components is the following,

$$\overline{u'^2} : \overline{v'^2} : \overline{w'^2} \approx 4 : 2 : 3$$

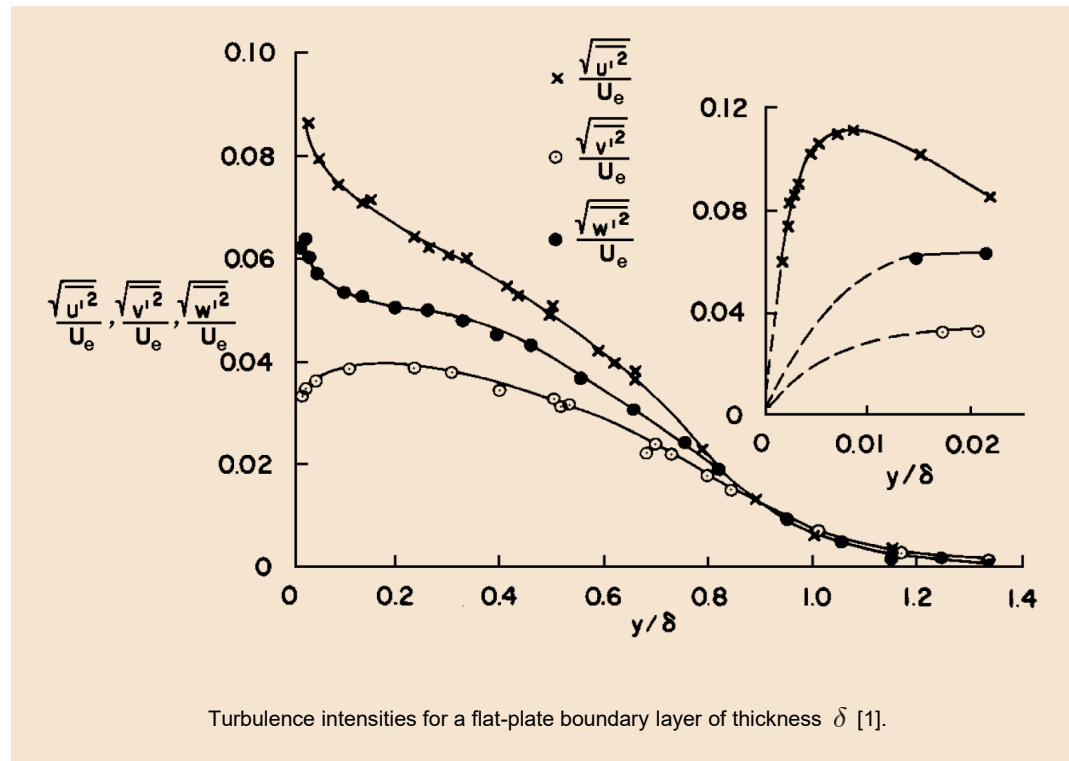


Correlation based on flat-plate boundary layer

- And recall that,

$$I = \frac{u_{\text{RMS}}}{\bar{u}}$$

$$u_{\text{RMS}} = \sqrt{\frac{1}{3} \left( \overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right)}$$



# Practical turbulence estimates

## Turbulence intensity – LIKE

- For the case of isotropic turbulence, the normal components of the Reynolds stress tensor are the same.

$$\overline{u'^2} = \overline{v'^2} = \overline{w'^2}$$

- Then, the turbulence intensity can be computed as follows,

$$I = \sqrt{\frac{\frac{2}{3}k}{\bar{u}^2}} = \frac{\sqrt{\frac{2}{3}k}}{\bar{u}}$$

- And recall that,

$$I = \frac{u_{\text{RMS}}}{\bar{u}} \quad u_{\text{RMS}} = \sqrt{\frac{1}{3} \left( \overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right)}$$

{

$k = \frac{1}{2} \left( \overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right)$

$k = \frac{3}{2} \overline{u'^2}$

←

For isotropic turbulence

# Practical turbulence estimates

## Turbulence intensity – LIKE

- To use the previous relations, you need to have some form of measurements or previous experience to base the estimate on.
- The turbulence intensity can also be estimated using empirical correlations.
- For instance, if you are working with pipes, there are many correlations that are expressed in the form of a power law,

$$I = C_1 Re_h^{-C_2}$$



Hydraulic Reynolds number

- One widely used correlation is the following one,

$$I = 0.16 Re_h^{-1/8}$$

- Remember, there are many forms of these correlations (for smooth and rough pipes).

# Practical turbulence estimates

## Turbulence intensity – LIKE

- If you are working with external aerodynamics, it might be a little bit more difficult to get rule of thumb estimates.
- However, the following estimates are acceptable,

	Low	Medium	High
$I_t$	1.0 %	5.0 %	10.0 %

**Low turbulence intensity (1%):** external flow around cars, ships, submarines, and aircrafts. Very high-quality wind-tunnels can also reach low turbulence levels, typically below 1.0%.

**Medium turbulence intensity (5%):** flows in not-so-complex devices like large pipes, fans, ventilation flows, wind tunnels, low speed flows, and fully-developed internal flows. Typical values are between 2.0% and 7.0%.

**High turbulence intensity (10%):** high-speed flow inside complex geometries like heat-exchangers and rotating machinery (turbines and compressors). Typical values are between 10.0% and 20.0%.



# Practical turbulence estimates

## Turbulent kinetic energy – **L**IKE

- The turbulent kinetic energy  $k$  is also known as TKE.
- If you have some estimates of the velocity fluctuations or normal Reynolds stresses , the TKE (per unit mass) can be computed as follows,

$$k = \frac{1}{2} \left( \overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right)$$

- Otherwise, you can use a rule of thumb turbulence intensity estimate and compute TKE as follows,

$$k = \frac{3}{2} (\overline{u}I)^2 \quad \text{where } \overline{u} \text{ is the freestream velocity}$$

- Instead of using  $\overline{u}$ , you can also use a value slightly higher that we will call turbulent freestream value  $u_{tur}$ ,

Arbitrary constant to correct velocity fluctuations

$$u_{tur} \approx C\overline{u} = \frac{11}{10}\overline{u}$$

# Practical turbulence estimates

## Energy dissipation rate – LIKE

- Once TKE is known, together with a crude estimate of the integral eddy length scale, the energy dissipation rate  $\epsilon$  (per unit mass) can be computed as follows,

$$\epsilon = C_{\mu} \frac{k^{3/2}}{l} \quad \text{where} \quad C_{\mu} = 0.09$$

- You can compute the specific dissipation rate once you know  $\epsilon$  and  $l$ , as follows,

$$\omega = \frac{\epsilon}{\beta^* k} \quad \text{where} \quad \beta^* = \frac{9}{100}$$

- You can compute the integral eddy length scale from specific dissipation rate as follows,

$$l = \frac{k^{1/2}}{\omega}$$

# Practical turbulence estimates

## Turbulent viscosity

- So far, we computed the integral eddy length scale, turbulence intensity, turbulent kinetic energy and energy dissipation rate.
- In the previous lecture, we saw that in turbulence modeling there is an extra ingredient, turbulent viscosity  $\mu_t$ .
- We can also get an estimate for this quantity. How do we estimate it depends on the turbulent model.
- For example, the  $k - \omega$  model computes the turbulent viscosity as follows,

$$\mu_t = \frac{\rho k}{\omega}$$

Where you can compute  $\omega$  from  $\epsilon$  as follows,

$$\omega = \frac{\epsilon}{\beta^* k} \quad \beta^* = \frac{9}{100}$$

- As usual, check the dimensional groups.
- Remember, the turbulent viscosity is not a physical quantity.
- Also, the turbulent viscosity is larger than the laminar viscosity (molecular viscosity).

# Practical turbulence estimates

## Turbulent viscosity

- We can also use a rule of thumb to get a fast estimate of the turbulent viscosity.
- Using the turbulence intensity  $I_t$ , we can get an estimate for the viscosity ratio  $\mu_t/\mu$  as follows,

	Low	Medium	High
$I_t$	1.0 %	5.0 %	10.0 %
$\mu_t/\mu$	1-2	10	100

**Low turbulence intensity (1%):** external flow around cars, ships, submarines, and aircrafts. Very high-quality wind-tunnels can also reach low turbulence levels, typically below 1.0%.

**Medium turbulence intensity (5%):** flows in not-so-complex devices like large pipes, fans, ventilation flows, wind tunnels, low speed flows, and fully-developed internal flows. Typical values are between 2.0% and 7.0%.

**High turbulence intensity (10%):** high-speed flow inside complex geometries like heat-exchangers and rotating machinery (turbines and compressors). Typical values are between 10.0% and 20.0%.

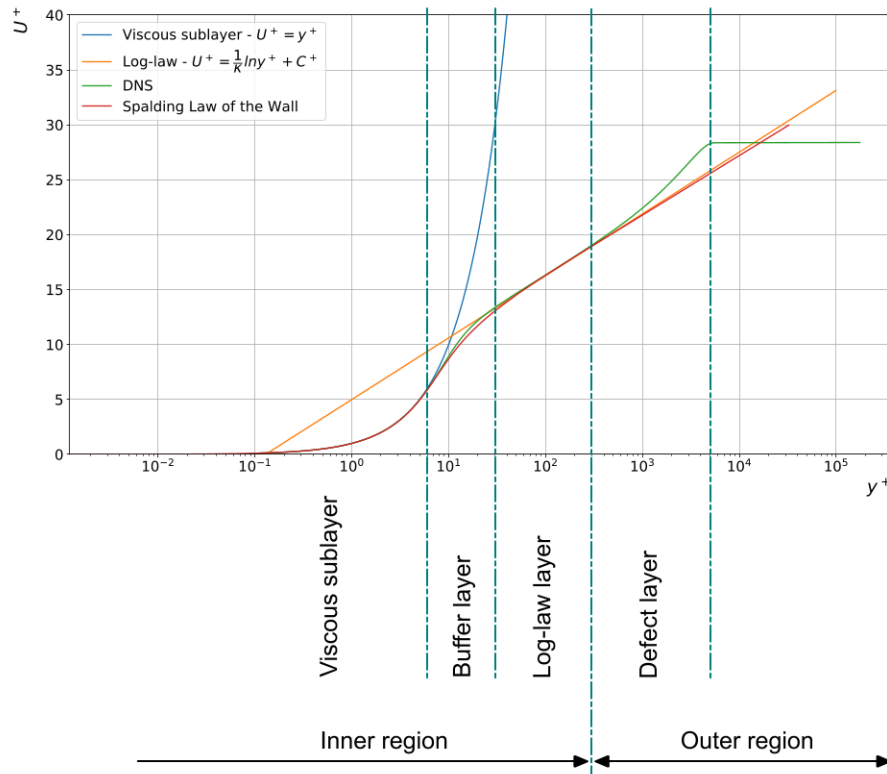
- We usually use these estimates when dealing with external aerodynamics.

## Estimation of $y^+$ and wall distance units

# Practical turbulence estimates

## Estimation of $y^+$

- Let us revisit the definition of  $y^+$ .
- $y^+$  is the non-dimensional distance normal to the wall.
- It is very similar to the Reynolds number, but it is non-dimensionalized using the shear velocity  $U_\tau$ .



$$y^+ = \frac{\rho \times U_\tau \times y}{\mu} = \frac{U_\tau \times y}{\nu}$$

$$U_\tau = \sqrt{\frac{\tau_w}{\rho}} \quad \leftarrow \text{wall shear stresses}$$

$$u^+ = \frac{U}{U_\tau}$$

In the previous equations:

- $y$  is the distance normal to the wall.
- $U_\tau$  is the shear velocity,
- $u^+$  relates the mean velocity to the shear velocity.

# Practical turbulence estimates

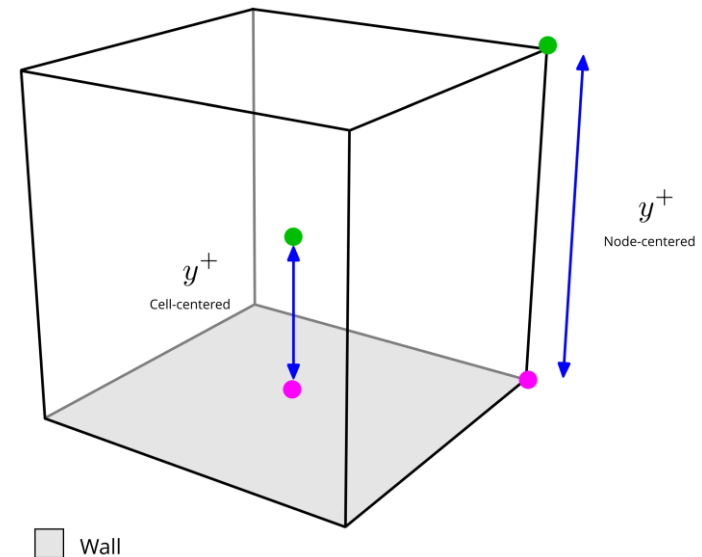
## Estimation of $y^+$

- $y^+$  is very important quantity in wall bounded turbulence modeling.
- We can use  $y^+$  to estimate the mesh resolution near the wall before running the simulation.
  - Have in mind that we do not know a-priori the wall shear stresses at the walls.
  - Therefore, we need to use correlations to get a rough estimate and generate the initial mesh.
  - The initial mesh is then generated according to the chosen near the wall treatment, namely, wall resolving, wall functions, or  $y^+$  insensitive.
  - Then, we run a precursor simulation to validate the mesh or to get a better estimate of  $y^+$  and determine if we need a finer or coarser mesh.
  - It is an iterative process, and it can be very time consuming, as it might require remeshing and rerunning the simulation to satisfy the near the wall treatment.

# Practical turbulence estimates

## Estimation of $y^+$

- It is worth mentioning that  $y^+$  can be defined in reference to the cell center or the cell nodes.
- From now on, we are going to assume that we are using a cell-centered solver.
  - Ansys Fluent, StarCCM+, OpenFOAM, code Saturne, SU2, are all cell-centered.
  - It is up to you to check with the solver's documentation to find out with what kind of formulation are you using.
- By the way,  $y^+$  is computed in the same way disregarding of the cell type used.
  - In the cell-centered formulation you need to know the location of the cell center.
  - In the node-centered formulation you need to know the location of the reference node. Usually, the node closest to the surface.

