Homework 2

Turbulence and CFD models: Theory and applications
Lectures 5 and 6

Question 1

Starting from the exact incompressible Navier-Stokes equations, derive the incompressible Reynolds-Averaged Navier-Stokes (RANS) equations,

$$\begin{aligned} \nabla \cdot (\bar{\mathbf{u}}) &= 0, \\ \frac{\partial \bar{\mathbf{u}}}{\partial t} + \nabla \cdot (\bar{\mathbf{u}}\bar{\mathbf{u}}) &= \frac{-\nabla \bar{p}}{\rho} + \nu \nabla^2 \bar{\mathbf{u}} + \frac{1}{\rho} \nabla \cdot \boldsymbol{\tau}^R \end{aligned} \tag{1}$$

where $\boldsymbol{\tau}^R = -\rho\left(\overline{\mathbf{u}'\mathbf{u}'}\right)$. Write down all steps, averaging rules, and vector identities used.

Question 2

Using the Boussinesq hypothesis,

$$\boldsymbol{\tau}^R = -\rho\left(\overline{\mathbf{u}'\mathbf{u}'}\right) = 2\mu_T \bar{\mathbf{D}}^R - \frac{2}{3}\rho k \mathbf{I} = \mu_T\left[\nabla \overline{\mathbf{u}} + (\nabla \overline{\mathbf{u}})^{\mathrm{T}}\right] - \frac{2}{3}\rho k \mathbf{I} \tag{2}$$

Derive the solvable incompressible Reynolds-Averaged Navier-Stokes (RANS) equations,

$$\begin{aligned} \nabla \cdot (\bar{\mathbf{u}}) &= 0 \\ \frac{\partial \bar{\mathbf{u}}}{\partial t} + \nabla \cdot (\bar{\mathbf{u}}\bar{\mathbf{u}}) &= -\frac{1}{\rho}\left(\nabla \bar{p} + \frac{2}{3}\rho \nabla k\right) + \nabla \cdot \left[\frac{1}{\rho}\left(\mu + \mu_t\right) \nabla \bar{\mathbf{u}}\right] \end{aligned} \tag{3}$$

What is the definition of effective viscosity μ_{eff}?

Write down all steps, averaging rules, and vector identities used.

Question 3

This task consists of a bibliographical research. You are asked to look into the literature and document a RANS turbulence model. You should describe the solvable equations, closure coefficients, auxiliary relations (damping functions, blending functions, and so on), recommended boundary conditions and initial conditions, limitations, deficiencies, palliatives, and best standard practices. You must include at least two references.

The following turbulence models are not accepted:

- Standard $k - \epsilon$ [1].
- Wilcox 1988 $k - \omega$ [2].
- Standard Spalart-Allmaras [3].

References

[1] B. E. Launder, D. B. Spalding. The Numerical Computation of Turbulent Flows. Computer Methods in Applied Mechanics and Engineering, Vol. 3, Issue 2, pp. 269-289, 1974.

[2] D. C. Wilcox. Reassessment of the Scale-Determining Equation for Advanced Turbulence Models. AIAA Journal, Vol. 26, No. 11, pp. 1299-1310, 1998.

[3] P. Spalart, S. Allmaras. A One-Equation Turbulence Model for Aerodynamic Flows. Recherche Aerospatiale, No. 1, pp. 5-21, 1994.

General guidelines

- You can write your report in English or Italian.
- Write down all the steps followed to derive the equations.
- You can use vector notation or index notation.

- You can also write the equations in their expanded Cartesian form.
- Write down all the averaging rules and vector identities used.
- It will be highly appreciated if you write your homework using LaTeX (this is not compulsory). You can use overleaf, which is a free online LaTeX editor.
  - `https://www.overleaf.com/`
- You can get a quick start on using LaTeX at this link:
  - `https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_minute`
- Do not forget to add the references. You should use at least two references (websites and Fluent documentation are allow).
- Use any of the bibliography styles described at the following link,
  - `https://www.overleaf.com/learn/latex/biblatex_bibliography_styles`
- You can cite Ansys Fluent as follows,
  - Ansys Fluent Academic Research, Release 2021, Help System, Ansys Fluent Theory Guide, Ansys, inc.
- Do not hesitate to contact me if you have any questions.

Deadline

The deadline to submit your homework is 23MAY2022. You must submit your homework via the Teams channel.