Turbulence and CFD models: Theory and applications

Roadmap

1. **Post-processing and analysis of turbulent simulations**

General remarks

- Post-processing turbulent simulations can be a very daunting task. Specially if we are dealing with scale resolving simulations (SRS).
- Turbulence simulations can be analyzed using quantitative and qualitative data, we will address the most common methods.
- From a qualitative point of view, most of the times we want to visualize the field variables at boundary surfaces (*e.g.*, at the walls), at cut-planes or at iso-surfaces.
- Most of the times we are interested in visualizing the vortical structures.
- Sometimes it is interesting to know the sense of rotation of the vortices, this can be done using the vorticity criterion or plotting velocity vectors.
- From a quantitative point of view, we are interested in plotting the time history of the forces and some other quantities of interest (such as mass flow, heat transfer coefficient, maximum and minimum values of transported quantities and so on).
- It is also of interest plotting the energy spectrum. This kind of plot is useful to determine if we are resolving well the spatial and temporal scales.
- More advanced post-processing includes detection of separation and reattachment points, shock wave detection, vortex core identifcation, computing derived fields (*e.g.*, Pope criterion, integral length scales) and so on.

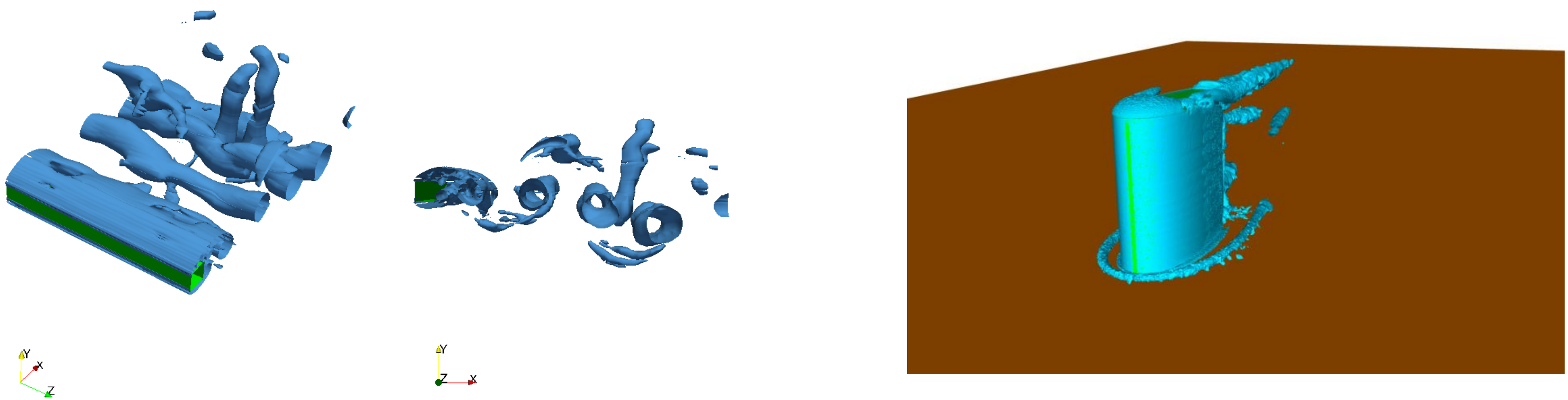
Post-processing and analysis of turbulent simulations

Q-criterion

- The Q-criterion is used to capture vortices. It is defined as,

$$Q = \frac{1}{2}\left[(\mathrm{tr}(\nabla \mathbf{u}))^2 - \mathrm{tr}\left(\nabla \mathbf{u} \cdot \nabla \mathbf{u}\right)\right]$$

- To visualize the vortical structures we plot the iso-surfaces of Q-criterion. The values to plot are positives and several order of magnitudes lower than the maximum value.

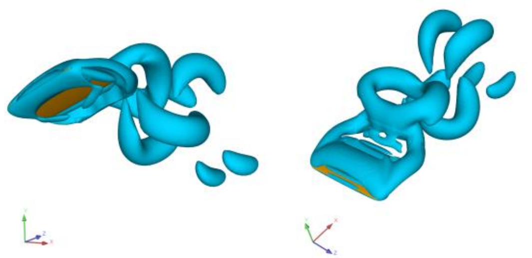


Iso-surfaces of Q-criterion

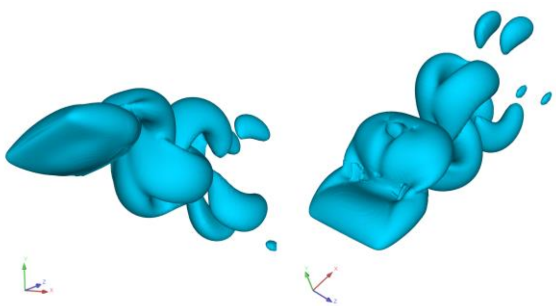
Post-processing and analysis of turbulent simulations

Vorticity criterion

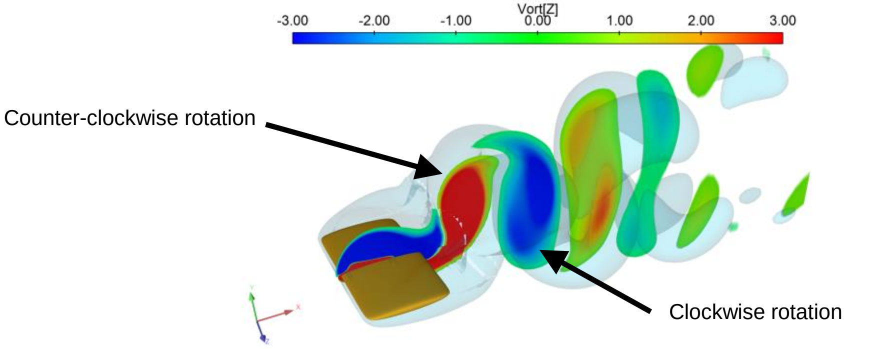
- The vorticity criterion is defined as $\omega = \nabla \times \mathbf{u}$