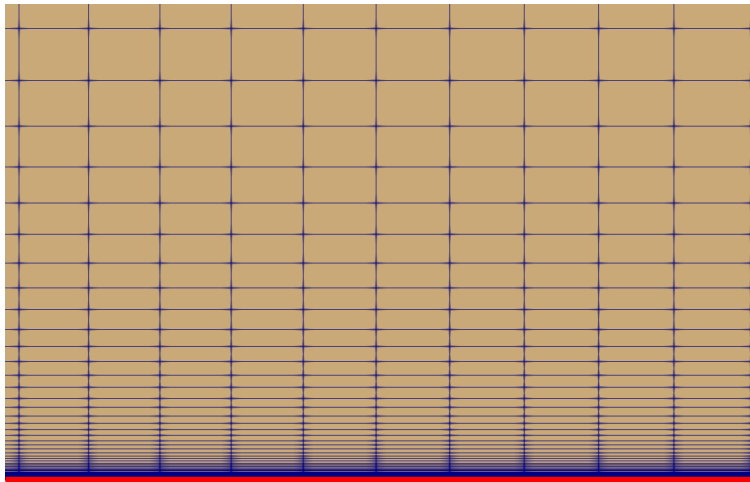




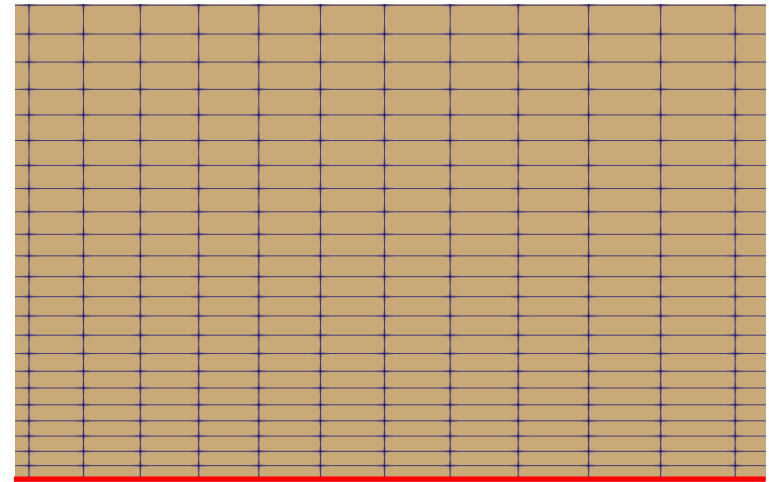
# Practical turbulence estimates

## Estimation of $y^+$

- At meshing time, to estimate the normal distance from the wall to the first cell center ( $y$ ), we use the well-known  $y^+$  definition.
- Where we set a target  $y^+$  value and then we solve for the quantity  $y$ .
  - If you choose a low target  $y^+$  (e.g., less than 10), you will have a mesh that is clustered towards the wall (small value of  $y$ ).
  - If you choose a large  $y^+$  value (e.g., more than 100), you will have a coarse mesh close to the walls (large value of  $y$ ).



Fine mesh towards the walls



Coarse mesh towards the walls

# Practical turbulence estimates

## Estimation of $y^+$

- At meshing time, to estimate the normal distance from the wall to the first cell center, we use the well-known  $y^+$  definition,

$$y^+ = \frac{\rho \times U_\tau \times y}{\mu} = \frac{U_\tau \times y}{\nu}$$

- The problem is that at meshing time we do not know the value of the shear velocity,

$$U_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

- So, how do we get an initial estimate of this quantity?
  - We can use correlations, as described in the next slide.

# Practical turbulence estimates

## Estimation of $y^+$

- At meshing time, to estimate the normal distance from the wall to the first cell center, you can proceed as follows,

1. 
$$Re = \frac{\rho \times U \times L}{\mu}$$

Compute the Reynolds number using the characteristic length of the problem.

2. 
$$C_f = 0.058 \times Re^{-0.2}$$

Compute the friction coefficient using any of the correlations available in the literature. There are many correlations available ranging from pipes to flat plates, for smooth and rough surfaces.  
This correlation corresponds to a smooth flat plate case, ideal for external aerodynamics.

3. 
$$\tau_w = \frac{1}{2} \times C_f \times \rho \times U_\infty^2$$

Compute the wall shear stresses using the friction coefficient computed in the previous step.

4. 
$$U_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

Compute the shear velocity using the wall shear stresses computed in the previous step.

5. 
$$y = \frac{\mu \times y^+}{\rho \times U_\tau}$$

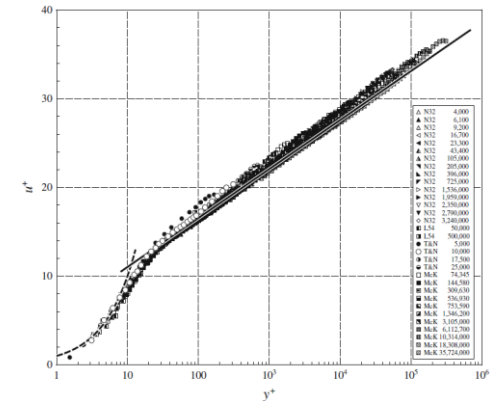
Set a target  $y^+$  value and solve for  $y$  using the flow properties and previous estimates.  
This value represents the distance from the wall to the first cell center.  
At this point, you can generate your mesh trying to respect this value.

You will find a simple calculator for the wall distance estimation in the following link: <http://www.wolfdynamics.com/tools.html?id=2>

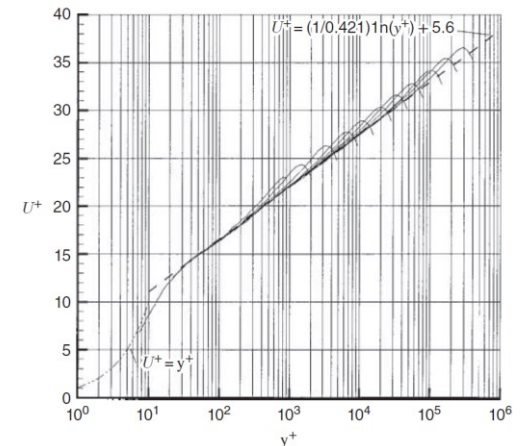
# Practical turbulence estimates

## $y^+$ additional comments – When to use wall functions

- Most of the times you will find in the literature a  $y^+$  upper limit of 200-300 for the applicability of wall functions.
- In reality, this upper limit depends on the Reynolds number, as shown in the figures.
- For high Reynolds number you can reach  $y^+$  values of up to 1000.
- Care must be taken if the  $y^+$  upper limit is below or close to 100.
  - In this case, you should not use wall functions because the overlap region is too narrow and is likely that you will cluster the cell centers in the buffer region or too close to it.
- Remember, when using wall functions it is recommended to cluster at least 5 cells in the log-law region in order to resolve the velocity and TKE profiles.



Dimensionless mean velocity profile  $u^+$  as a function of the dimensionless wall distance  $y^+$  for turbulent pipe flow with Reynolds numbers between 4000 and 3600000 [1].



Mean velocity profiles in pipe flow showing the collective approach to a log law. The curves are for Reynolds numbers between  $Re = 31 \times 10^3$  and  $Re = 18 \times 10^6$  [2].

[1] F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer. 2016.

[2] B. McKeon, J. Li, W. Jiang, J. Morrison, A. Smits. Further observations on the mean velocity distribution in fully developed pipe flow. 2004.

# Practical turbulence estimates

## $y^+$ guidelines – Correction of the mesh

- Remember, the thickness of the thermal sublayer for a high Prandtl number fluid (e.g., water) is much less than the momentum sublayer thickness.
- Therefore, the mesh requirements close to the walls need to be corrected for the thinner thermal boundary layer.
- The  $y^+$  value can be corrected as follows,

$$y^+ \approx \frac{1}{\sqrt{Pr}}$$

- A similar situation exists when working with multispecies transport and using the Schmidt number (Sc).
- If the Schmidt number is considerably larger than unity, then the thickness of the thermal diffusion sublayer is much less than the momentum sublayer thickness.
- In this situation, the  $y^+$  value can be corrected as follows,

$$y^+ \approx \frac{1}{\sqrt[3]{Sc}}$$

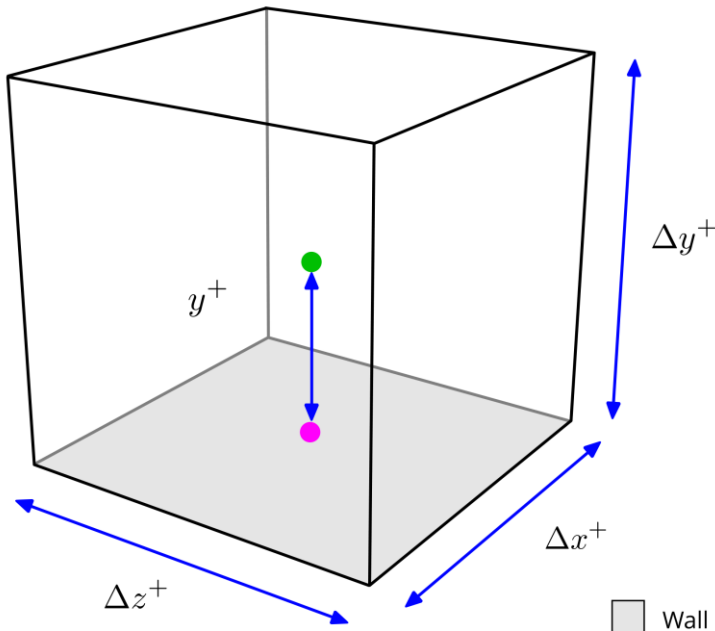
# Practical turbulence estimates

## $y^+$ guidelines – Limiting values and solution monitoring

- $y^+$  always needs to be monitored during the simulation.
  - Have in mind that it is quite difficult (if not impossible) to get a uniform  $y^+$  value at the walls.
  - We usually monitor the average  $y^+$  value. If this value covers approximately 80% of the wall, we can take the mesh as a good one.
  - Otherwise, we need to refine or coarse the mesh to get a more uniform distribution of  $y^+$ .
  - It is also important to monitor the maximum values of  $y^+$ . It is not a good practice to have values larger than 1000.
  - Values of  $y^+$  up to 300 are fine.
  - Values of  $y^+$  larger than 300 and up to a 1000 are acceptable if they do not cover a large surface area (no more than 20% of the total wall area), or if they are not located in critical zones.
  - It is also important to monitor the minimum  $y^+$ , as some models might have problems with low  $y^+$  values.
  - Use common sense when accessing  $y^+$  value.

# Practical turbulence estimates

## Wall distance units $x^+ - y^+ - z^+$



- Similar to  $y^+$ , the wall distance units can be computed in the stream-wise ( $\Delta x^+$ ) and span-wise ( $\Delta z^+$ ) directions.
- The wall distance units in the stream-wise and span-wise directions can be computed as follows:

$$\Delta x^+ = \frac{U_\tau \Delta x}{\nu} \qquad \Delta z^+ = \frac{U_\tau \Delta z}{\nu}$$

- And recall that  $y^+$  is computed at the cell center, therefore:

$$\Delta y^+ = 2 \times y^+$$

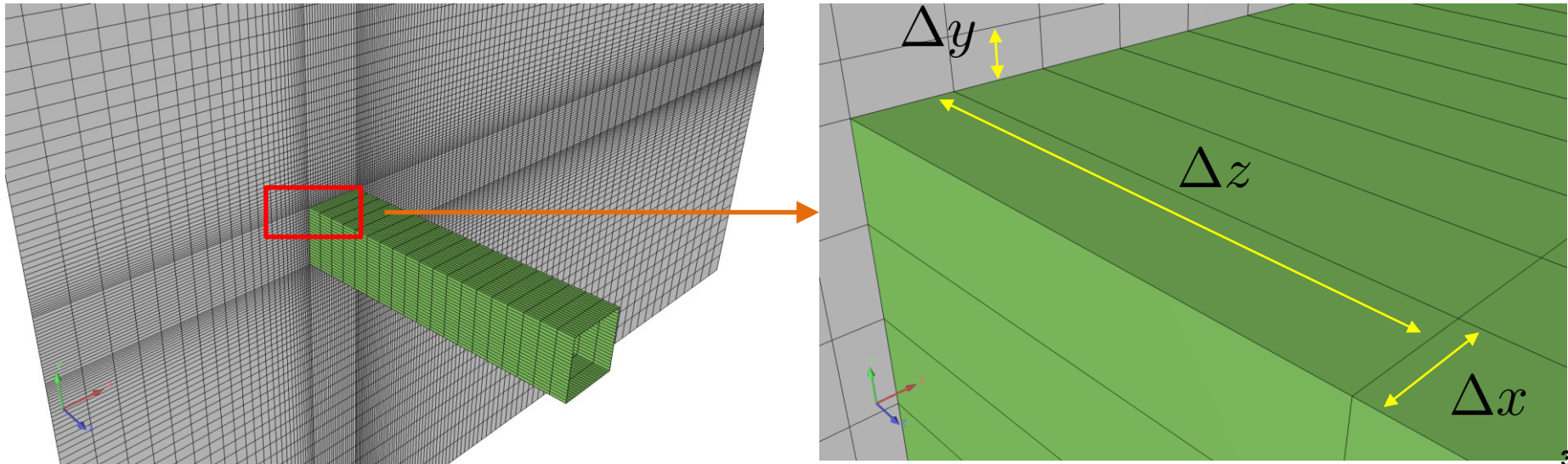
$$(\Delta x^+, \Delta y^+, \Delta z^+) = \left( \frac{x}{l_\tau}, \frac{y}{l_\tau}, \frac{z}{l_\tau} \right)$$

where  $l_\tau = \frac{\nu}{U_\tau}$   
Viscous length

# Practical turbulence estimates

## Wall distance units $x^+ - y^+ - z^+$

- It is not only about the  $y^+$  value.
- In SRS simulations  $x^+$  and  $z^+$  are also important.
- The  $x^+$  and  $z^+$  values strongly depend on the surface mesh aspect ratio.
- The aspect ratio is defined as the ratio between the longest edge and shortest edge.
- Recommended values of surface mesh aspect ratio for SRS simulations are 10 or less.
- Meshing applications usually have adequate tools for mesh diagnostics.
- This is a rough estimate based on experience.





