Matching Mach number and Reynolds number

- In high-speed aerodynamics, the important nondimensional parameters that we need to match are Mach number and Reynolds number.

$$M_\infty = \frac{U_\infty}{a} \qquad Re_L = \frac{\rho U L}{\mu} = \frac{U L}{\nu}$$

- Setting one of these numbers is easy.
- But if we want to keep the dynamic similarity, matching both numbers is not very straightforward.
- In order to maintain dynamic similarity, we need to change the reference pressure and temperature.
- The question is, what reference values do we use in order to match the Mach number and the Reynolds number?

Matching Mach number and Reynolds number

- Let us start with the definition of the Mach number M and the Reynolds number Re (based on a characteristics length L),

$$M_\infty = \frac{U_\infty}{a} \qquad\qquad Re_L = \frac{\rho U L}{\mu} = \frac{UL}{\nu}$$

- And recall that the speed of sound a is defined as,

$$a = \left[\left(\frac{\partial p}{\partial \rho} \right)_S \right]^{\frac{1}{2}} = \sqrt{\gamma \frac{p}{\rho}} = \sqrt{\gamma R_g T}$$

- Therefore, the Mach number can be written as,

$$M_\infty = \frac{U_\infty}{a} = \frac{U_\infty}{\sqrt{\gamma (p/\rho)}} = \frac{U_\infty}{\sqrt{\gamma R_g T}}$$

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- At this point the question is, how does temperature and pressure affect the Mach number and Reynolds number?
- Recall that the speed of sound and the dynamic viscosity are both functions of temperature.
- Also recall that the density is a function of pressure and temperature, and they can be related via a thermodynamics equation of state (i.e., the ideal gas law).
- Therefore, we can express the Mach number and the Reynolds number as follows,

$$M_\infty = \frac{U_\infty}{a(T)} \qquad Re_L = \frac{\rho(P,T)U_\infty L}{\mu(T)}$$

- We can now plug the Mach number equation into the Reynolds number expression by solving for the freestream velocity U_∞.

$$Re_L = \frac{\rho(P,T)M_\infty a(T) L}{\mu(T)}$$

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- By using the definition of the speed of sound we obtain the following relationship,

$$Re_L = \frac{\rho(P,T) M_\infty \sqrt{\gamma R_g T} L}{\mu(T)}$$

- At this point we can use the ideal gas law to relate density to pressure and temperature as follows,

$$P = \rho R_g T$$

- By substituting the ideal gas law into the previous Reynolds number relation, we obtain the following equation,

$$Re_L = \frac{P M_\infty \sqrt{\gamma R_g T} L}{R_g T \mu(T)} = \frac{P M_\infty \sqrt{\gamma} L}{\sqrt{RT} \mu(T)}$$

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- In this expression,

$$Re_L = \frac{PM_\infty\sqrt{\gamma R_g T}L}{R_g T\mu(T)} = \frac{PM_\infty\sqrt{\gamma}L}{\sqrt{RT}\mu(T)}$$

- We are only missing the dynamic viscosity dependence on temperature.
- This dependence can be computed using, for example, Sutherland's law,

$$\mu = \frac{C_1 T^{\frac{3}{2}}}{(T + C_2)}$$

- This expression corresponds to Sutherland's law with two coefficients.
- One of the many models available in the literature.

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- At this point, we can solve for the pressure to obtain the following expression that relates the Mach number and Reynolds number,

$$P = \frac{Re_L \sqrt{RT} \mu(T)}{M_\infty \sqrt{\gamma} L}$$

- By choosing a reference temperature, we know everything on the right-hand side and can directly solve for P.
- Note that the temperature is given in Kelvins.

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- The opposite approach is also possible, but it requires more work as we need to use an iterative method (e.g., Newton's method) to solve for T.

$$\sqrt{T}\mu(T) = \frac{PM_\infty\sqrt{\gamma}L}{Re_L\sqrt{R}}$$

- Either of the previous approaches are valid.
- However, the first approach is easier to implement.