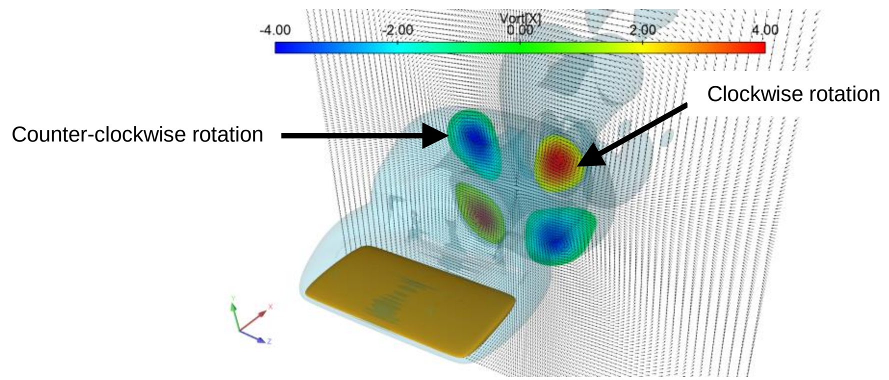


Q-criterion



Vorticity magnitude





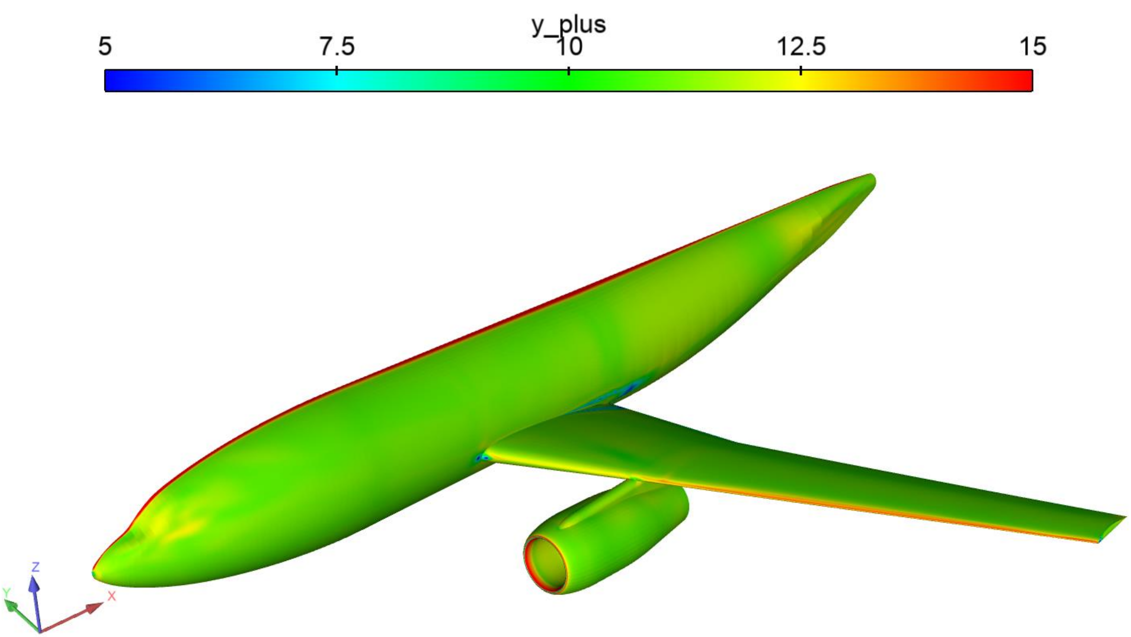
- Comparison of Q-criterion (top figure) and vorticity criterion (bottom image).
- Notice that both methods show the vortical structures, but the vorticity criterion has the disadvantage of also showing the shear layers near the body and between the vortices.

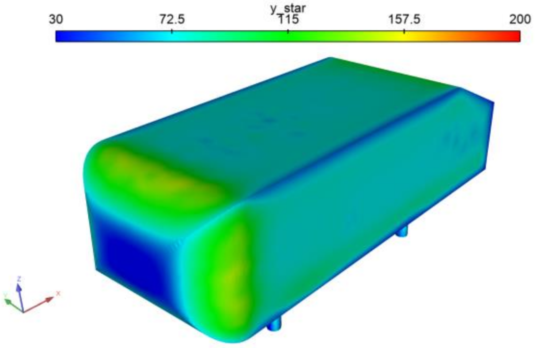
- The vorticity criterion is capable of showing the rotation of the vortices.
- In the top figure contours of ω_z are plot. The red vortices are rotating in counter-clockwise sense.
- In the bottom figure contours of ω_x are plot. The red vortices are rotating in clockwise sense.

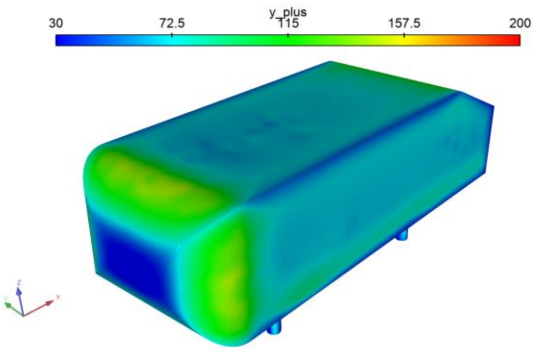
Post-processing and analysis of turbulent simulations

y^+ value and y^*

- The y^+ and y^* values can be plotted at the walls.
- Recall that it is almost impossible to have a uniform y^+ or y^* value at the walls.
- When evaluating these quantities, we should check that the average value is roughly speaking close to our target value.
- Be sure not to have high peaks in large areas (y^+ values of more than 1000).

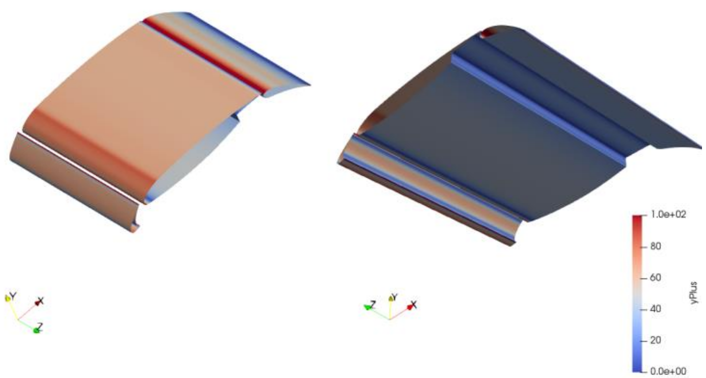






Minimum, maximum, and averaged values

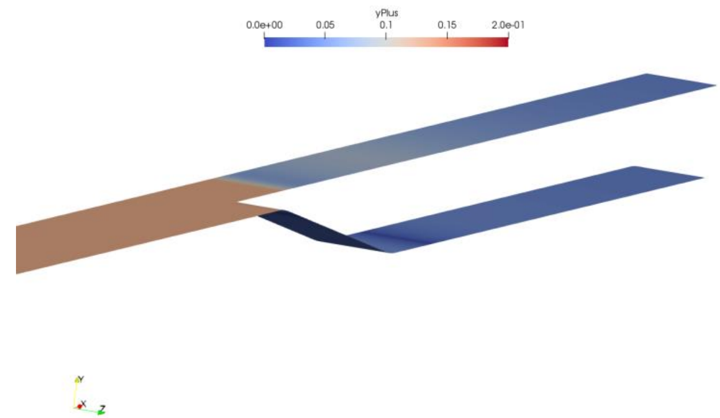
- Besides the mean values, it is also recommended to compute (and monitor) the minimum and maximum values of the field variables.
- For example, if at any point of the simulation a quantity is oscillating too much or the minimum or maximum value is unrealistic, you might stop the simulation and revise the case setup.
- In the figure below, we show this scenario for y^+. But you can do it with any quantity, *e.g.*, pressure, velocity, temperature, turbulent kinetic energy, and so on.
- Remember, there are some quantities that are strictly bounded, so it is a good idea to monitor those quantities.
- For example, in the images below we monitored the minimum, maximum, and average values of y^+.



airfoil y+ : min = 3.3170682, max = 122.32767, average = 42.357341

flap y+ : min = 9.6251989, max = 447.31831, average = 47.411466

slat y+ : min = 14.072073, max = 305.59193, average = 93.392662



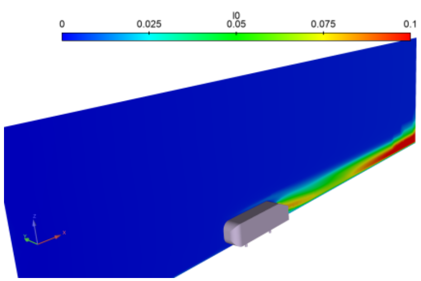
walls y+ : min = 0.00135130, max = 0.290177, average = 0.0664195

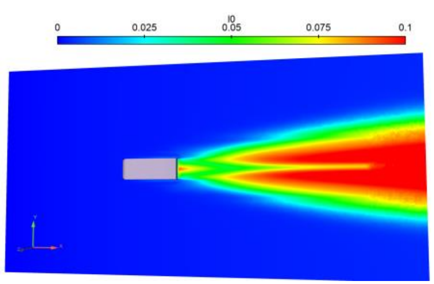
Integral length scales l_0

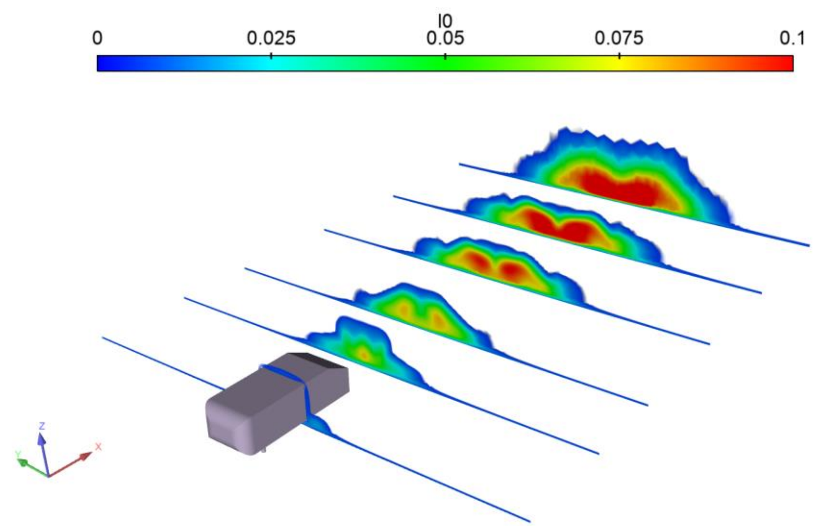
- The integral length scales l_0 can be computed from the turbulent variables, as follows,

$$l_0 = \frac{k^{1.5}}{\epsilon} \quad \text{or} \quad l_0 = \frac{k^{0.5}}{C_\mu \omega} \quad \text{where} \quad C_\mu = 0.09$$

- You will need to use a two-equation model ($k-\epsilon$ family or $k-\omega$ family).
- Alternatively, you can compute the integral length scales using two-point correlations.