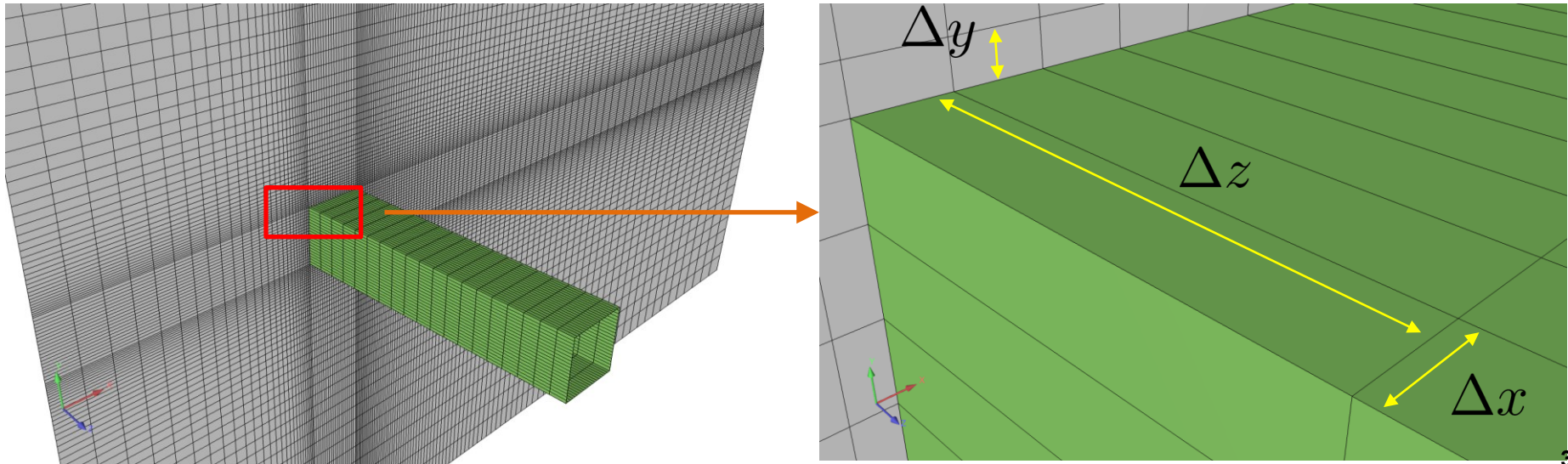
# Practical turbulence estimates

## Wall distance units $x^+ - y^+ - z^+$

- Besides the surface aspect ratio, you should also check the ratio between surface cell base length and the cell height. This aspect ratio can be approximated as follows,

$$AR = \frac{\sqrt{\text{Surface cell area}}}{\Delta y}$$

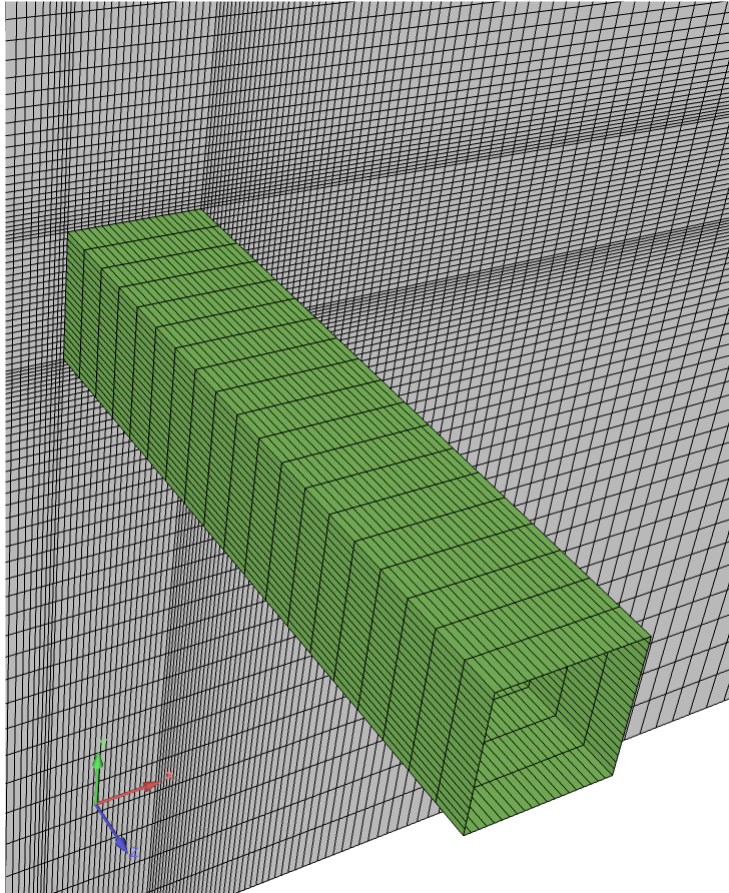
- Recommended values are 20 or less.
- As you can see, this aspect ratio can impose strong requirements on the mesh.
- Again, this is a rough estimate based on experience, and that should be used when conducting SRS simulations



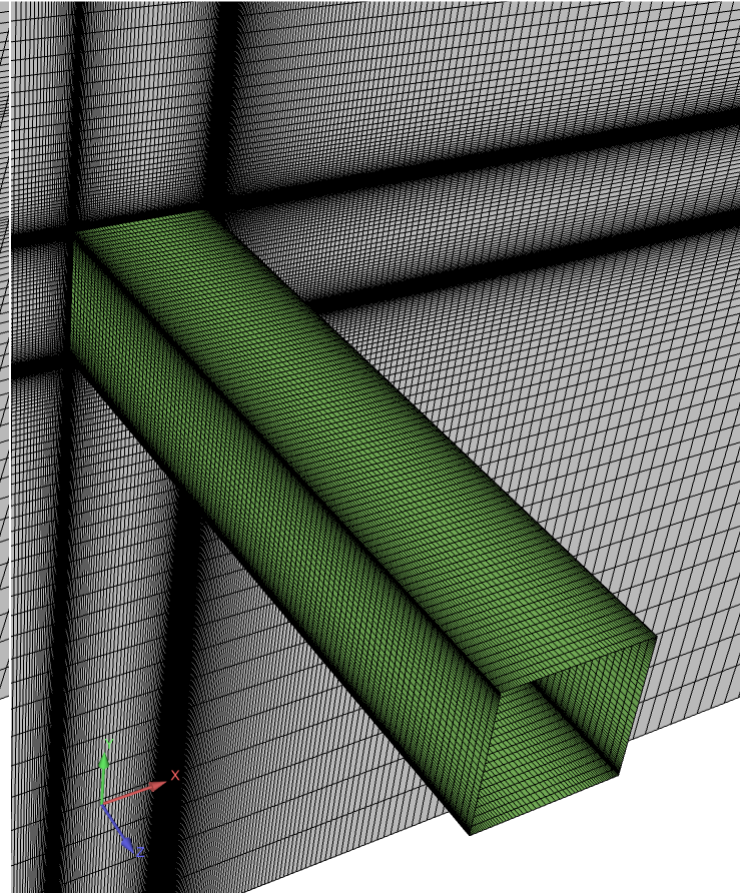
# Practical turbulence estimates

## Wall distance units $x^+ - y^+ - z^+$

- The wall distance units  $x^+$  and  $z^+$  values strongly depend on the surface mesh aspect ratio.



Surface mesh with large aspect ratio and large ratio between surface cell base length and the cell height.



Surface mesh with low aspect ratio and low-to-moderate ratio between surface cell base length and the cell height

# Practical turbulence estimates

## Wall distance units $x^+ - y^+ - z^+$

- The stream-wise and span-wise wall distance units requirements are important when conducting scale-resolving simulations (DES, LES, DNS).
- Typical requirements for LES are (these are recommendations based on different references):

$$\Delta x^+ < 50, \Delta z^+ < 50 \quad \text{for} \quad y^+ < 5$$

Wall resolving

$$\Delta x^+ < 4\Delta y^+, \Delta z^+ < 4\Delta y^+ \quad \text{for} \quad 30 \leq y^+ \leq 300$$

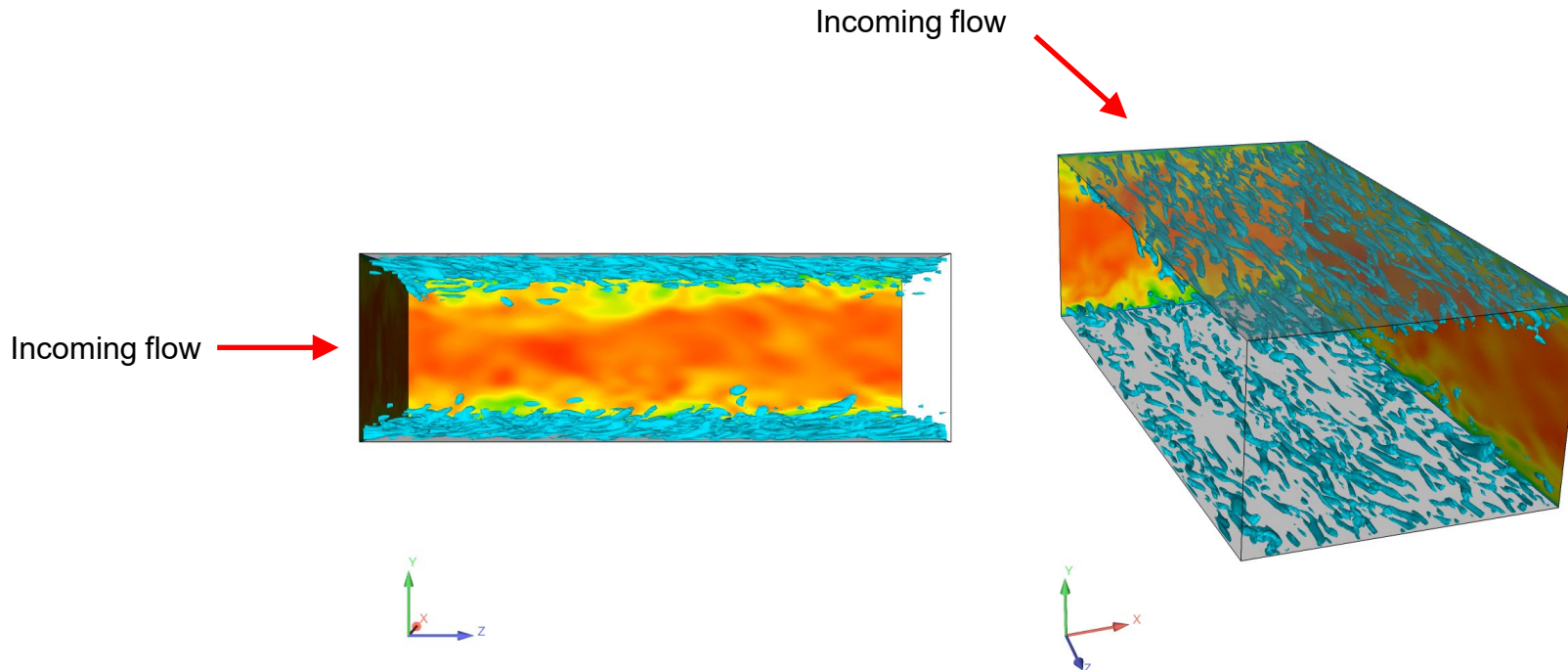
Wall modeling

- As you can see, these requirements translate in much finer surface meshes and low surface aspect ratios.
- We will talk more about this in Lecture 10.
- RANS simulations only have requirements respect to  $y^+$ . They do not have strict requirements when it comes to the span-wise and stream-wise directions.
  - Therefore, they are more affordable.

# Practical turbulence estimates

## Wall distance units $x^+ - y^+ - z^+$

- These strict requirements in SRS simulation are due to the fact that we need to resolve the structures close to the walls (streaks).
- And this require fine meshes in all space directions.



Wall bounded flow – Channel flow

<http://www.wolfdynamics.com/training/turbulence/channel1.mp4>



# Practical turbulence estimates

## Wall distance units – A few mesh resolution guidelines and rough estimates

- The mesh is everything in CFD, and when it comes to turbulence modeling, it is extremely important to have meshes with good quality and acceptable resolution.
- Some general guidelines for meshes to be used with RANS/DES/LES:
  - Resolve well the curvature.
  - Allow a smooth transition between cells of different sizes (at least 3 cells).
  - Identify the integral scales and try to cluster at least 5 cells in the domain regions where you expect to find these scales.
  - If you have no restrictions, use a wall resolving approach, that is, fine meshes in the direction normal to the walls.
  - It is recommended to avoid hanging nodes (in Lecture 3 we illustrated what they are).



# Practical turbulence estimates

## Wall distance units – A few mesh resolution guidelines and rough estimates

- The mesh is everything in CFD, and when it comes to turbulence modeling, it is extremely important to have meshes with good quality and acceptable resolution.
- Some guidelines specific to RANS meshes:
  - When it comes to RANS, the most important metric for mesh resolution is the  $y^+$  value.
  - Choose your wall treatment and mesh your domain according to this requirement.
  - If you are doing 3D simulations, there are no strict requirements when it comes to the span-wise and stream-wise directions.
  - But as a rule of thumb rule, you can use  $\Delta x^+$  and  $\Delta z^+$  values as high as 300 times the value of  $\Delta y^+$  and less than a 1000 wall distance units.

# Practical turbulence estimates

## Wall distance units – A few mesh resolution guidelines and rough estimates

- Some guidelines specific to LES meshes:
  - When it comes to LES meshes, it is recommended to use wall functions.
  - Otherwise, the meshing requirements are similar to those of DNS.
  - Recommended wall distance units values are,

$$\Delta x^+ < 50, \Delta z^+ < 50 \quad \text{for} \quad y^+ < 5$$

Wall resolving

$$\Delta x^+ < 4\Delta y^+, \Delta z^+ < 4\Delta y^+ \quad \text{for} \quad 30 \leq y^+ \leq 300$$

Wall modeling

- If you are doing DNS simulations, the requirements for wall distance units in all directions are in the order of 1.
  - You might be able to go as high as 10 for  $\Delta x^+$  and  $\Delta z^+$ .