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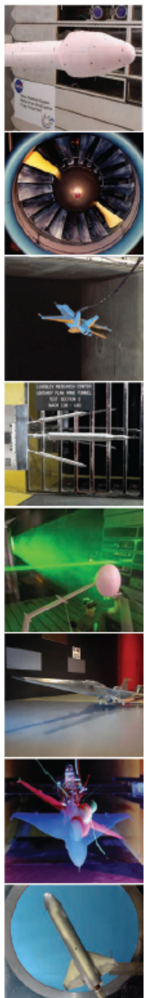
- The main takeaway of this discussion is that when working with scaled models (in physical experiments or in numerical simulations) and in order to keep the dynamic similarity between the Mach number and the Reynolds number, you need to define the right temperature and pressure values.
- When conducting numerical simulations, you must know the reference pressure and temperature, and the working fluid used in the experimental facility.
  - Read carefully the experiment specifications or reference publication.
- A big advantage of numerical simulations is that there is no need to work with scaled models.
- But when comparing numerical simulations with the experimental values, you always need to respect the dynamic similarity because not necessarily the physical experiments are conducted full scale.

Matching Mach number and Reynolds number

- NASA wind tunnel testing guide.
- https://www.nasa.gov/sites/default/files/atoms/files/nasa-2018-gftd_trifold-3-20-2019-508.pdf