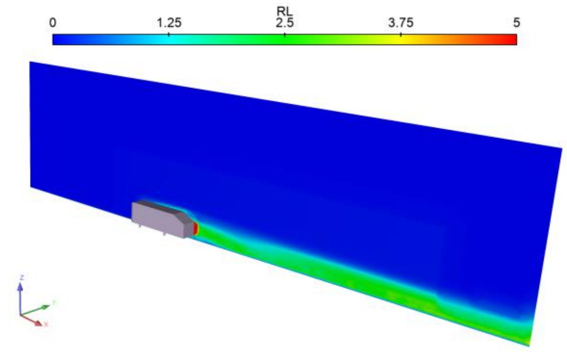


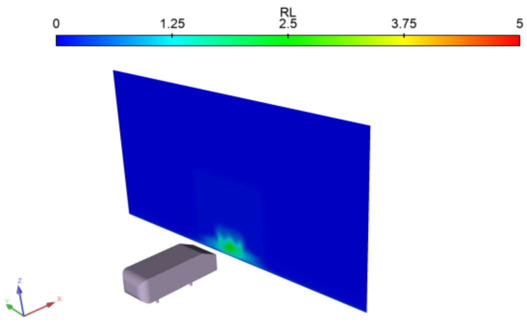


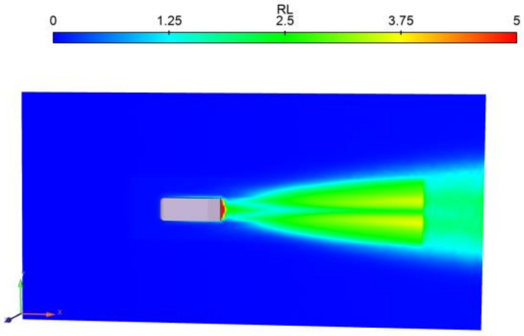
Post-processing and analysis of turbulent simulations

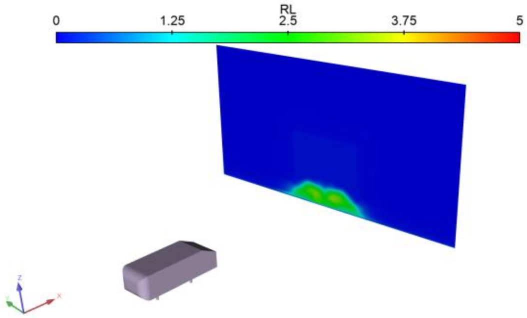
Ratio of integral length scale to grid length scale R_L

- The ratio of integral length scale to grid length scale R_L, can used to determine if the mesh density is enough to resolve the integral scales.
- In regions where R_L is less than a given criterion ($R_L < 5$), the mesh requires refinement.









Ratio of integral length scale to grid length scale R_L

- The ratio of integral length scale to grid length scale R_L, can be computed as follows,

$$R_l = \frac{l_0}{\Delta}$$

where Δ can be approximated as follows

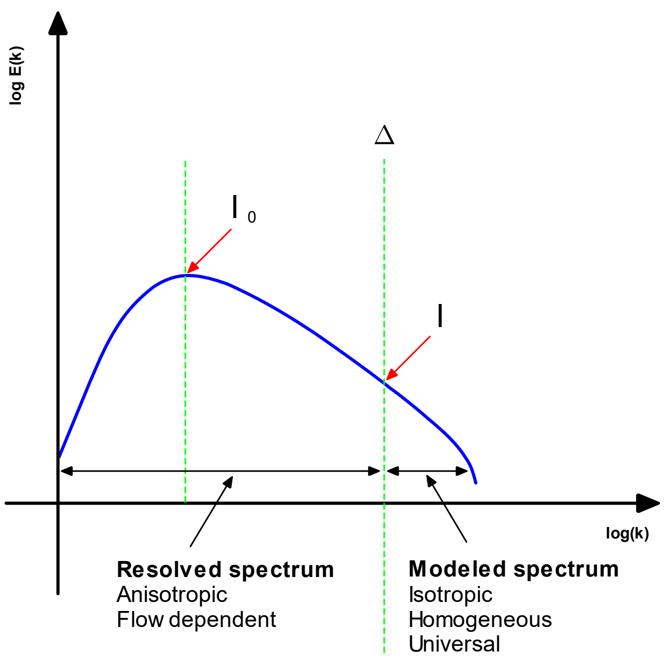
$$\Delta \approx \sqrt[3]{\text{cell volume}}$$

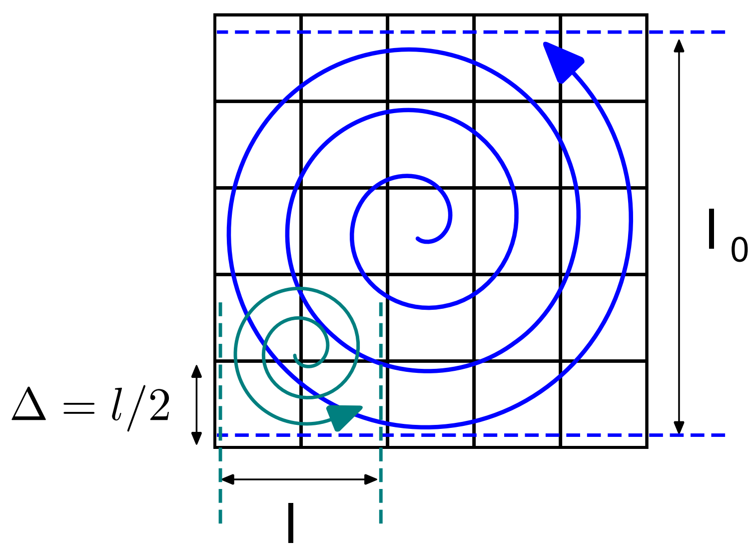
This approximation is accurate if the aspect ratios are modest (less than 1.2)

- The recommended value is $R_L > 5\text{-}10$.
- Where 5 should be considered the lowest limit for DES/LES.
- For accurate LES simulations, the integral length scales must be sufficiently resolved. Therefore, the recommended value is 10 or more.
- For RANS/URANS and VLES, it is enough to use 3-5 cells across integral length scales.

Ratio of integral length scale to grid length scale R_L

- To resolve the integral length scales l_0, at least 5 cells must be used across the eddies (explanation in next slide).
- To resolve an eddy with a length scale l (where l is the smallest scales that can be resolved with the grid or ⍰), at least a couple of cells need to be used in each direction.
- Remember, eddies cannot be resolved down to the molecular dissipation limit (it is too expensive).