# Practical turbulence estimates

## Wall distance units – A few mesh resolution guidelines and rough estimates

- Some guidelines specific to DES meshes:
  - Usually in DES simulations wall resolving meshes are used with RANS requirements similar to RANS.
  - That is, there are no specific requirements on the streamwise and spanwise wall units values.
  - However, it is extremely important to resolve well the integral length scales, as in LES simulations.
  - What is very tricky in DES, is how to control the transition from the RANS mesh close to the walls to the fine-medium mesh far from the walls.
  - You might encounter a mismatch between the RANS mesh and the LES mesh, because the RANS mesh tends to be coarser in the streamwise and spanwise directions.
  - This transition, if not well controlled can cause grid induced separation and stability problems.

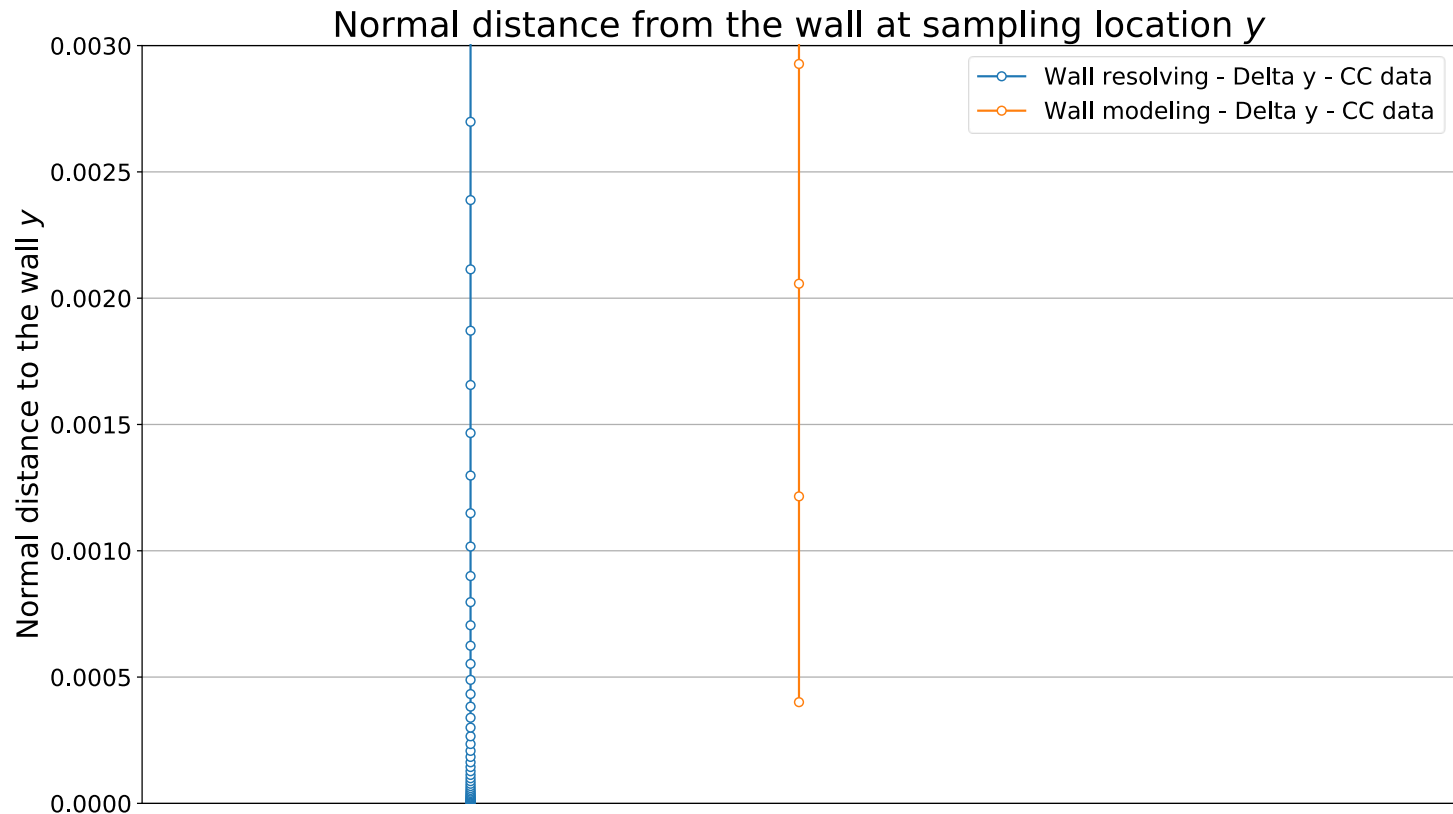


**Profiles close to the walls – Dependence  
on the  $y^+$  value**

# Practical turbulence estimates

## Profiles close to the walls – Dependence on the $y^+$ value

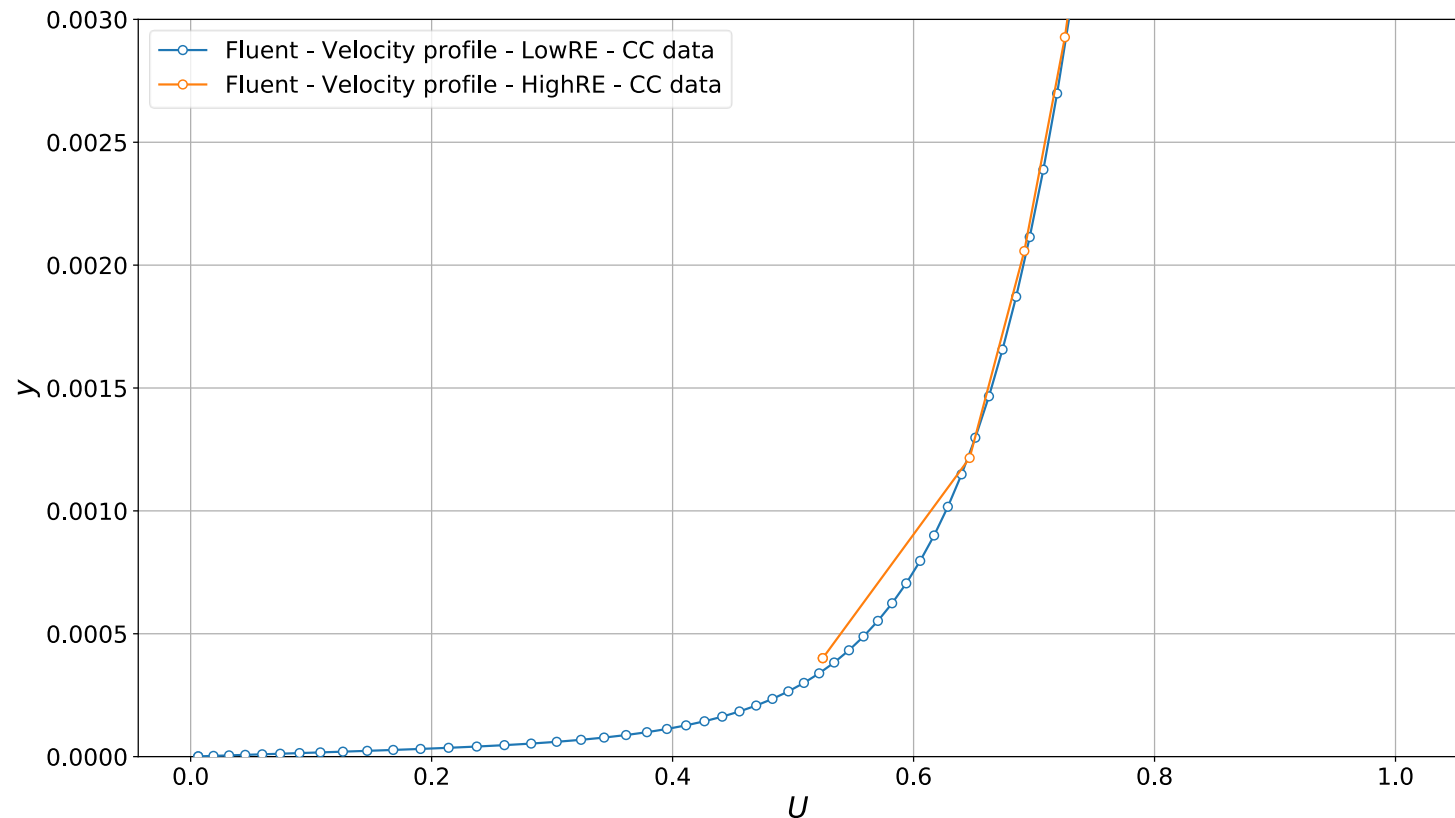
- Distance normal to wall  $y$  (in meters).
- Each circle represents a cell center.



# Practical turbulence estimates

## Profiles close to the walls – Dependence on the $y^+$ value

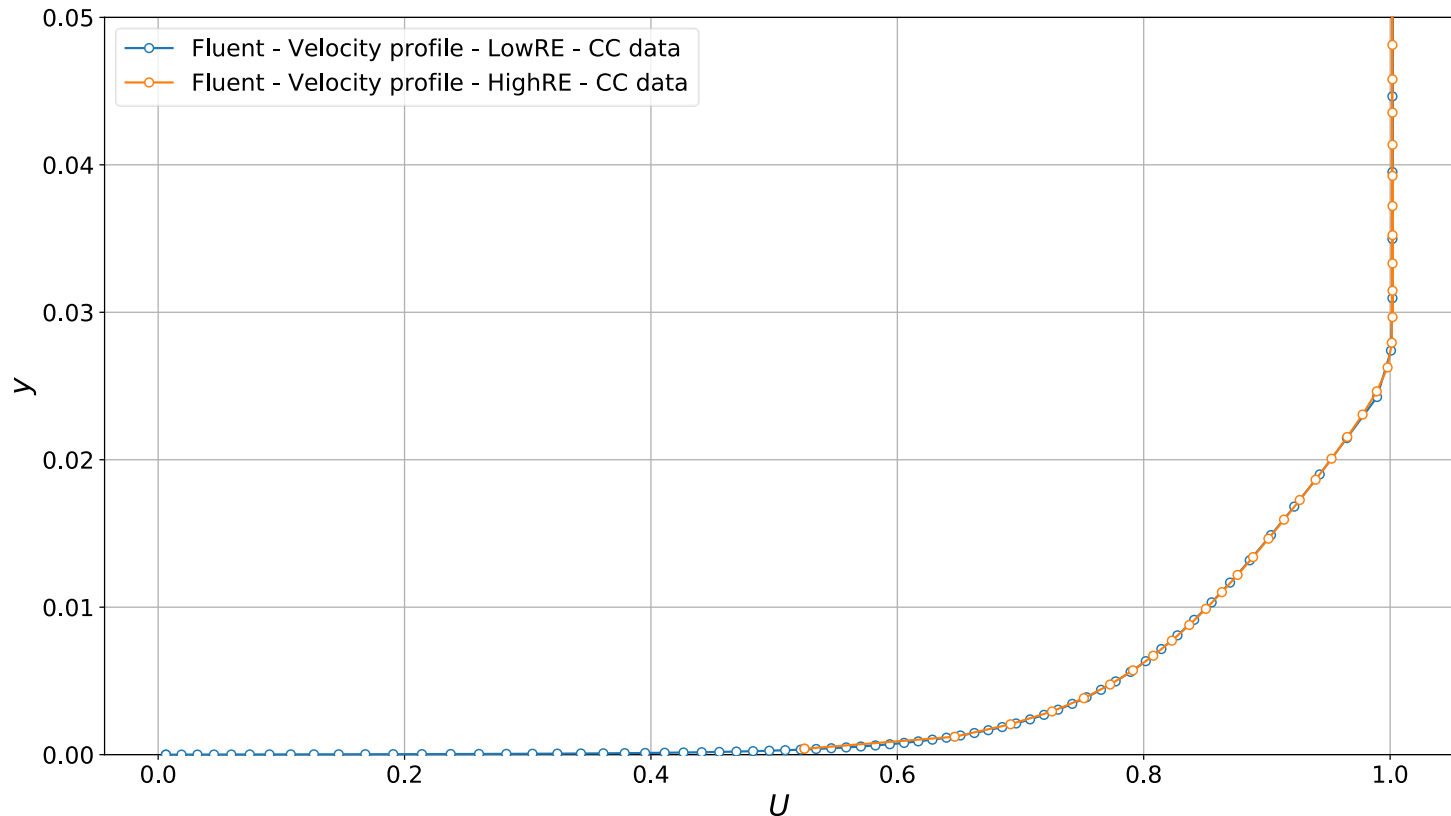
- Velocity profile close to the wall –  $y$  vs.  $U$
- Each circle represents a cell center.



