

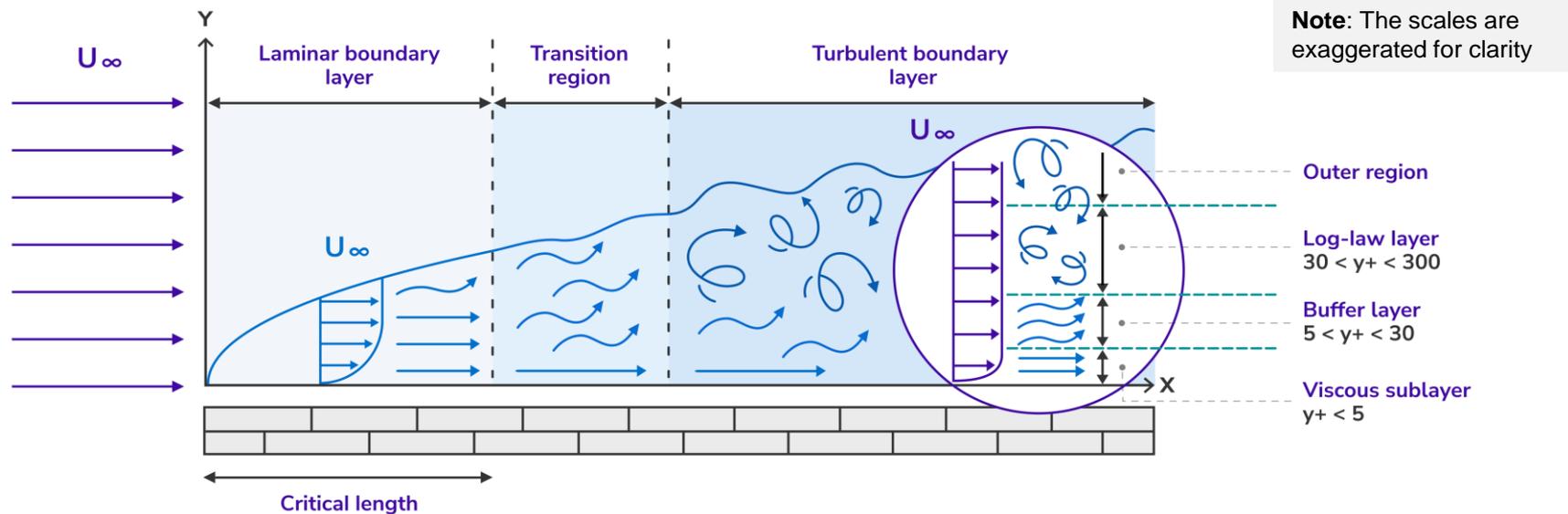
Roadmap to Lecture 6

Part 3

- ~~1. The Reynolds stress model~~
- ~~2. Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress~~
- 3. Transition models – Review of the $\gamma - Re_\theta$ model**

Transition models – Review of the $\gamma - Re_\theta$ model

Boundary layer – Laminar, transitional, and turbulent flow



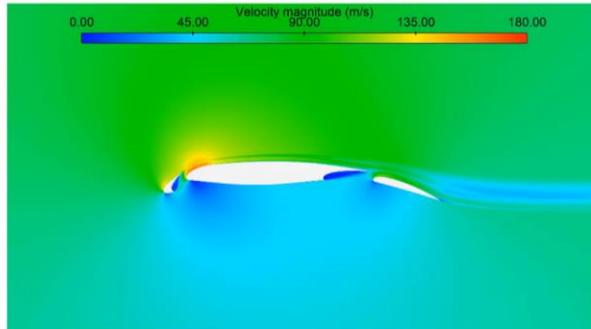
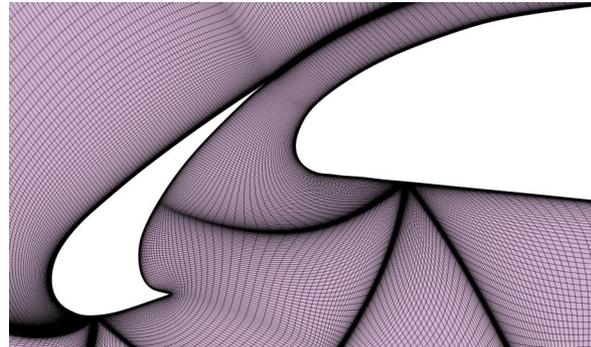
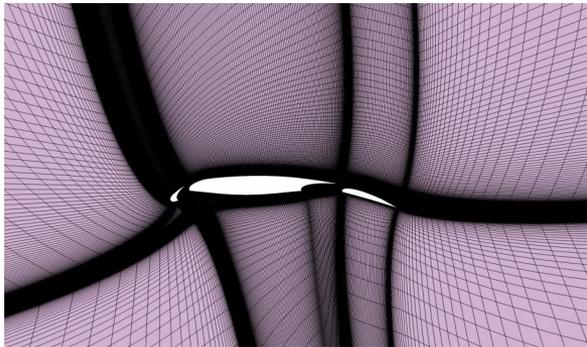
- In this case, a laminar boundary layer starts to form at the leading edge.
- As the flow proceeds further downstream, large shear stresses and velocity gradients develop within the boundary layer.
- At one point, the flow will undergo a transition from laminar to turbulent.
- What is happening in the transition region is not well understood, the flow can become laminar again or can become turbulent.
- Shear stresses, heat transfer rate, mixing rate, and velocities profiles, are very different in each region of the boundary layer (laminar, transitional, or turbulent).