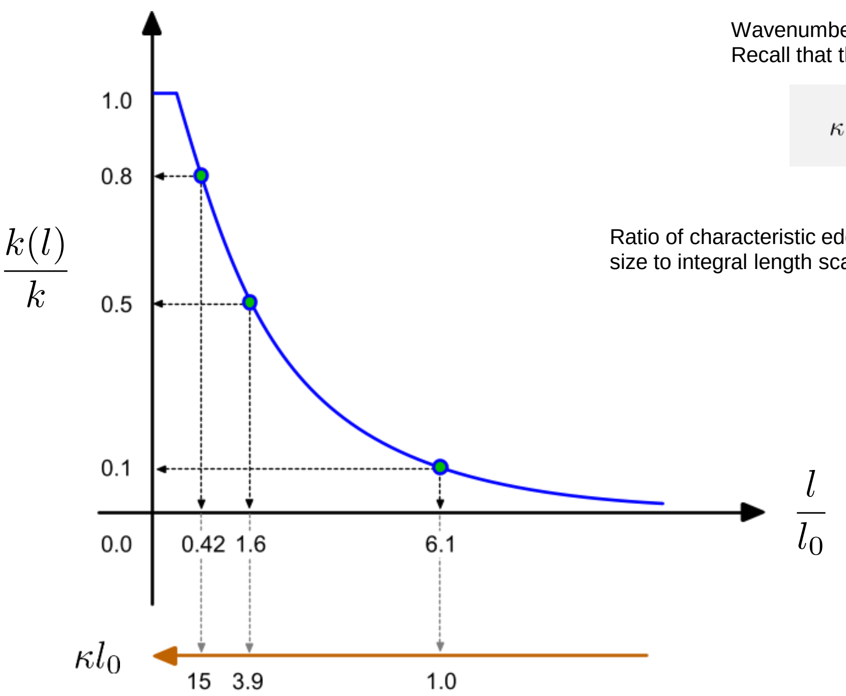




Post-processing and analysis of turbulent simulations

Ratio of integral length scale to grid length scale R_L

- From the energy spectrum plot, we can infer that the mesh resolution determines the fraction of the turbulent kinetic energy directly resolved.
- So, let us suppose that we want to resolve 80% of the turbulent kinetic energy *k(l)* in an SRS simulation.
- Then, the grid must resolve the eddies whose sizes are larger than roughly half the size of the characteristics eddy size (⍰ = *l* /2) up to the integral scales.
- Then, approximately 5 cells will be needed across the integral length scale l_0.



Cumulative turbulent kinetic energy against lengths scale of eddies. The figure has been adapted from Turbulent Flows by S. Pope

Wavenumber times integral length scales. Recall that the wavenumber is defined as,

$$\kappa = \frac{2\pi}{l} = \frac{2\pi f}{U}$$

Ratio of integral length scale to grid length scale. The grid cut-off dimension is defined as,

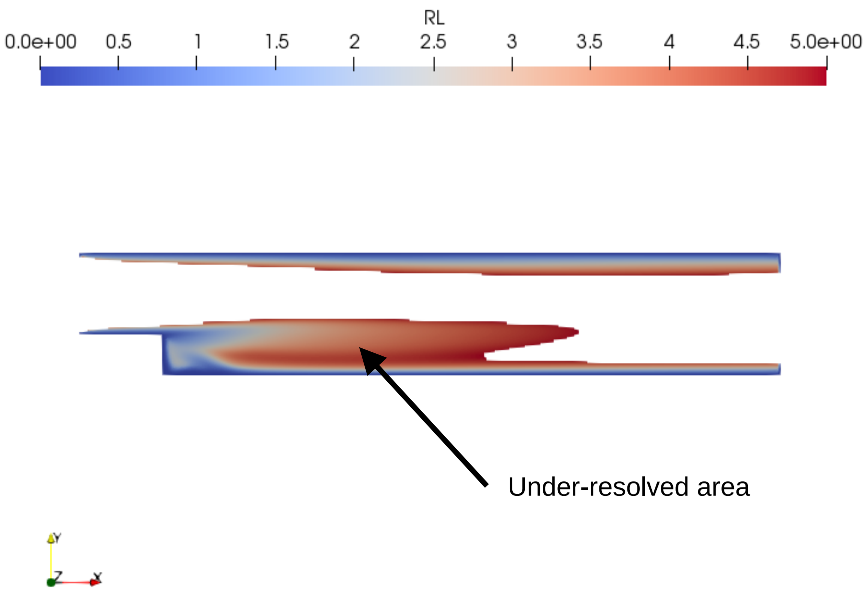
$$\Delta = l/2$$

Ratio of characteristic eddy size to integral length scale

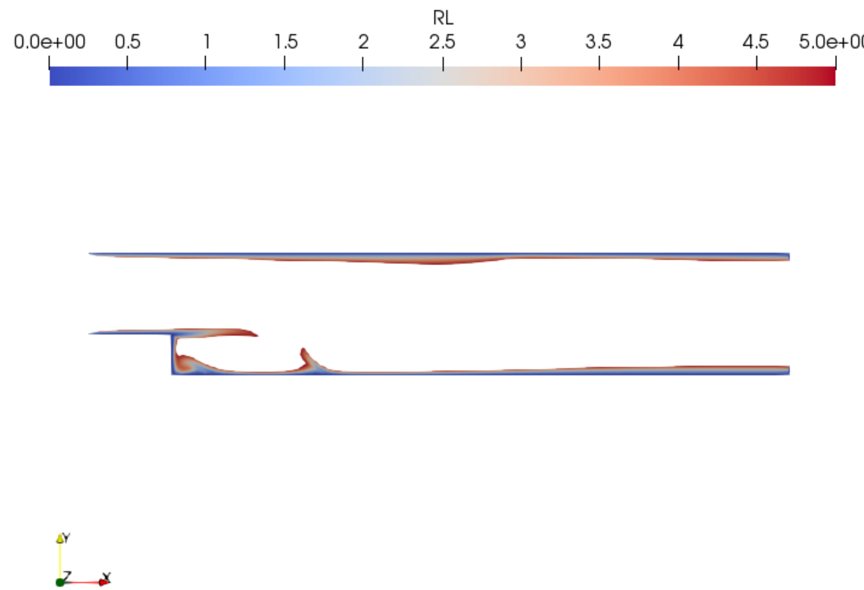
	l/l_0	κl_0	l_0/Δ
$k(l) = 0.1k \ (10\%)$	≈ 6.1	≈ 1.0	≈ 0.33
$k(l) = 0.5k \ (50\%)$	≈ 1.6	≈ 3.9	≈ 1.25
$k(l) = 0.8k \ (80\%)$	≈ 0.42	≈ 15	≈ 4.8
$k(l) = 0.9k \ (90\%)$	≈ 0.16	≈ 38	≈ 12.5

Characteristics turbulent kinetic energy and length scales of the energy spectrum. For a rigorous explanation of these results, please refer to Turbulent Flows by S. Pope

Identification of integral length scale and grid length scale



Coarse mesh



Fine mesh

- To identify integral length scales and grid length scales you can plot contours of these quantities at different locations/planes in the domain.
- The lowest limit of R_L can be clipped so that the well resolved areas do not appear.
- In this case we are clipping (showing) $0 < R_L < 5$.
- Under-resolved areas (the areas shown), will need finer meshes or local mesh adaption.
- Near-wall regions always pose challenges. In these areas is better to quantify the y^+ value.

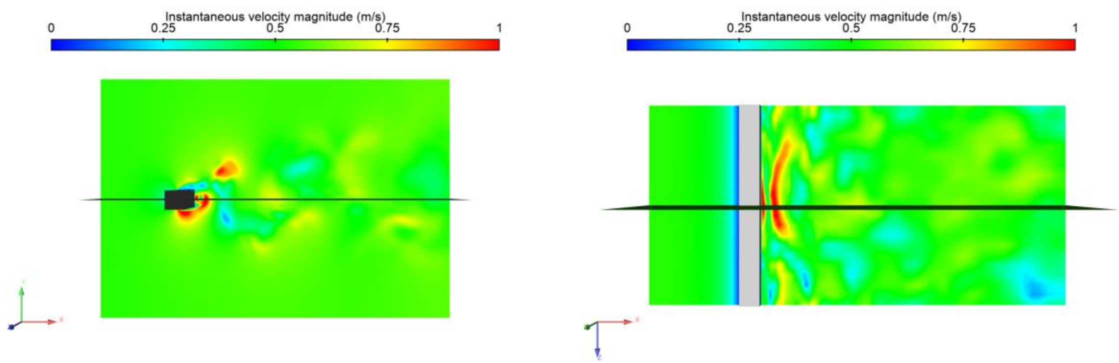
Unsteady statistics – Mean values

- Add animation showing that mean do not change anymore

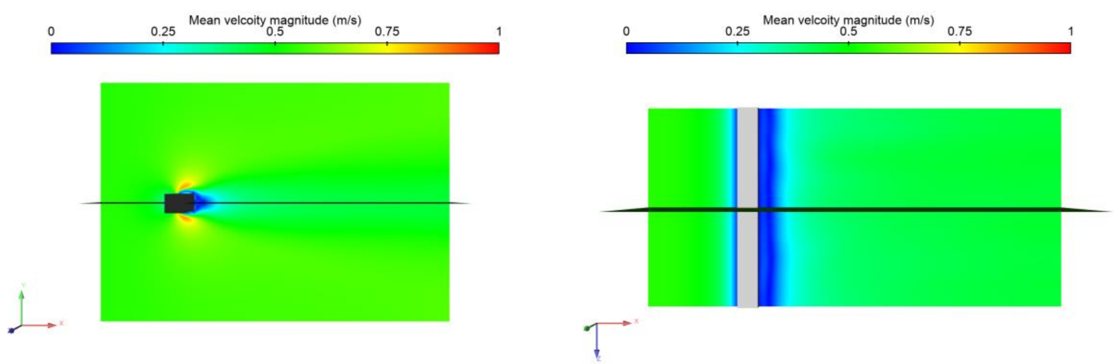
- Check previous lecture that slides seems to be differnte.