# Practical turbulence estimates

## Profiles close to the walls – Dependence on the $y^+$ value

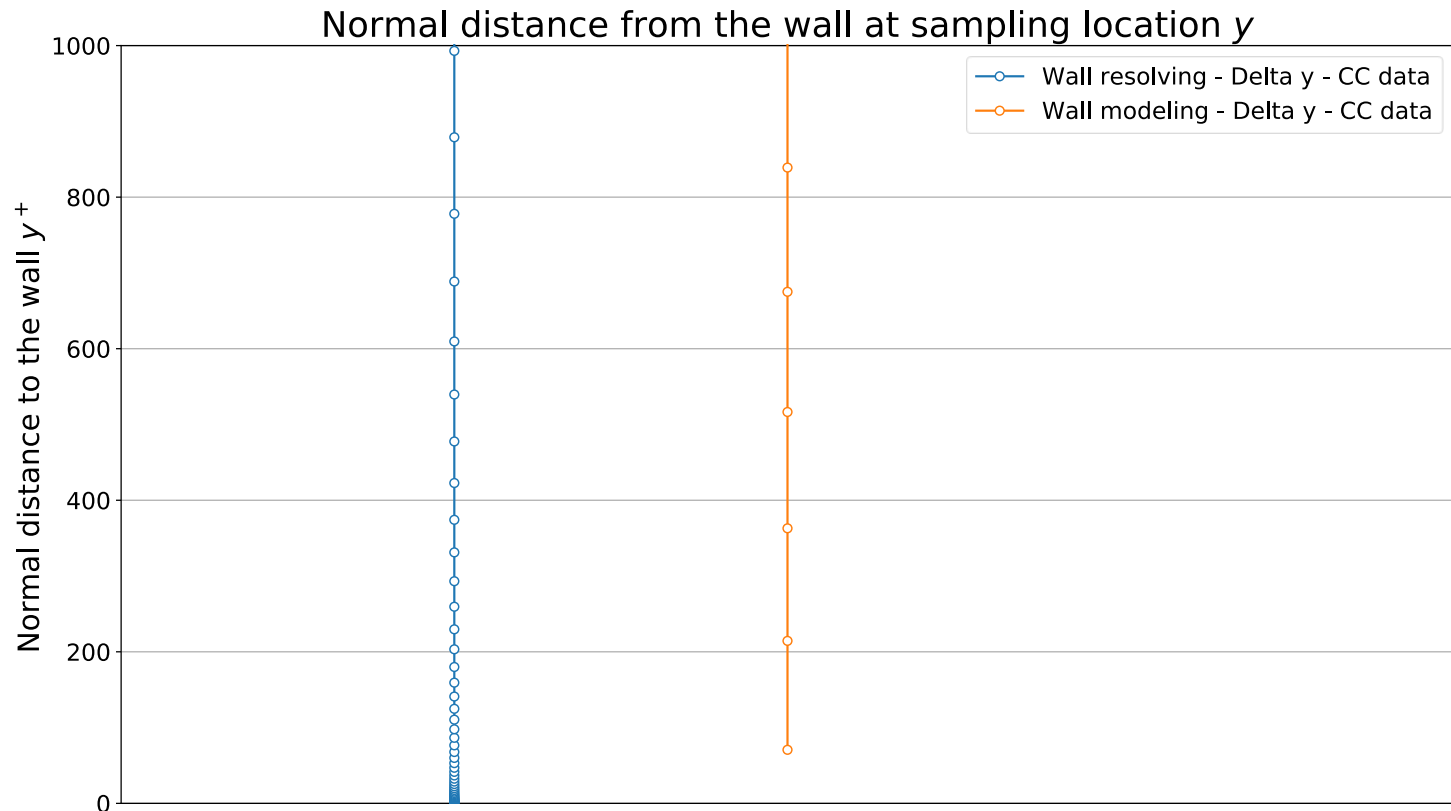
- Velocity profile close to the wall –  $y$  vs.  $U$
- Each circle represents a cell center.



# Practical turbulence estimates

## Profiles close to the walls – Dependence on the $y^+$ value

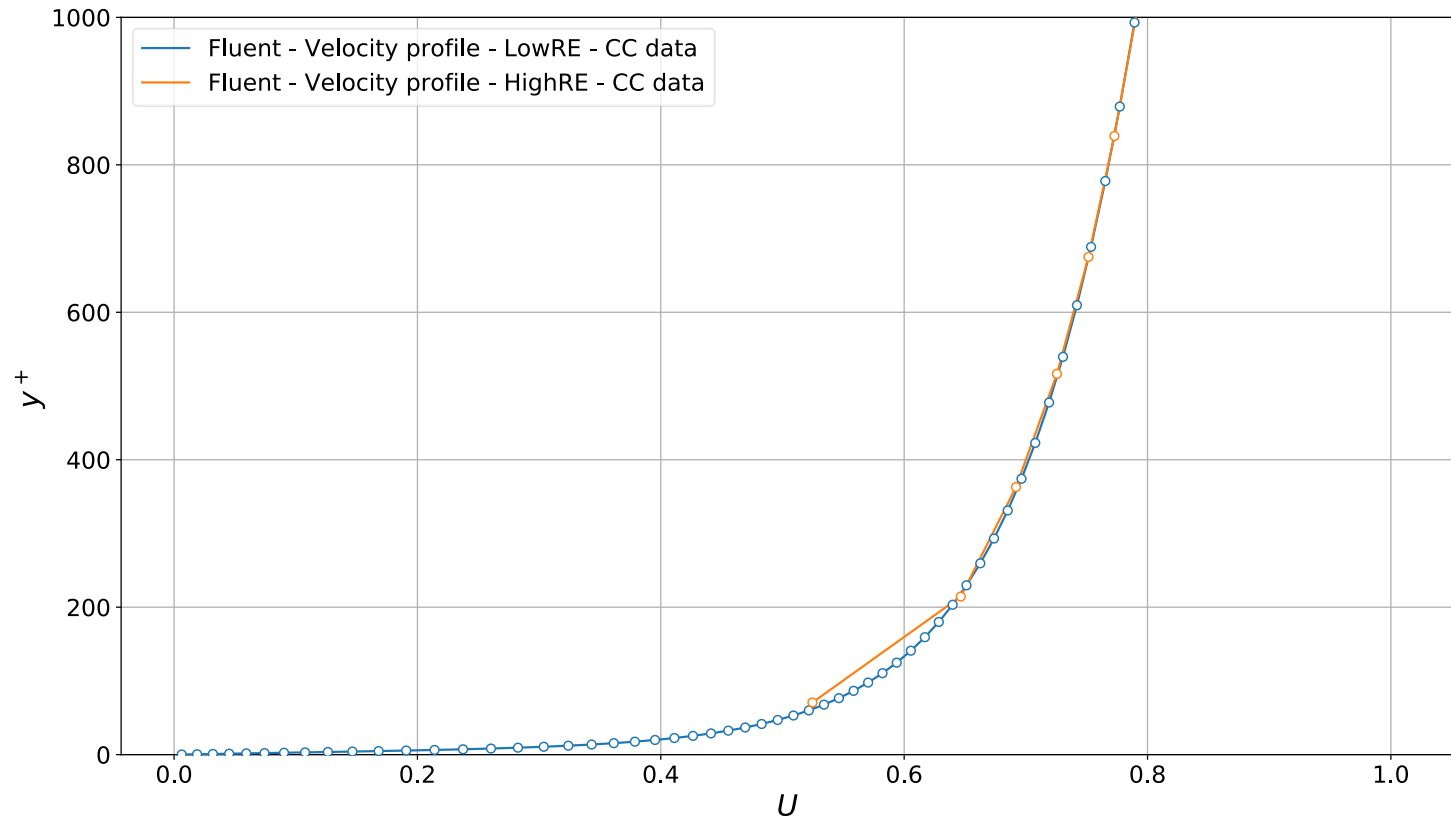
- Non-dimensional distance normal to wall  $y^+$ .
- Each circle represents a cell center.



# Practical turbulence estimates

## Profiles close to the walls – Dependence on the $y^+$ value

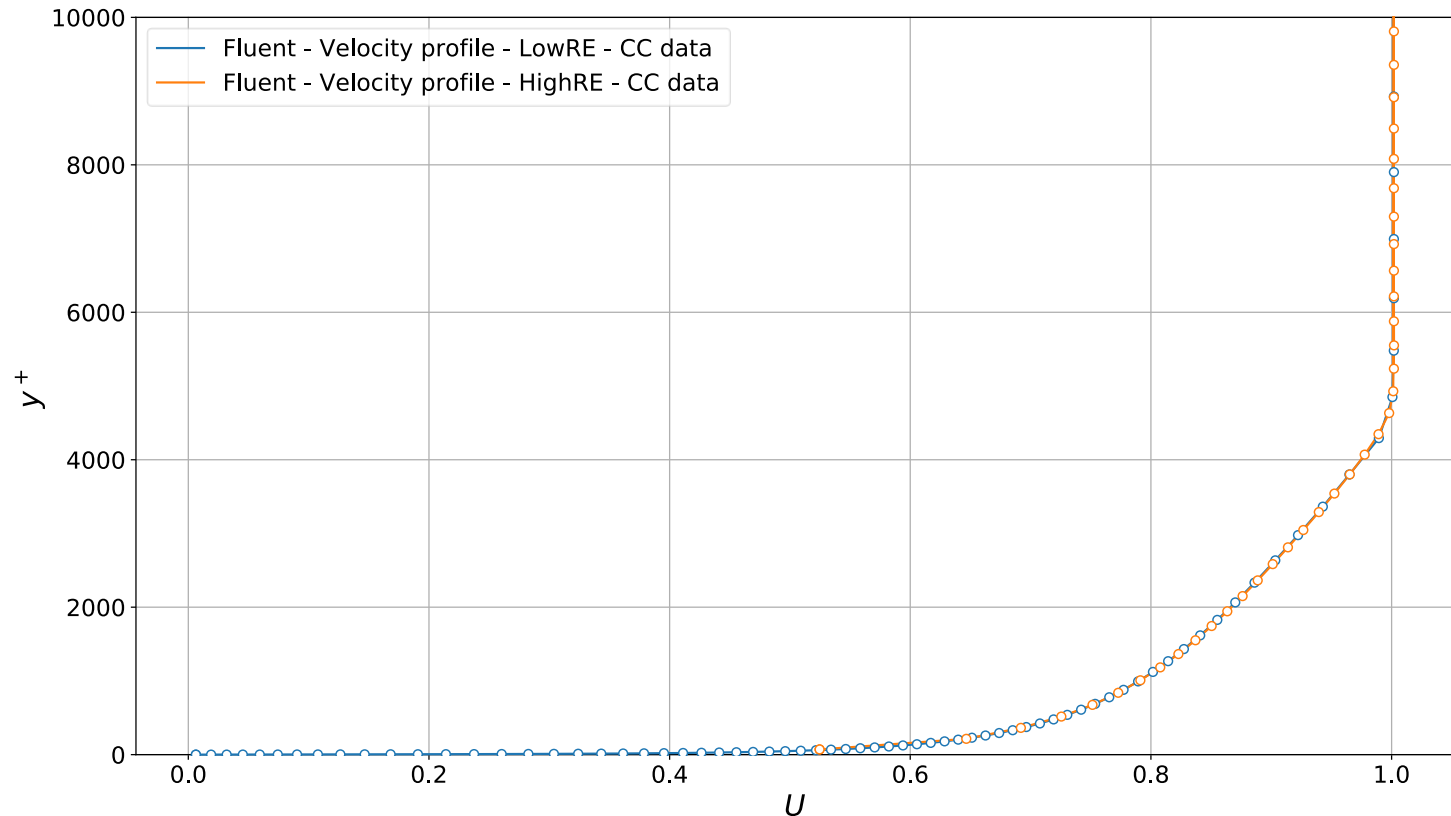
- Velocity profile close to the wall –  $y^+$  vs.  $U$
- Each circle represents a cell center.



# Practical turbulence estimates

## Profiles close to the walls – Dependence on the $y^+$ value

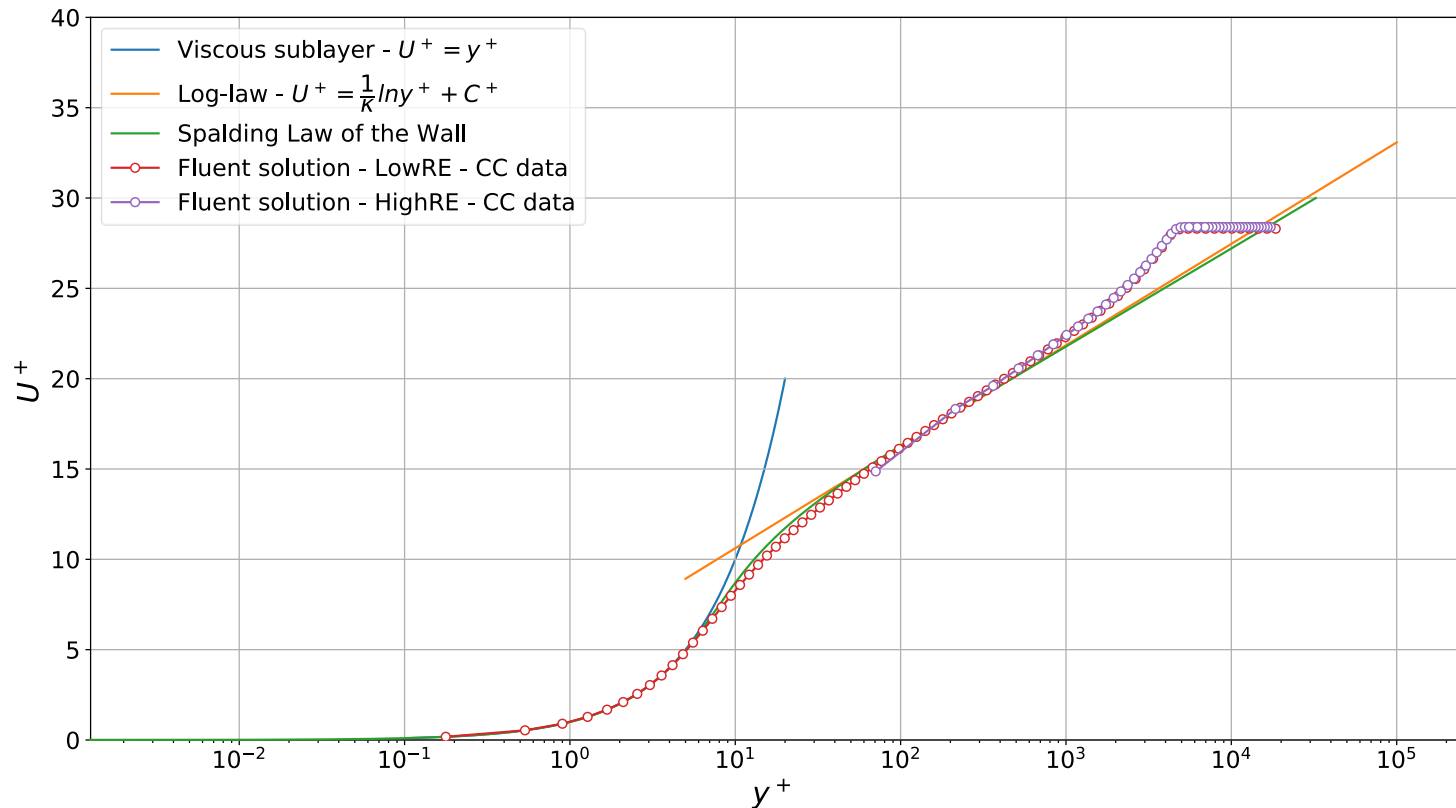
- Velocity profile close to the wall –  $y^+$  vs.  $U$
- Each circle represents a cell center.