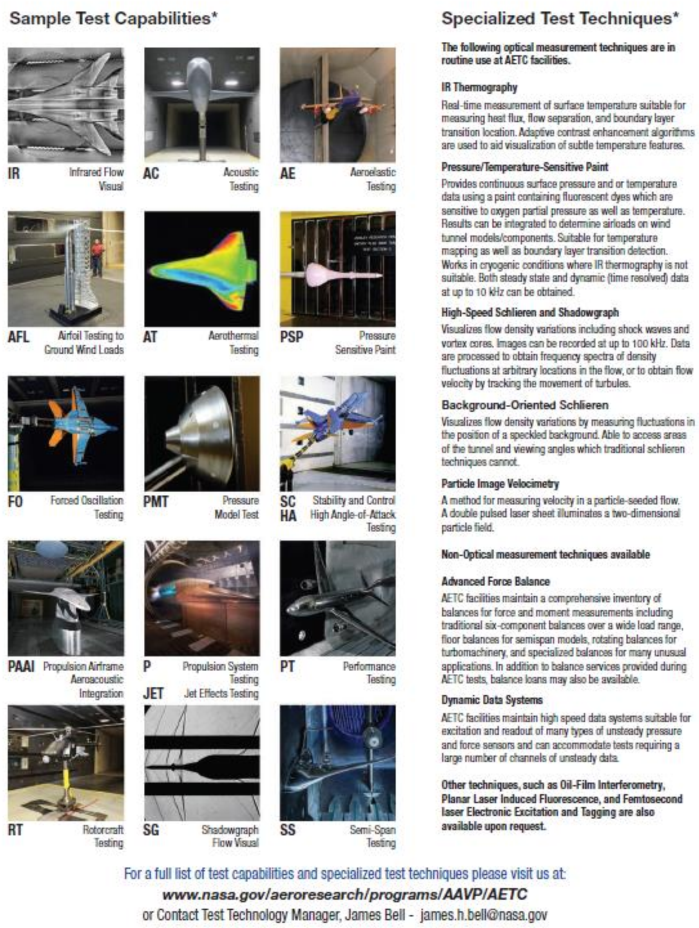
Facility Capabilities at a Glance

Facility		Speed	Site	Reynolds Number	Test Section Size	Total Pressure	Total Temperature	Test Gas	Type	Sample Test Capabilities
SUBSONIC SPEED RANGE										
14-by 22 Foot Subsonic Tunnel (14 x 22)		Mach 0 to 0.3 (348 ft/s)	NASA LaRC	0 to 2.2 x 10^6 per ft	14.5' H x 21.75' W x 50' L	Atmospheric	Ambient	Air	Closed Circuit, open or closed test section	AC, FF, FO, GE, SC, HA, P, JET, R, SS, PAAI
20-Foot Vertical Spin Tunnel (VST)		0 to 90 ft/s	NASA LaRC	0 to 0.55 x 10^6 per ft	25' H x 20' W	Atmospheric	Ambient	Air	Closed-throat, annular return	FF, FO, SC, HA
9-by 15 Foot Low Speed Wind Tunnel (LSWT)		Mach 0 to 0.21	NASA Glenn	0 to 1.4 x 10^6 per ft	9' H x 15' Wx 33' L	0 to 72 psf	Ambient to 550°R	Air	Atmospheric	PSP, Aero, UHB, PIV, PAAI
Icing Research Tunnel (IRT)		Mach 0.05 to 0.50	NASA Glenn	0 to 3.6 x 10^6 per ft	6' H x 9' W x 20' L	0 to 230 psf	Ambient to -35°	Air	Closed Return - Atmospheric	In-flight icing tests and simulations
TRANSONIC SPEED REGIME										
Transonic Dynamics Tunnel (TDT)	Air Mode: Heavy Gas Mode:	Mach 0 to 1.2 Mach 0 to 1.2	NASA LaRC	0.01 to 3.0 x 10^6 per ft 0.1 to 9.6 x 10^6 per ft	16' H x 16' W	0.5 psia to atmospheric	70° to 130°	Air R-134a	Closed Circuit	AE, FO, SC, R, SS
National Transonic Facility (NTF)	Air Mode: Cryogenic Mode:	Mach 0.1 to 1.05 Mach 0.1 to 1.20	NASA LaRC	1 to 23 x 10^6 per ft 4 to 140 x 10^6 per ft	8.2' H x 8.2' W x 25' L	14.7 to 120 psia	+70° to +130° -250° to +130°	Air Nitrogen	Closed Circuit	PSP, TSP, Model Deformation Systems, SC, HA, JET, PT, SS
11-by 11 Foot Unitary Plan Transonic Wind Tunnel		Mach 0.2 to 1.45	NASA Ames	0.3 to 9.6 x 10^6 per ft	11' H x 11'W x 22' L	432-4608 psfs	110 ± 20°F	Air	Closed Return	PSP, PIV, OF, IR, SI
SUPERSONIC SPEED REGIME										
4-Foot Supersonic Unitary Plan Wind Tunnel (UPWT)	Test Section 1: Test Section 2:	Mach 1.5 to 2.9 Mach 2.3 to 4.6	NASA LaRC	0.5 to 11.4 x 10^6 per ft 0.5 to 8.4 x 10^6 per ft	4' H x 4' W x 7' L	0 to 10 atmospheres	100° to 300°	Dry Air	Closed Circuit	AT, FO, SC, HA, JET, PT, SS
9-by 7 Foot Unitary Plan Wind Tunnel		Mach 1.55 to 2.55	NASA Ames	0.9 to 5.6 x 10^6 per ft	9' H x 7' W x 18' L	634-3888 psfs	110 ± 20°F	Air	Closed Return	IR, PSP, OF, ADST
10-by 10 Foot Foot Wind Tunnel		Mach 0 to 0.36, 2.0 to 3.5	NASA Glenn	0.12 to 3.4 x 10^6 per ft	10' H x 10' W x 40' L	20 to 720 psf	520 to 1140°R	Air	Open or Closed Circuit	PSP, SS, PIV, PT
8-by 6 Foot Supersonic Wind Tunnel		0 to 0.1, 0.25 to 2.0	NASA Glenn	1.5 to 5.5 x 10^6 per ft	8' W x 6' H x 23.5' L	100 to 1340 psf	520 to 720°R	Air	Open or Closed Circuit, Atmospheric	PSP, SS, P
Propulsion Systems Lab		0 to 3.5, 0 to 6.0 w/ topping heater	NASA Glenn	n/a	12' x 12' x 39'	150 psig	850°F	Air	Non Vitiated	Altitude icing simulations
HYPERSONIC SPEED REGIME										
Langley Aerothermal Dynamics Laboratory (LAL)										
20-Inch Mach 6 Air Tunnel		Mach 6	NASA LaRC	0.5 to 8.0 x 10^6 per ft	20" H x 20.5" W	30 to 475 psia	760° to 940°R	Dry Air	Blow Down	AT, JET, TSP, PSP, High AOA, IR, High Speed Schlieren, BOS, PLIF, Oil Flow
15-Inch Mach 6 High Temperature Tunnel		Mach 6	NASA LaRC	0.5 to 6.0 x 10^6 per ft	14.6" diameter open jet	50 to 450 psia	870°-1260° R	Dry Air	Blow Down	AT, JET, TSP, PSP, High AOA, IR, High Speed Schlieren, BOS, PLIF, Oil Flow
31-Inch Mach 10 Air Tunnel		Mach 10	NASA LaRC	0.5 to 2.2 x 10^6 per ft	31" H x 31" W	150 to 1450 psia	1850°R	Dry Air	Blow Down	AT, JET, TSP, PSP, High AOA, IR, High Speed Schlieren, BOS, PLIF, Oil Flow
8-Foot High Temperature Tunnel (8-ft HTT)		Mach 3, 5 Mach 4, 5, and 7	NASA LaRC	0.44 to 5.09 x 10^6 per ft	54.5" diameter Mach 3, 5 96" diameter Mach 4, 5, & 7	50 to 4000 psia	850° to 4000°	Air	Blow Down	AT, P