Unsteady statistics – Mean values

- When running SRS simulations, remember to enable the unsteady statistics.



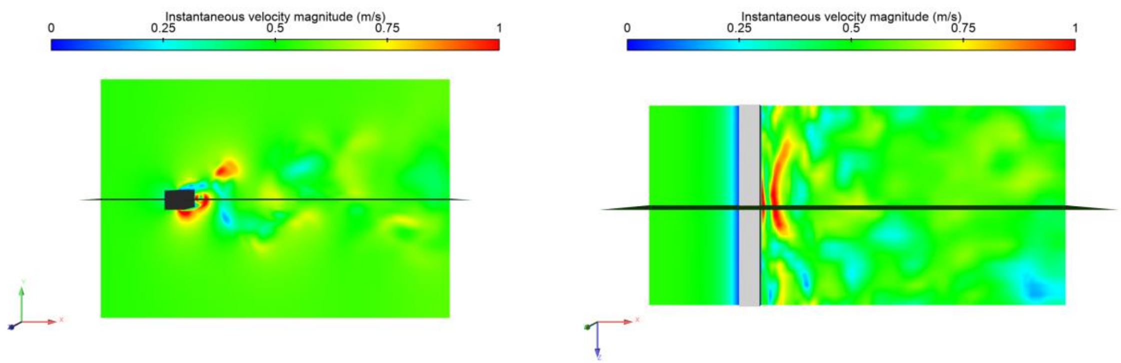
Instantaneous quantity



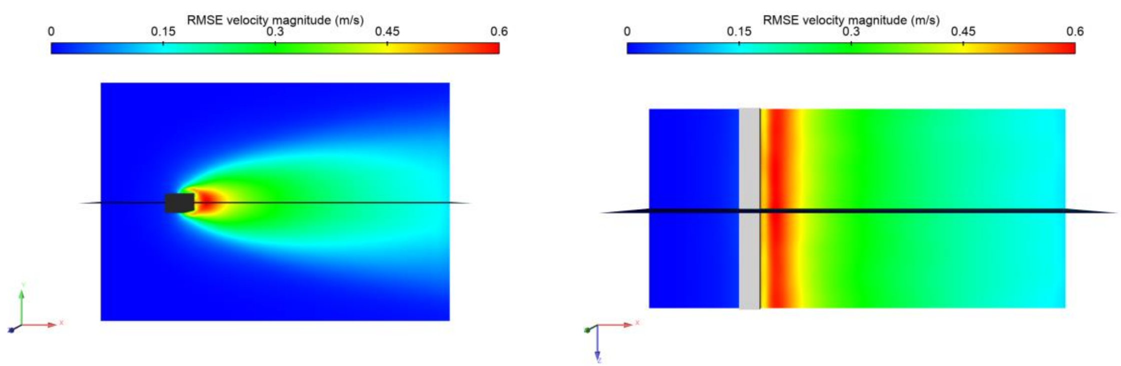
Mean quantity

Unsteady statistics – Fluctuating values (RMS)

- When running SRS simulations, remember to enable the unsteady statistics.



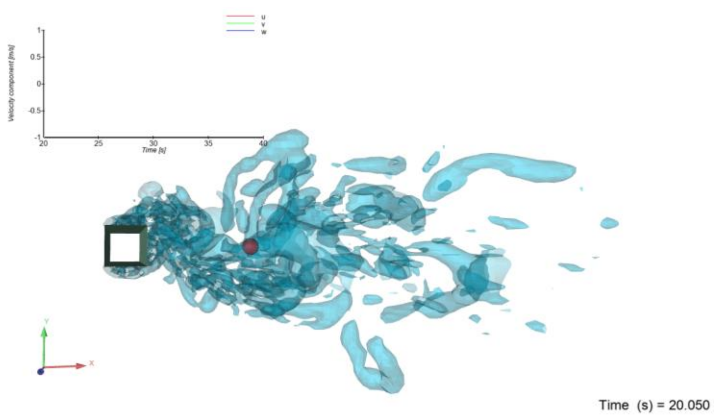
Instantaneous quantity



Fluctuating quantity (RMS)

Flow statistics

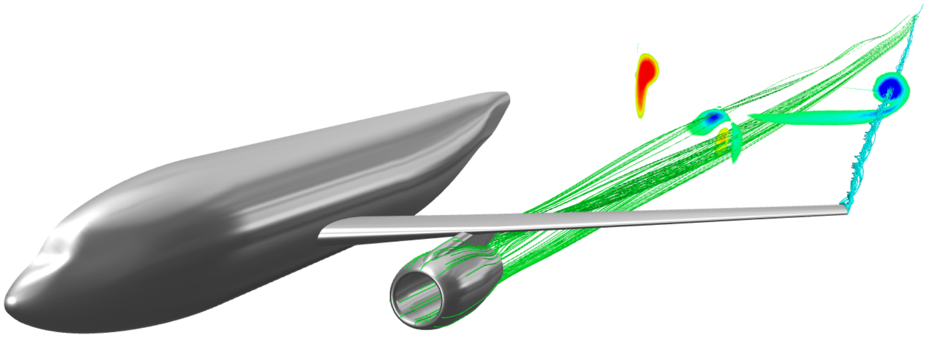
- A final comment on the flow statistics.
- The flow statistics can be computed for unsteady and steady flows.
  - If you compute unsteady statistics, they depend on time.
  - If you compute steady statistics, they depend on the iteration number.
- The flow statistics can be saved as a field that can be visualized at a latter time (as the plots shown in the two previous slides).
- They can also be saved in a text file and post-processed at a latter time.
  - We usually save the statistics in a text file when we are interested in doing local sampling (probes and sampling).
- Remember, you can compute the flow statistics for any primitive variable or derived quantity.

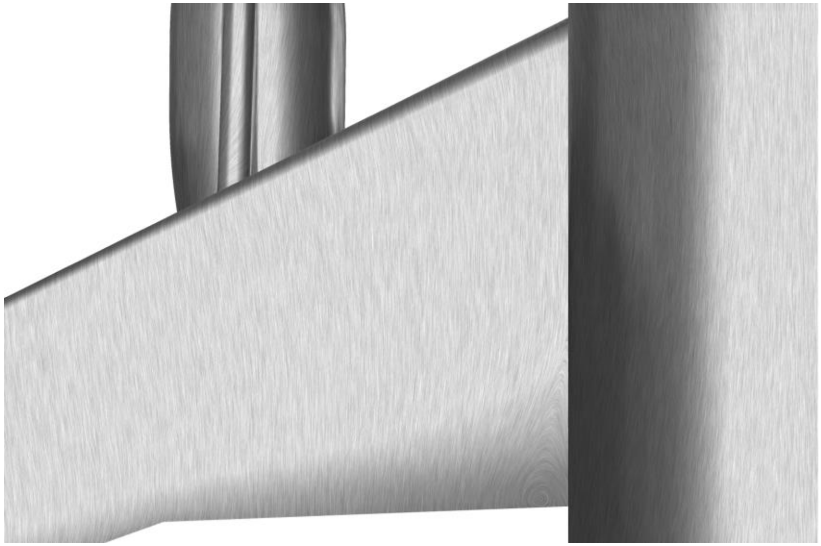


http://www.wolfdynamics.com/training/turbulence/image19.gif

Streamlines and oil lines

- We can also track vortices by using streamlines.
- Streamlines can be released from any location or surface.
- Another way to visualize the flow on the walls is by using oil lines, which are basically streamlines that remain attached to the walls.