# Practical turbulence estimates

## Profiles close to the walls – Dependence on the $y^+$ value

- Normalized velocity profile close to the wall –  $U^+$  vs.  $y^+$
- Each circle represents a cell center.

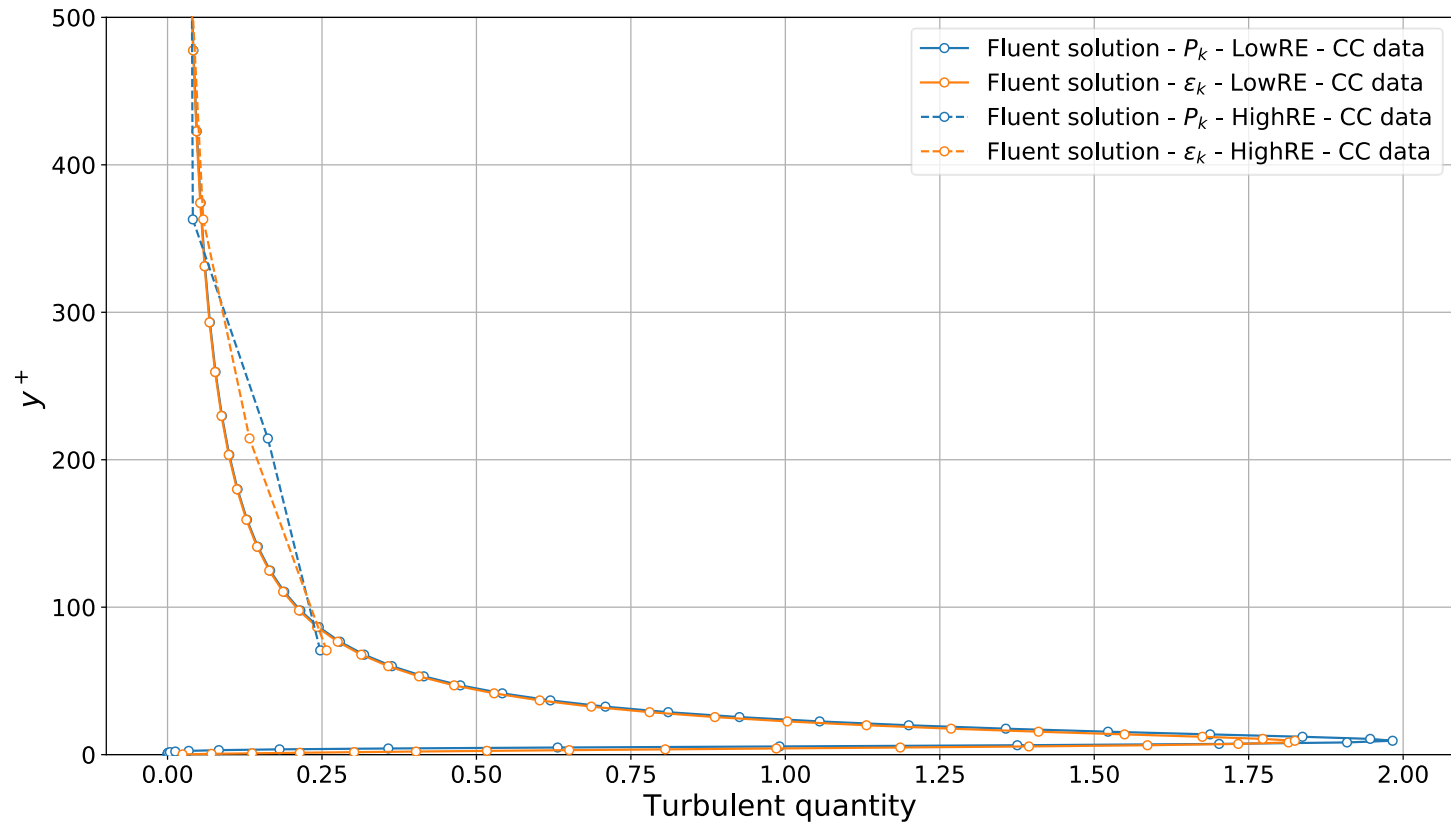




# Practical turbulence estimates

## Profiles close to the walls – Dependence on the $y^+$ value

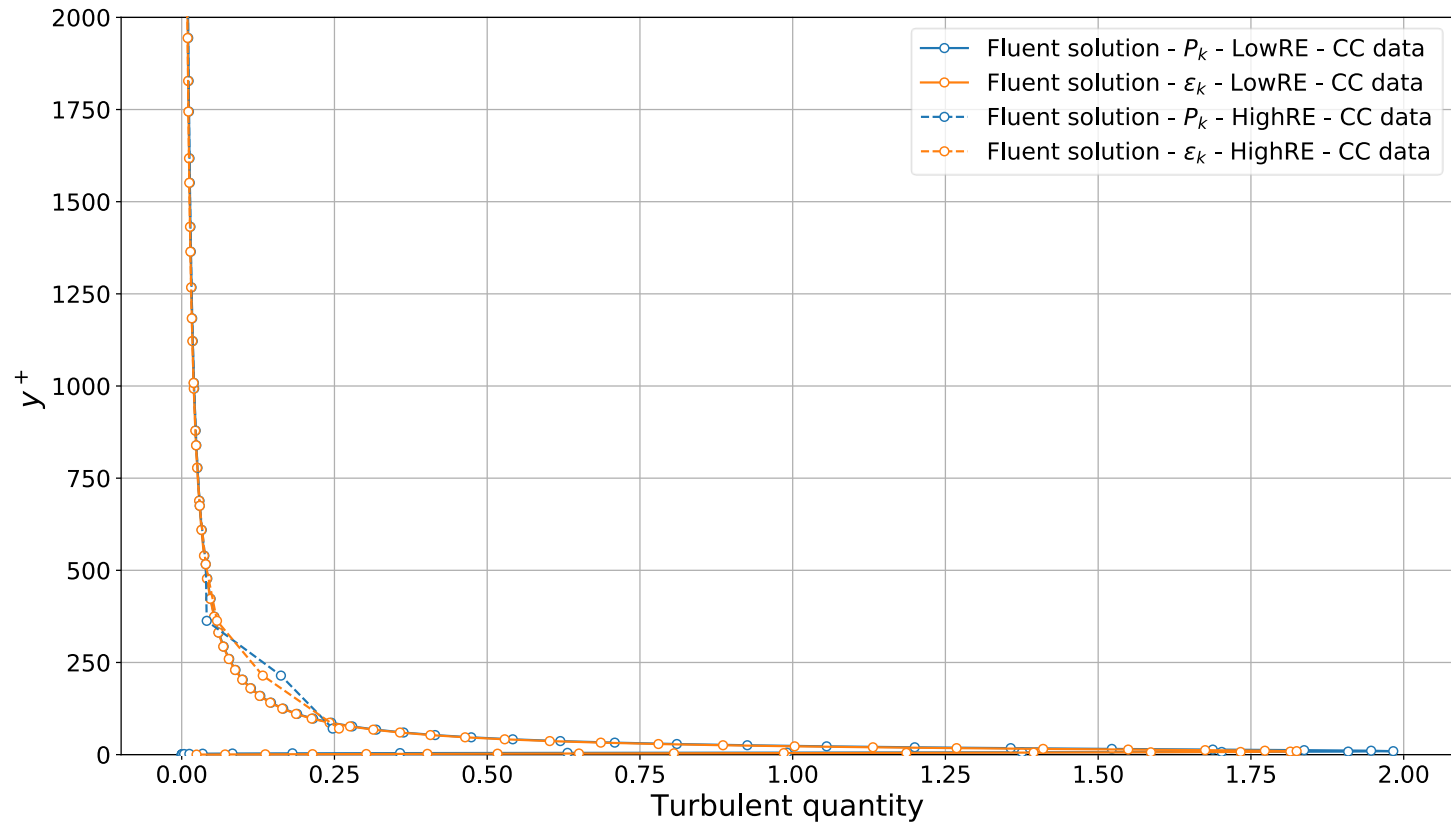
- Turbulent production and dissipation close to the walls.
- Each circle represents a cell center.



# Practical turbulence estimates

## Profiles close to the walls – Dependence on the $y^+$ value

- Turbulent production and dissipation close to the walls.
- Each circle represents a cell center.



# Summary of turbulence length scales

# Practical turbulence estimates

## Summary of turbulence length scales

- The Kolmogorov scales are summarized as follows,

$$\eta = \left( \frac{\nu^3}{\epsilon} \right)^{1/4}$$

Length scale

$$\tau_\eta = \left( \frac{\nu}{\epsilon} \right)^{1/2}$$

Time scale

$$v_\eta = (\nu\epsilon)^{1/4}$$

Velocity scale

- The Taylor microscales are summarized as follows,

$$\lambda = \left( \frac{10\nu k}{\epsilon} \right)^{1/2}$$

Length scale

$$\tau_\lambda = \left( \frac{15\nu}{\epsilon} \right)^{1/2}$$

Time scale

$$u_\lambda = \frac{\lambda}{\tau_\lambda}$$

Velocity scale

# Practical turbulence estimates

## Summary of turbulence length scales

- Taylor [1] suggests that the integral length scales can be approximated as follows,

$$l \approx \frac{k^{3/2}}{\epsilon}$$

- This estimate can be improved by using experimental data [2],

$$l = C_\mu \frac{k^{3/2}}{\epsilon} \quad \text{where} \quad C_\mu = 0.09$$

- You can express the previous relation in function of  $\omega$  as follows,

$$l = \frac{k^{1/2}}{\omega} \quad \text{where} \quad \omega = \frac{\epsilon}{\beta^* k} \quad \text{and} \quad \beta^* = \frac{9}{100}$$

[1] G. I. Taylor. Statistical theory of turbulence. Proceedings of the Royal Society of London. 1935.

[2] D. Wilcox. Turbulence Modeling for CFD. DCW Industries Inc., 2010.

# Practical turbulence estimates

## Summary of turbulence length scales

- The eddies turnover time is the ratio between the integral length scales and the velocity  $k^{1/2}$  (the measure of the velocity fluctuations around the mean), and it can be computed as follows,

$$\tau_{turnover} \sim \frac{l}{k^{1/2}} \qquad \tau_{turnover} = \frac{C_\mu k}{\epsilon}$$

- The eddy turnover time is a measure of the time it takes an eddy to interact with its surroundings.
- The integral eddy velocity is the ratio of its integral length scale and its turnover time,

$$u_l = \frac{l}{\tau_{turnover}} = k^{1/2}$$

- If you assume isotropic turbulence then,  $u_l = \left(\frac{2k}{3}\right)^{1/2}$

# Practical turbulence estimates

## Summary of turbulence length scales

- The different length scales can be related as follows,

$$\frac{l_0}{\eta} \sim Re_T^{3/4}$$

$$\frac{\lambda}{l_0} = \sqrt{10} Re_T^{-1/2}$$

$$\frac{\lambda}{\eta} = \sqrt{10} Re_T^{1/4}$$

$$\lambda = \sqrt{10} \eta^{2/3} l_0^{1/3}$$

- Support equations:

$$Re_T = \frac{k^{1/2} l_0}{\nu} = \frac{k^2}{\epsilon \nu}$$

$$l_0 = \frac{k^{3/2}}{\epsilon}$$

$$k = \frac{1}{2} \left( \overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right)$$

$$\lambda = \left( \frac{10 \nu k}{\epsilon} \right)^{1/2}$$

# Practical turbulence estimates

## Summary of Reynolds numbers

- The different Reynolds numbers, based on different length scales, can be summarized as follows,

$$Re = \frac{UL}{\nu}$$

Flow Reynolds number

$$Re_\lambda = \frac{u'\lambda}{\nu}$$

Taylor Reynolds number

$$Re_\eta = \frac{\eta u_\eta}{\nu} = 1$$

Kolmogorov Reynolds number

- The turbulent Reynolds number is related to the integral scales. It is a few order of magnitude lower than the flow Reynolds number (on the order of 100 to 10000).

$$Re_T = \frac{k^{1/2}l_0}{\nu} = \frac{k^2}{\epsilon\nu}$$

- The turbulence and Taylor Reynolds numbers can be related as follows

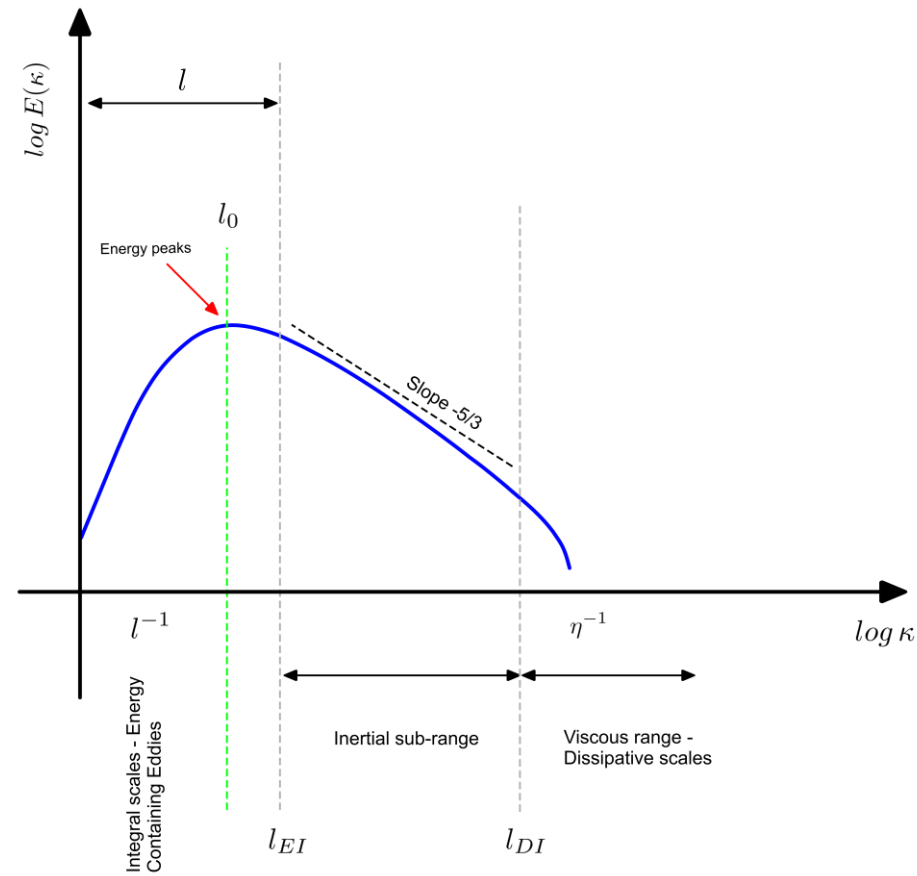
$$Re_\lambda = \left( \frac{20}{3} Re_T \right)^{1/2}$$