The Right Facility at the Right Time

Matching Mach number and Reynolds number

- NASA wind tunnel testing guide.
- https://www.nasa.gov/sites/default/files/atoms/files/nasa-2018-gftd_trifold-3-20-2019-508.pdf



Sample Test Capabilities*

IR Infrared Flow Visual
AC Acoustic Testing
AE Aeroelastic Testing
AFL Airfoil Testing to Ground Wind Loads
AT Aerothermal Testing
PSP Pressure Sensitive Paint
FO Forced Oscillation Testing
PMT Pressure Model Test
SC Stability and Control
HA High Angle-of-Attack Testing
PAAI Propulsion Airframe Aeroacoustic Integration
P Propulsion System Testing
JET Jet Effects Testing
PT Performance Testing
RT Rotorcraft Testing
SG Shadowgraph Flow Visual
SS Semi-Span Testing

Specialized Test Techniques*

The following optical measurement techniques are in routine use at AETC facilities.

IR Thermography

Real-time measurement of surface temperature suitable for measuring heat flux, flow separation, and boundary layer transition location. Adaptive contrast enhancement algorithms are used to aid visualization of subtle temperature features.

Pressure/Temperature-Sensitive Paint

Provides continuous surface pressure and or temperature data using a paint containing fluorescent dyes which are sensitive to oxygen partial pressure as well as temperature. Results can be integrated to determine airloads on wind tunnel models/components. Suitable for temperature mapping as well as boundary layer transition detection. Works in cryogenic conditions where IR thermography is not suitable. Both steady state and dynamic (time resolved) data at up to 10 kHz can be obtained.

High-Speed Schlieren and Shadowgraph

Visualizes flow density variations including shock waves and vortex cores. Images can be recorded at up to 100 kHz. Data are processed to obtain frequency spectra of density fluctuations at arbitrary locations in the flow, or to obtain flow velocity by tracking the movement of turbules.

Background-Oriented Schlieren

Visualizes flow density variations by measuring fluctuations in the position of a speckled background. Able to access areas of the tunnel and viewing angles which traditional schlieren techniques cannot.

Particle Image Velocimetry

A method for measuring velocity in a particle-seeded flow. A double pulsed laser sheet illuminates a two-dimensional particle field.

Non-Optical measurement techniques available

Advanced Force Balance

AETC facilities maintain a comprehensive inventory of balances for force and moment measurements including traditional six-component balances over a wide load range, floor balances for semispan models, rotating balances for turbomachinery, and specialized balances for many unusual applications. In addition to balance services provided during AETC tests, balance loans may also be available.

Dynamic Data Systems

AETC facilities maintain high speed data systems suitable for excitation and readout of many types of unsteady pressure and force sensors and can accommodate tests requiring a large number of channels of unsteady data.

Other techniques, such as Oil-Film Interferometry, Planar Laser Induced Fluorescence, and Femtosecond laser Electronic Excitation and Tagging are also available upon request.

For a full list of test capabilities and specialized test techniques please visit us at:
www.nasa.gov/aeroresearch/programs/AAVP/AETC
or Contact Test Technology Manager, James Bell - james.h.bell@nasa.gov

Matching Mach number and Reynolds number

- A few links to wind tunnel facilities around the world:
  - https://www.nasa.gov/centers/langley/news/factsheets/WindTunnel.html
  - https://www.nasa.gov/centers/ames/orgs/aeronautics/windtunnels/index.html
  - https://www.etw.de/wind-tunnel/overview
  - https://www.onera.fr/en/windtunnel