Pope criterion

- A good LES simulation aims at resolving 80% of the turbulent energy spectrum.
- A way to measure the quality of a LES simulation is by using the Pope criterion [1, 2], which is a simple measure of the fraction of the turbulent kinetic energy in the resolved motions. This criterion is defined as follows,

$$M(\mathbf{x},t) = \frac{k_{SGS}(\mathbf{x},t)}{k_{SGS}(\mathbf{x},t) + k_{RES}(\mathbf{x},t)} \qquad \text{where} \qquad k_{RES} = 0.5\left(u'^2 + v'^2 + w'^2\right)$$

where k_{SGS} is the turbulent kinetic energy of the SGS eddies and k_{RES} is the kinetic energy of the resolved eddies (determined by the grid size).

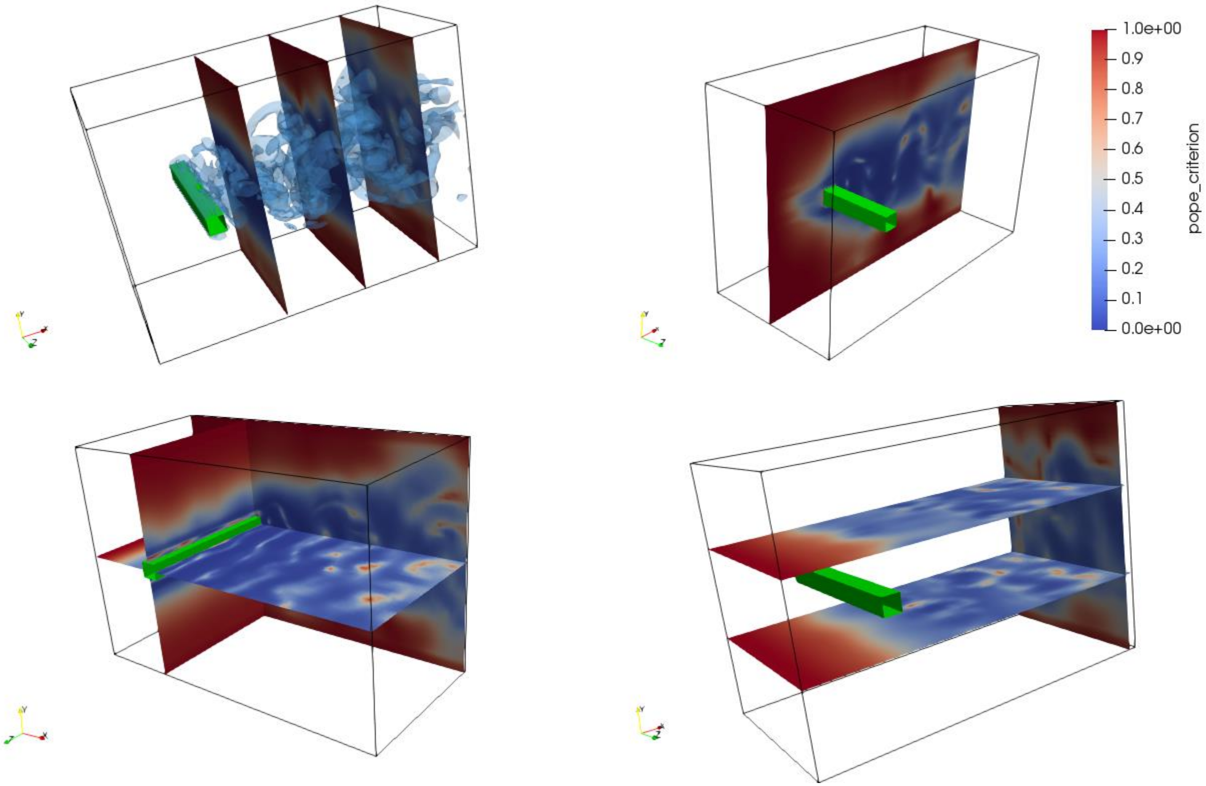
- To compute M, a methodology to find k_{SGS} is required. Therefore, the use of one equation kinetic energy transport models is recommended.
- If you are using the Smagorinsky model, you can approximate k_{SGS} as follows,

$$k_{SGS} = \frac{(C_s \Delta |\tilde{\mathbf{S}}|)^2}{0.3} = \frac{1}{0.3}\left(\frac{\nu_t}{C_s \Delta}\right)^2$$

[1] S. Pope. Turbulent Flows. Cambridge University Press. 2014.
[2] S. Pope. Ten questions concerning the large-eddy simulation of turbulent flows. New J. of Physics. 2004.

Pope criterion

- The values of M are between 0 and 1. A value of M = 0 corresponds to a DNS simulation, and a value of M = 1 corresponds to a RANS simulation.
- The Pope criterion can be visualized by plotting it in several cut planes of the domain.



Pope criterion

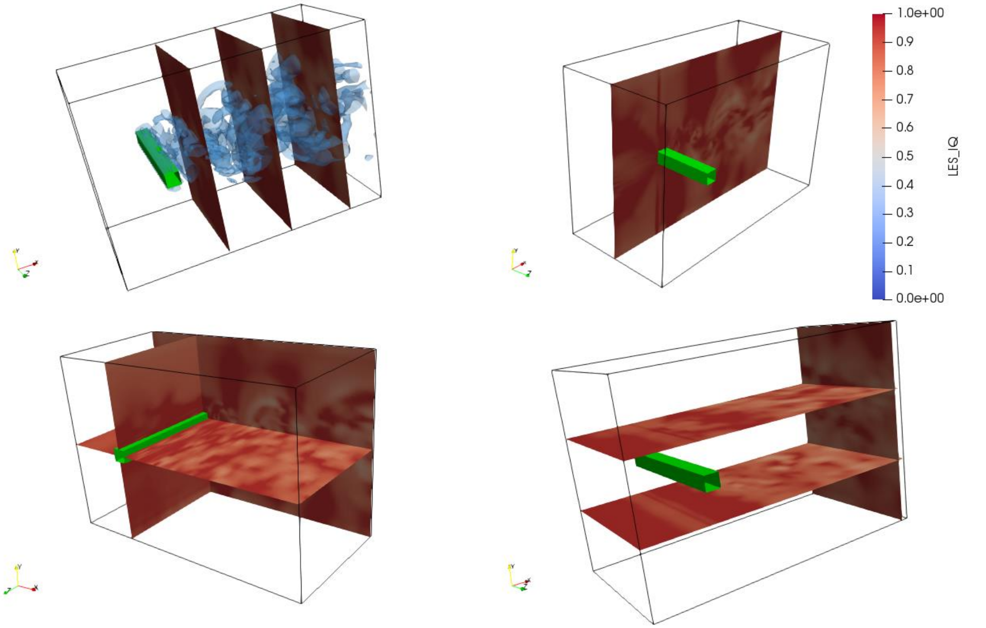
Post-processing and analysis of turbulent simulations

Celik LES index quality

- Another method to determine the quality of a LES simulation, is the LES index quality proposed by Celik [1]. This metric is defined as follows:

$$LESIQ = \frac{1}{1 + 0.05\left[\frac{(\nu + \nu_{SGS})}{\nu}\right]^{0.53}}$$

- Where the value of LESIQ is between 0 and 1.
- The higher the value of LESIQ, the better the resolution.
- Recommended values are 0.8 and above.
- This metric can be visualized by plotting it in several cut planes of the domain.