# Practical turbulence estimates

## Summary of turbulence length scales

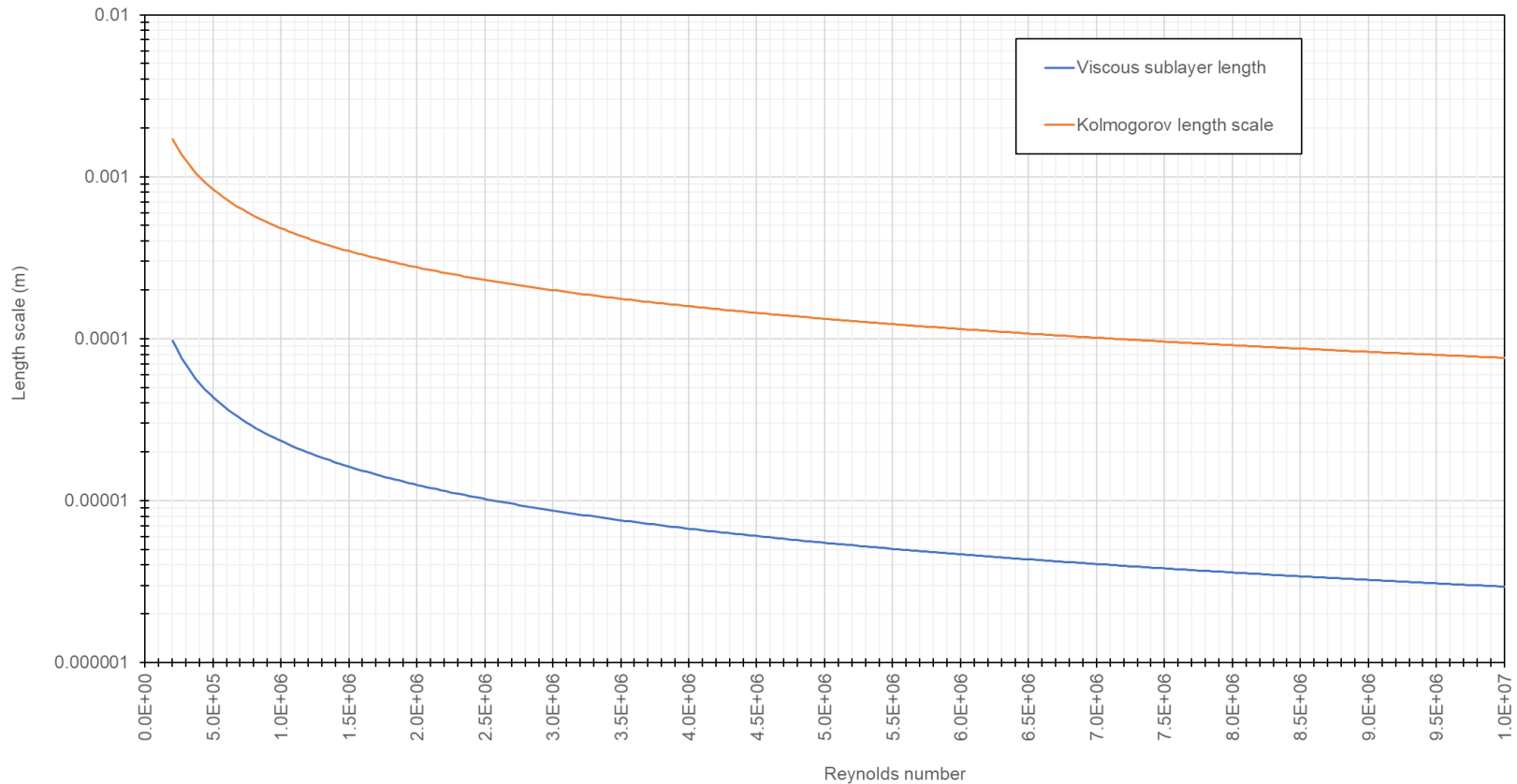
- During the previous lectures, sometimes we used the notation  $l_0$  and sometimes we used the notation  $l$ .
  - $l_0$  represents the largest integral eddy.
  - $l$  represents the average of the integral eddies.
- Some authors use  $l_0$  and some other authors use  $l$ .
- In our explanations, we assumed that these scales are interchangeable without loss of generality.
- Have in mind that  $Re_\tau$  is computed using the integral scale  $l_0$  and the TKE (related to the velocity fluctuations around the mean velocity).



**Can the Kolmogorov eddies become smaller than the viscous sublayer length?**

# Practical turbulence estimates

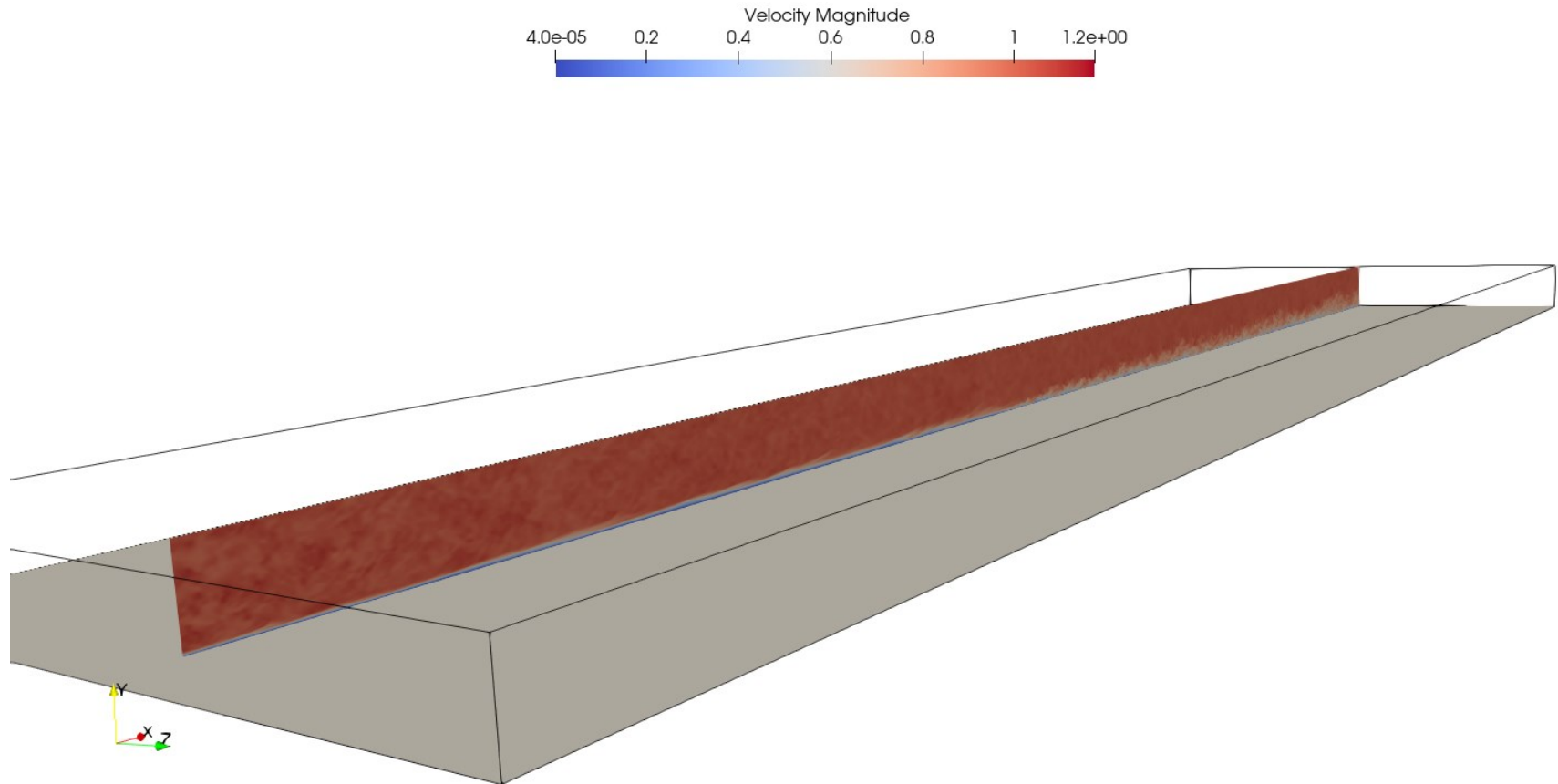
**Can the Kolmogorov eddies become smaller than the viscous sublayer length?**



- In a few words, no.
- In this case, the viscous sublayer is always at least one order of magnitude thinner than the Kolmogorov eddies.
- The viscous sublayer cannot accommodate Kolmogorov eddies.

# Practical turbulence estimates

**Can the Kolmogorov eddies become smaller than the viscous sublayer length?**

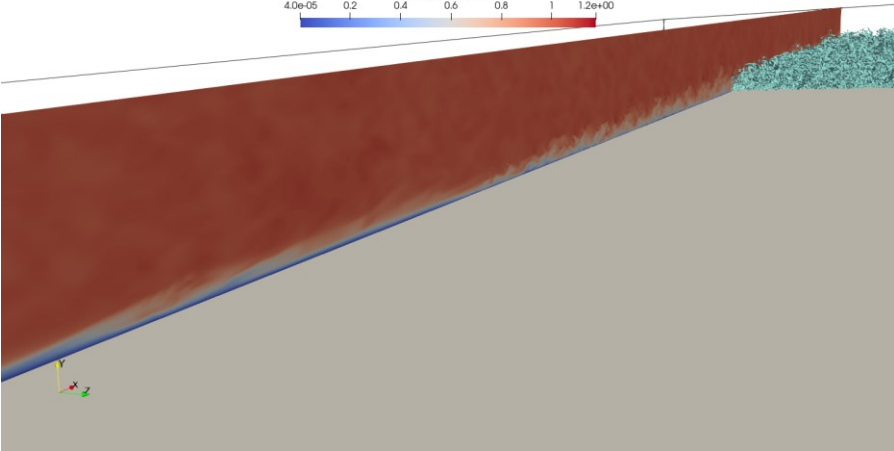


- As dissipation takes place at the viscous sublayer, it cannot accommodate Kolmogorov eddies.
- The viscous sublayer will always adapt so it is thinner than the Kolmogorov eddies.

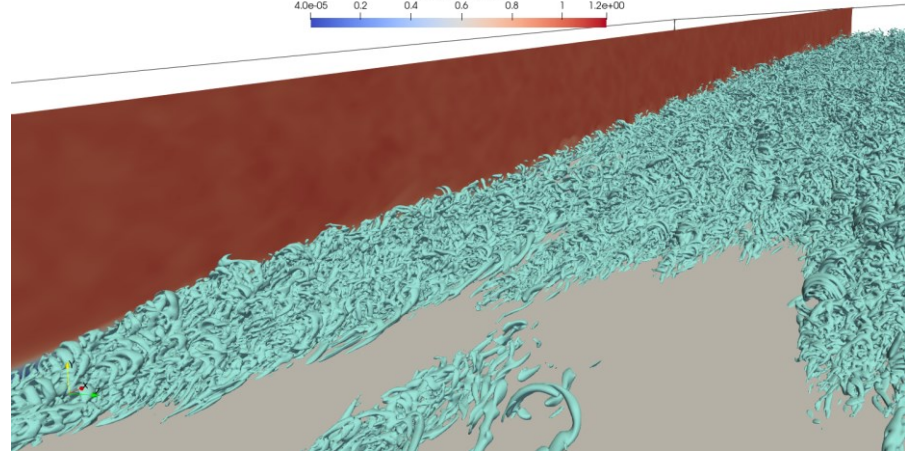
# Practical turbulence estimates

**Can the Kolmogorov eddies become smaller than the viscous sublayer length?**

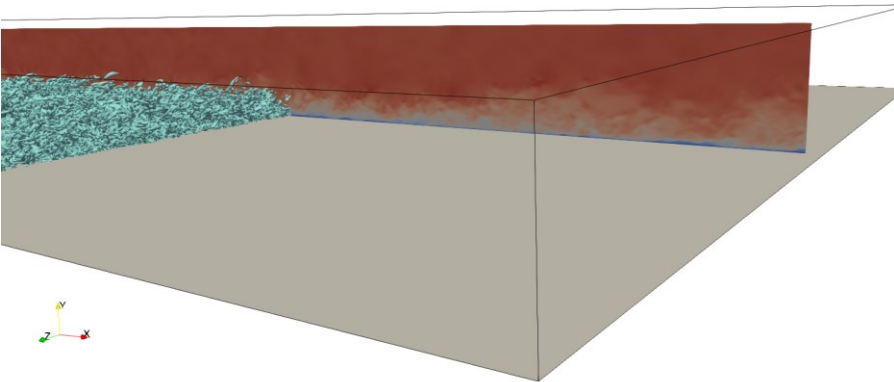
Velocity Magnitude  
4.0e-05 0.2 0.4 0.6 0.8 1 1.2e+00



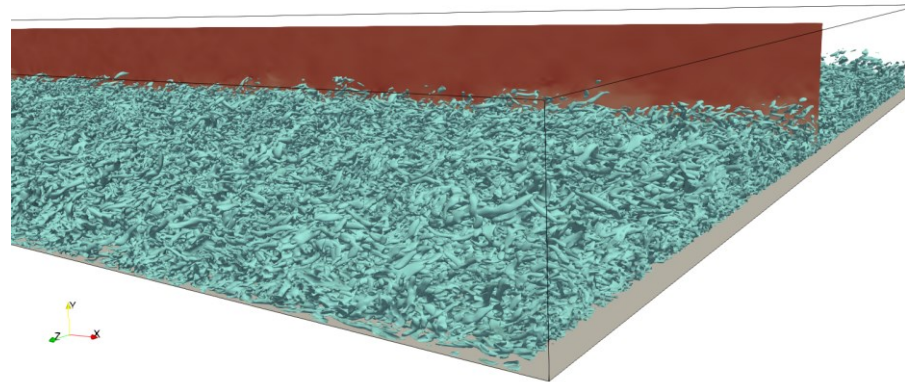
Velocity Magnitude  
4.0e-05 0.2 0.4 0.6 0.8 1 1.2e+00



