2D Laplace equation in a square domain

$$T = \sin(\pi x)$$

$$T = 0$$

$$\nabla^2 T = 0$$

$$T = 0$$

$$T = 0$$

$$T = 0$$

1.0 m

ш Ч In this case we are solving the following equation (general transport equation),

$$\underbrace{\frac{\partial \rho \phi}{\partial t}}_{\text{Time derivative}} + \underbrace{\nabla \cdot (\rho \mathbf{u} \phi)}_{\text{Convective term}} - \underbrace{\nabla \cdot (\rho \Gamma_{\phi} \nabla \phi)}_{\text{Difussion term}} = \underbrace{S_{\phi}}_{\text{Source term}}$$

 We will set the velocity to zero, the diffusion coefficient to one, and no source terms. Therefore, we will solve the following equation,

$$\frac{\partial \rho \phi}{\partial t} - \nabla \cdot \left(\rho \Gamma_{\phi} \nabla \phi \right) = 0$$

- This is the Poisson-Laplace equation, which has a smooth solution.
- With the given boundary and initial conditions, this case has an analytical solution.
- Run the case using different under-relaxation factors.
- Run the case in steady and unsteady mode.
- Run the case using different meshes.

2D Laplace equation in a square domain

- We are solving for the Laplacian operator, which is a smooth operator.
- This operator appears in many governing equations, e.g., NS momentum equations, transport equations of the turbulent quantities, energy equations.
- This operator is especially sensitive to secondary gradients due to mesh quality issues.



• This problem has the following analytical solution:

$$T(x,y) = \frac{\sin(\pi x) \times \sinh(\pi y)}{\sinh(\pi)}$$