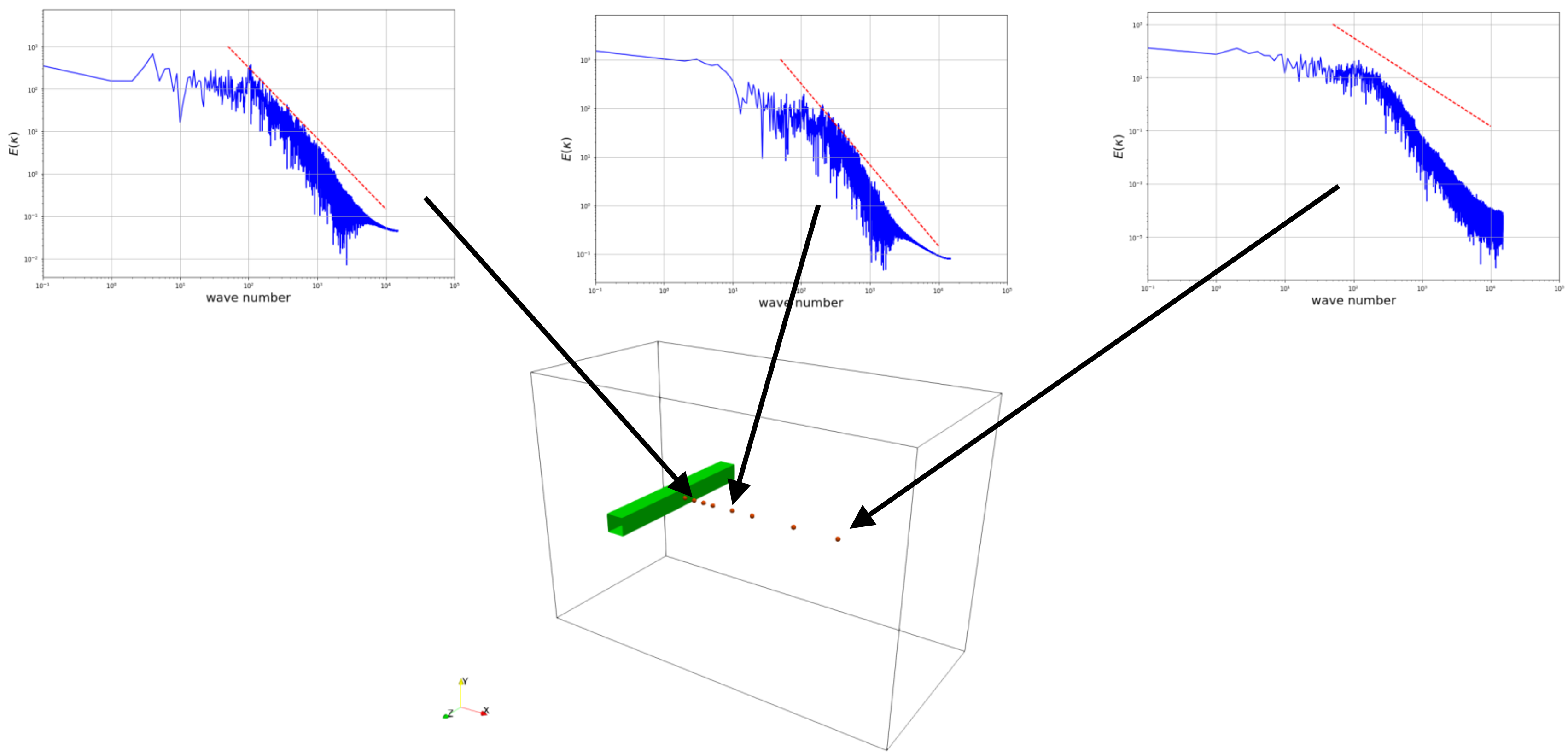


[1] Celik, Cehreli, Yavuz. Index of Resolution Quality for Large Eddy Simulations. J. of Fluids Engineering. 2005.

Post-processing and analysis of turbulent simulations

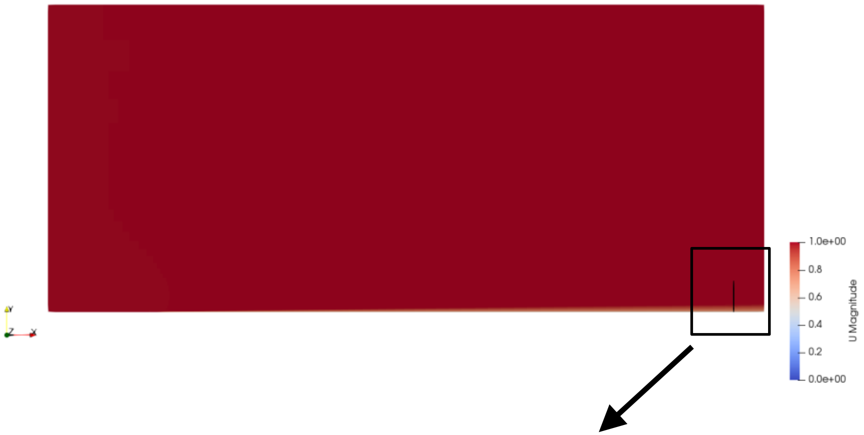
Turbulent power spectrum

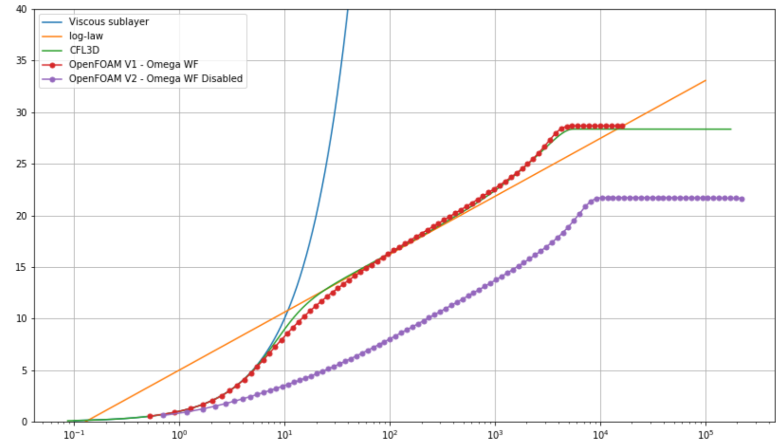
- The turbulent power spectrum represents the distribution of the kinetic energy across the various length scales.
- It is a direct indication of how energy is dissipated with eddies size.
- These plots are local and are obtained by sampling in time the kinetic energy in various locations of the domain (a lot of data needs to be gathered) and signal processing methods.

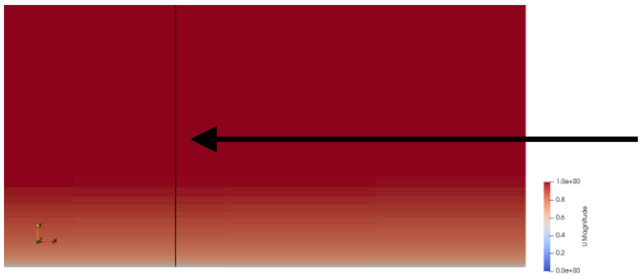


Plot of the law-of-the wall

- We can plot the law-of-the wall in any arbitrary sampling line.
- We know that the use of the non-dimensional velocity u^+ and non-dimensional distance from the wall y^+, results in a predictable boundary layer profile for a wide range of flows.
- By plotting the velocity in terms of the non-dimensional variables u^+ and y^+, we can compare the profiles obtained from the simulations with the theoretical profiles.