Velocity Magnitude  
4.0e-05 0.2 0.4 0.6 0.8 1 1.2e+00



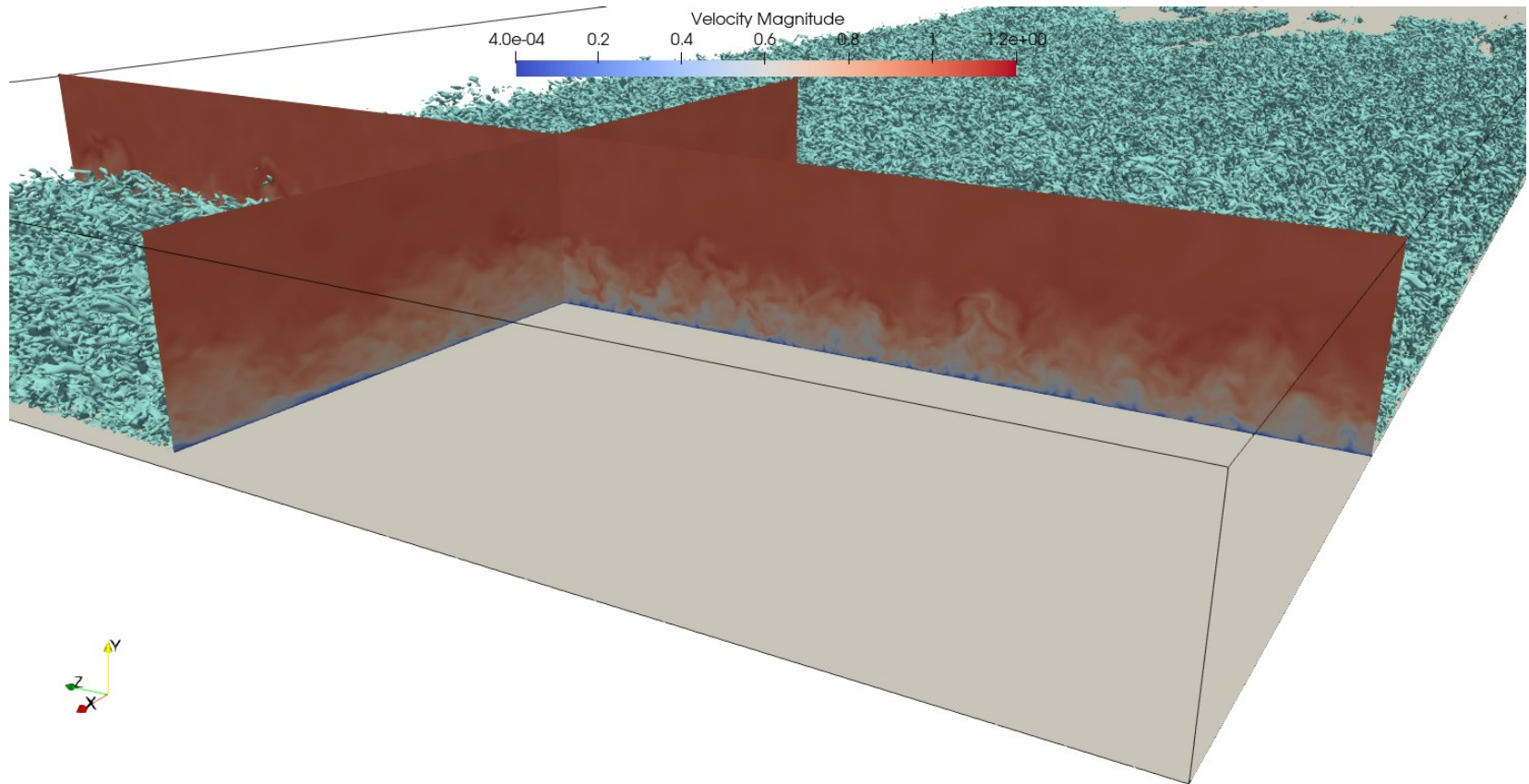
Velocity Magnitude  
4.0e-05 0.2 0.4 0.6 0.8 1 1.2e+00



- As dissipation takes place at the viscous sublayer, it cannot accommodate Kolmogorov eddies.
- The viscous sublayer will always adapt so it is thinner than the Kolmogorov eddies.

# Practical turbulence estimates

**Can the Kolmogorov eddies become smaller than the viscous sublayer length?**

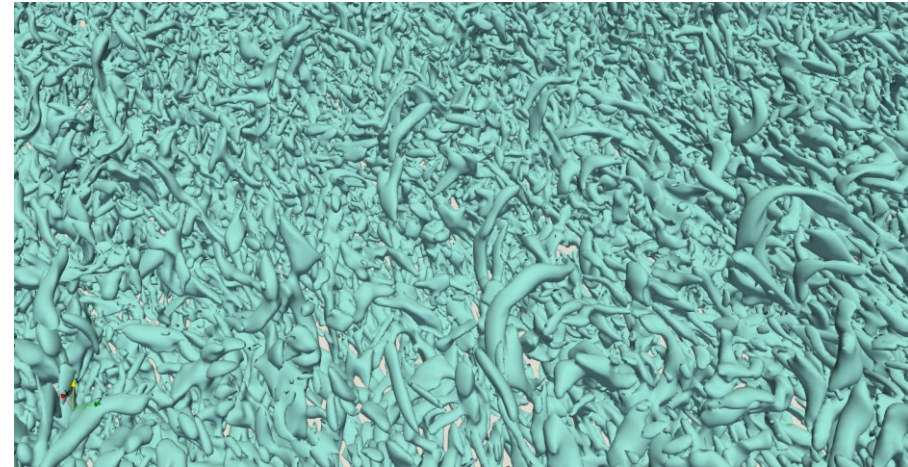
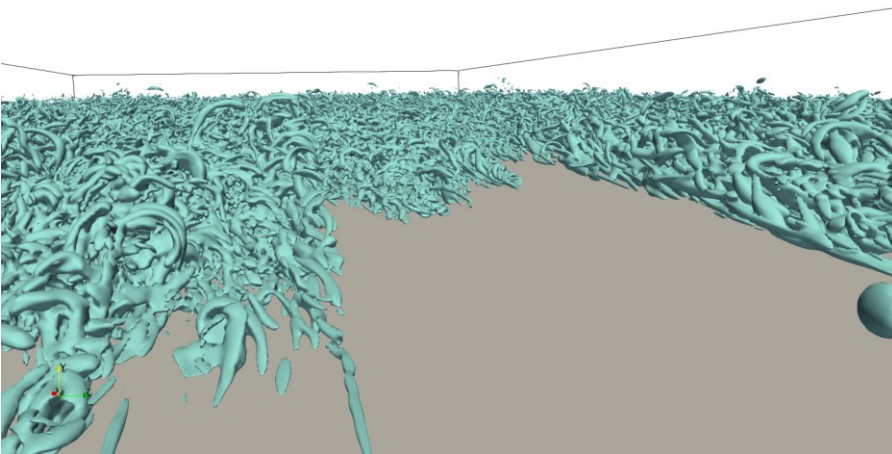
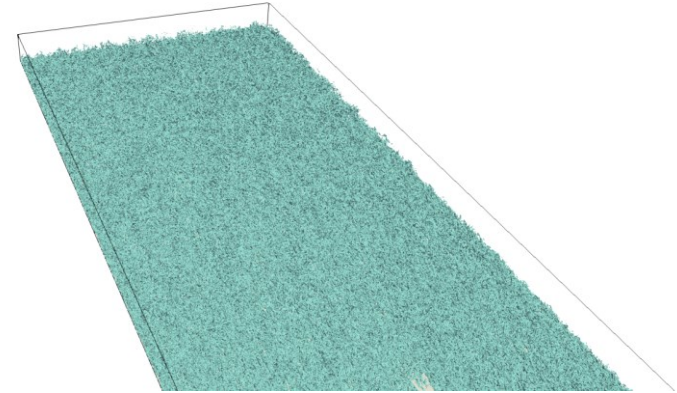
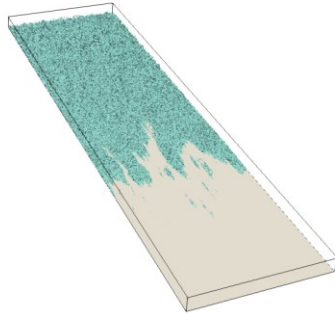


- As dissipation takes place at the viscous sublayer, it cannot accommodate Kolmogorov eddies.
- The viscous sublayer will always adapt so it is thinner than the Kolmogorov eddies.



# Practical turbulence estimates

**Can the Kolmogorov eddies become smaller than the viscous sublayer length?**



- To give you an idea how time consuming the postprocessing of large-scale simulations is:
  - We are looking at one timestep of a DNS simulation. The input file is about 17 GB, and it required about 110 GB of RAM memory, a GPU of 16 GB, 16 cores, and about 5 minutes to open and manipulate the data (mesh size approximately 1.5 billion grid points).

# Practical turbulence estimates

## Can the Kolmogorov eddies become smaller than the viscous sublayer length?

- Turbulence is a continuum phenomenon.
- The Kolmogorov scales are well above the molecular scales.
- A parameter used to determine when the continuum hypothesis is not valid anymore is the Knudsen number.
- The Knudsen number is the ratio of the mean free path\* to a characteristic dimension (e.g., body length, turbulent scales, and so on).

$$Kn = \frac{\lambda}{L}$$


Mean free path

- Although there is no definitive criterion, the continuum flow model starts to break down when the Knudsen number is roughly of the order of 0.1.
- At this Knudsen number values, the flow is rarefied and can not be treated as a continuum.
- In the case of  $Kn > 0.1$ , different equations need to be used.
  - In these lectures, we will deal only with the continuum equations.

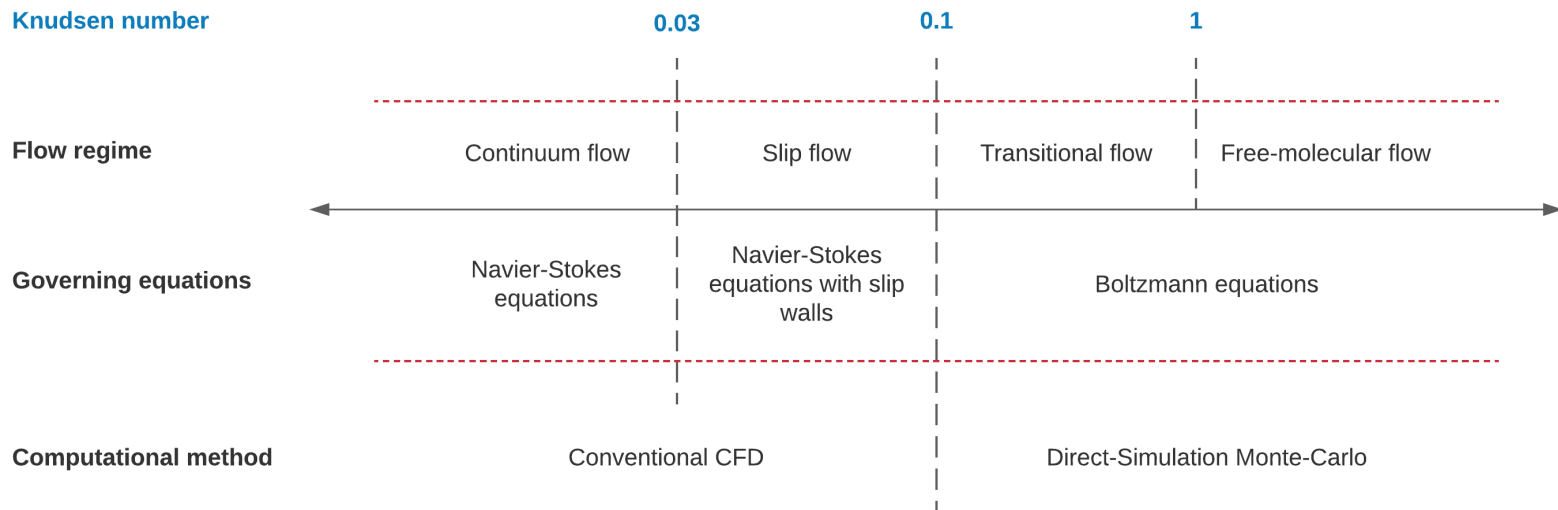
\* The mean free path is the average distance travelled by a moving particle (such as an atom, a molecule, a photon) between successive impacts (collisions), which modifies its direction or energy or other particle properties.



# Practical turbulence estimates

## Can the Kolmogorov eddies become smaller than the viscous sublayer length?

- Although there is no definitive criterion, the continuum flow model starts to break down when the Knudsen number is roughly of the order of 0.1.
- For Knudsen numbers up to approximately 0.03 the continuum hypothesis is valid.
- For Knudsen numbers between 0.03 and 0.1, the Navier-Stokes equations are valid in the freestream, but we need to use slip conditions at the walls.
- For Knudsen numbers larger than 0.1 the Navier-Stokes equations cannot be used anymore.



# Practical turbulence estimates

## Can the Kolmogorov eddies become smaller than the viscous sublayer length?

- Taking air under atmospheric conditions as an example, the mean free path is  $6.5 \times 10^{-8}$  m (65 nanometers)
- The mean time between successive collision of a molecule is  $10^{-10}$  s.
- In comparison, the smallest geometric length scale in a flow is seldom less than 0.1 mm (or  $10^{-4}$  m).
- Which for flow velocities up to  $100 \text{ m}\cdot\text{s}^{-1}$ , yields a flow timescale larger than  $10^{-6}$  s.
- Thus, even for this example of a flow with small length and time scales, these flow scales exceed the molecular scales by three or more orders of magnitude.
- This separation can be seen in the plot below, where for very large Reynolds number, the Kolmogorov scales are well above the molecular scales.