Transition models – Review of the $\gamma - Re_{\theta}$ model

- Maybe, the most challenging topic of turbulence modeling is the prediction of transition to turbulence.
- It is a very complex phenomenon and difficult to predict, even in experiments.
- Trying to predict transition to turbulence in CFD requires very fine meshes and well calibrated models, different from the traditional turbulence models we have described so far.
- Traditional turbulence models do not discriminate between laminar and turbulent regimes.
- To some extension, we can say that they assume that the boundary layer is turbulent in all its extension.
- But assuming that the boundary layer is entirely turbulent might not be a good hypothesis, as in some regions the boundary layer might still be laminar, so we may be overpredicting drag forces, overpredicting heat transfer rate, predicting wrong separation points, or predicting wrong mixing rates.
- The main transition to turbulence mechanism are,
 - Natural transition.
 - Bypass transition.
 - Separation induced transition (laminar separation bubbles).
 - Crossflow transition (due to spanwise effects).

Transition models – Review of the $\gamma - Re_{\theta}$ model

- In some applications, transition to turbulence is preceded by laminar separation bubbles (LSB).
- LSB are laminar recirculation areas that separate from the wall and reattach in a very short distance and are very sensitive to disturbances.
- After the LSB, the flow becomes turbulent.
- LSB are very difficult to predict. Specialized turbulence models are needed.