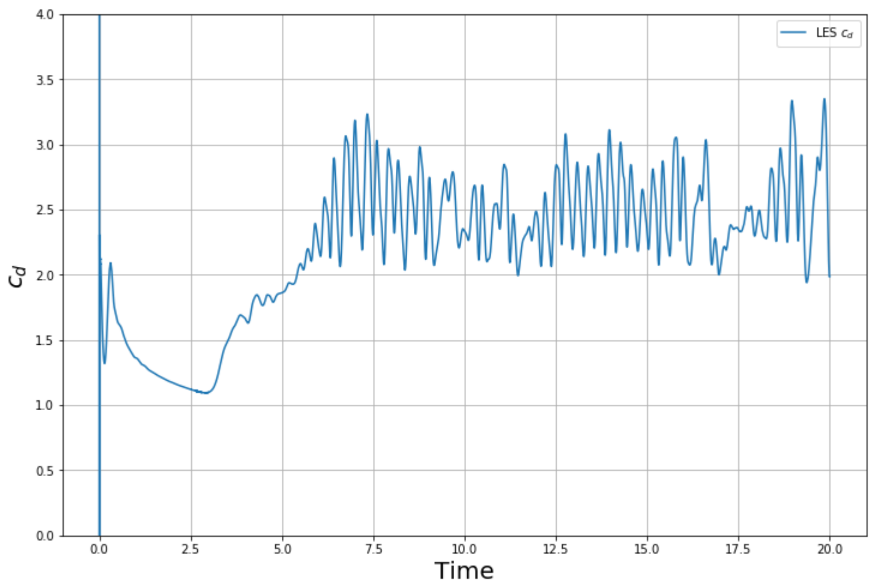
Sampling line

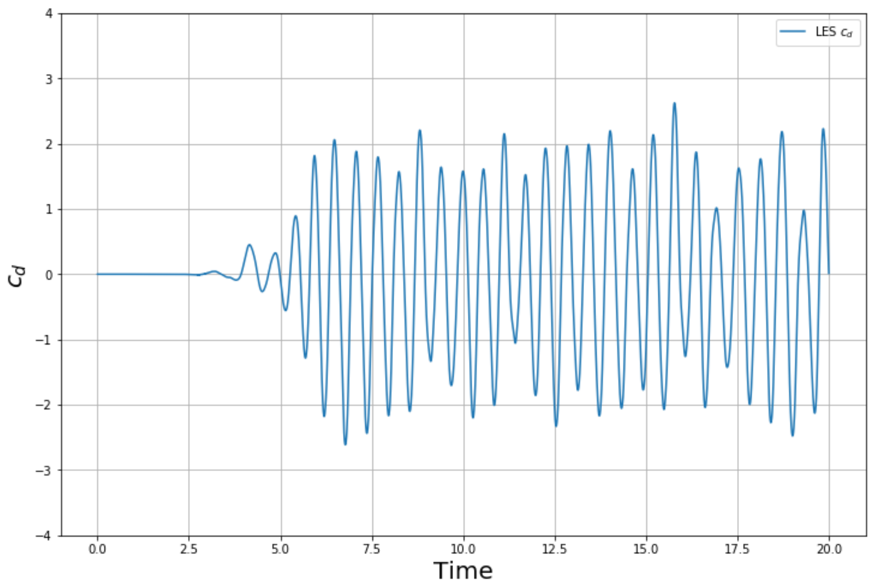
- This is hardcore validation and it is done for very academic cases or when it is requested by the application.
- For industrial cases, most of the times you do not need to do it, and if you do it, be sure to do the sampling in a region where the flow is attached and far from recirculation areas or other walls.

Post-processing and analysis of turbulent simulations

Sampling of integral quantities

- It is important to compute integral quantities when running turbulent simulations.
- You can sample the integral quantities in time and compute the descriptive statistics of the signal.
- Many integral quantities can be sampled, such as, mass flow, heat transfer rate, shear stresses, friction coefficient and so on.





Mean value	2.495
Standard deviation	0.286
Variance	0.0822
RMS	2.512

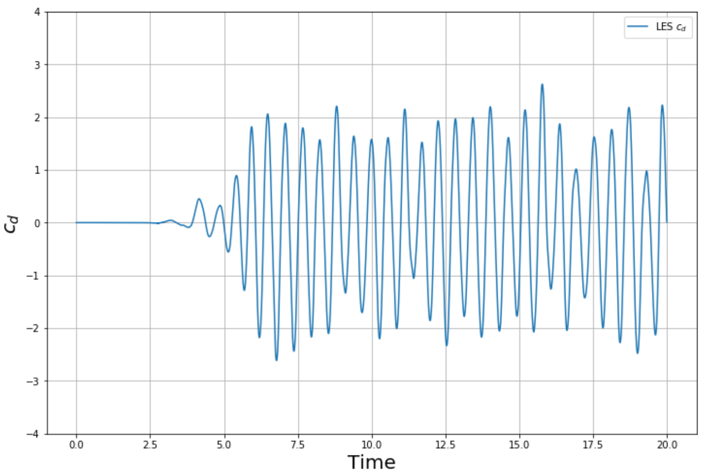
Mean value	-0.010
Standard deviation	1.355
Variance	1.837
RMS	1.355

Post-processing and analysis of turbulent simulations

Sampling of integral quantities – Dominant frequency

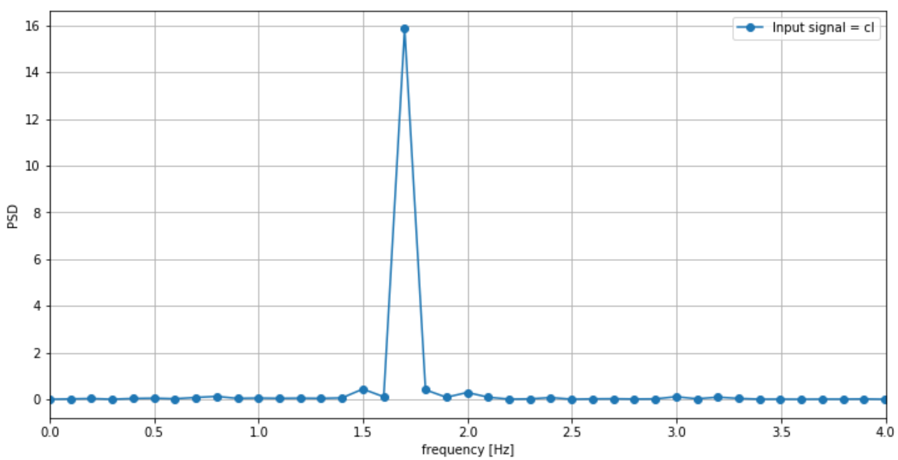
- In many unsteady simulations there is vortex shedding.
- The shedding frequency can be computed from the time signal of a sampled integral quantity (*e.g.*, forces).
- The dominant frequency can be computed using signal processing methods (e.g. periodogram).

Input signal



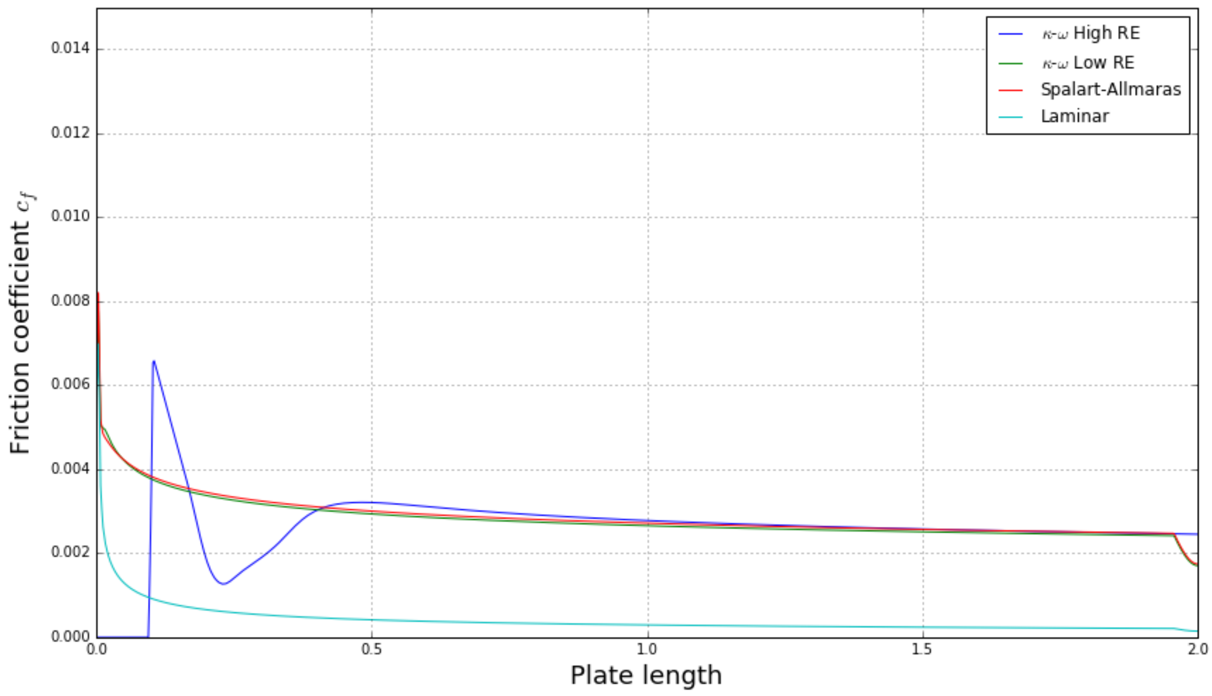
Signal processing

Power spectral density of the input signal



Plot of local quantities at the walls

- We can also compute local quantities at the walls and plot its behavior.
- Quantities that can be computed: friction coefficient, shear stresses, y^+, pressure distribution, pressure coefficient, temperature distribution, and on.



Friction coefficient plot along a surface – Comparison with other numerical results and empirical correlations.