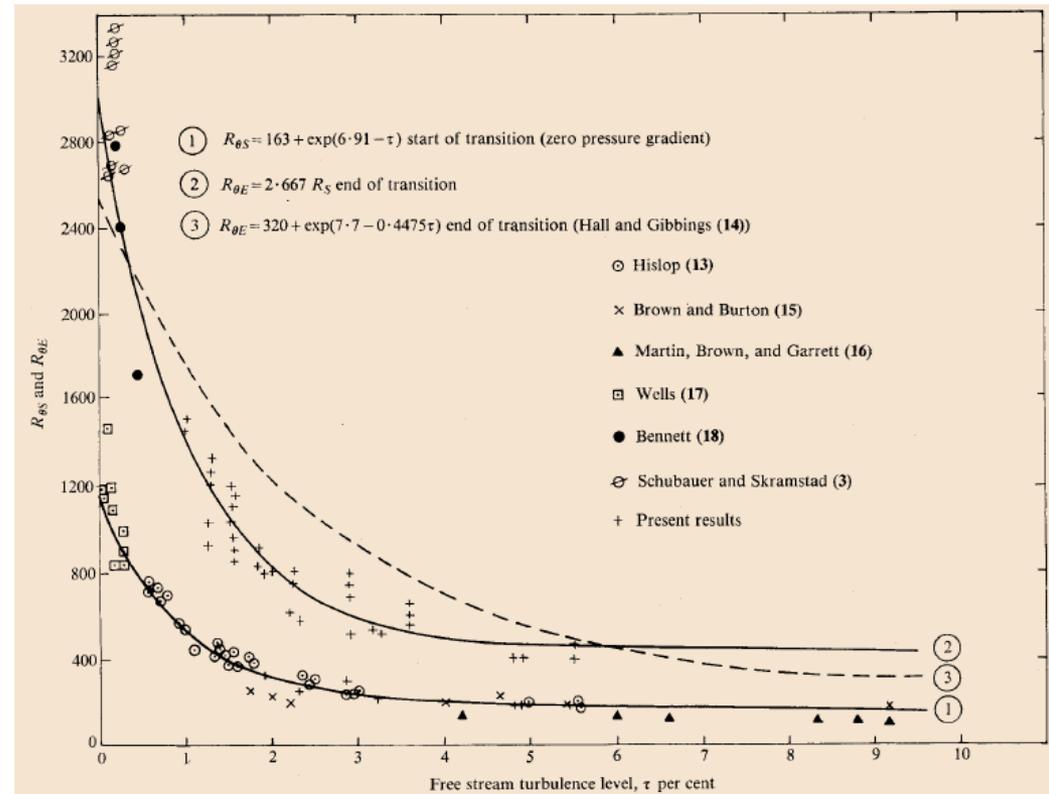


Transition models – Review of the $\gamma - Re_{\theta}$ model

- Transition to turbulence is elusive and difficult to solve.
- It requires very fine meshes, and additional computational resources as it solves additional transport equations.
- Transition models are based on correlations to model the mechanism of transition to transition.
- These correlations are then connected to transport equations.
- Transition onset can be affected by:
 - Free-stream turbulence intensity.
 - Pressure gradient λ_{θ} .
 - Separation.
 - Mach number.
 - Reynolds number.
 - Wall heating.
 - Surface conditions.
- The correlations used in transition models contain all the physics, no need to model individual effects.
- This is a big simplification; therefore, the correlations need to be accurate for good results.

Transition models – Review of the $\gamma - Re_{\theta}$ model

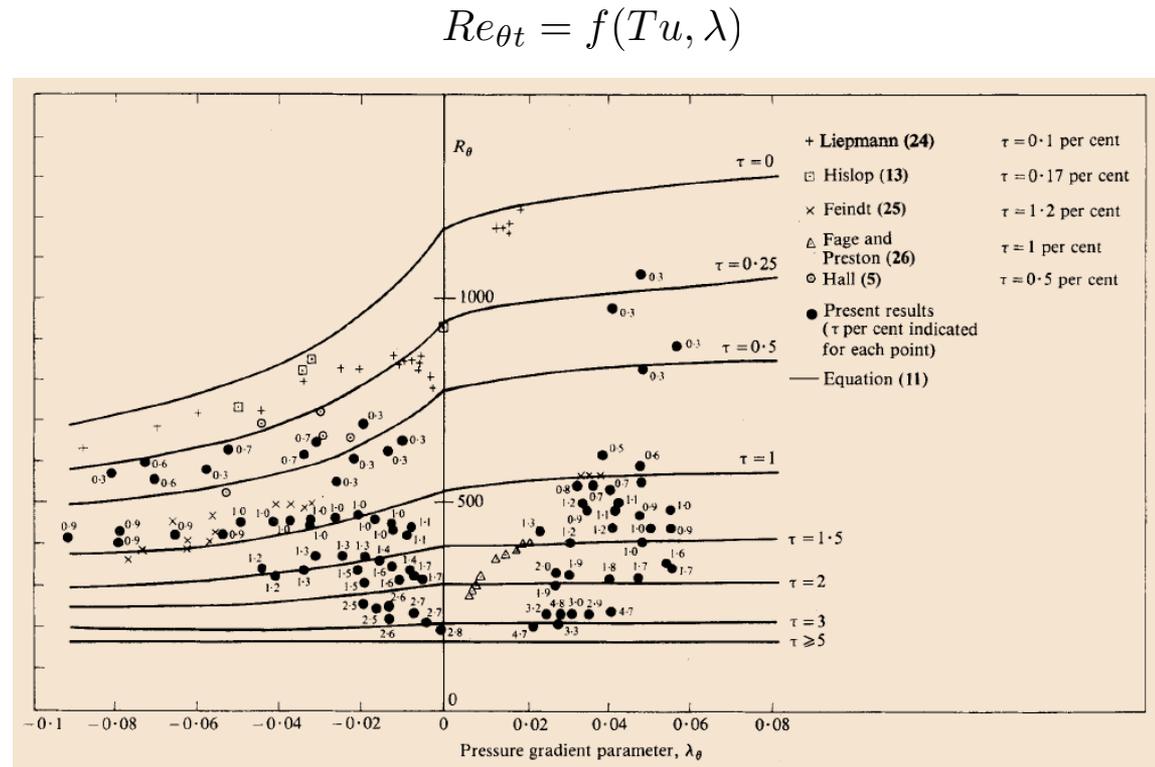
- Transition models are based on correlations to model the mechanism of transition to turbulence.
- Different models will have different correlations.
- In the figure, the momentum thickness Reynolds number at start and end of the transition for a zero-pressure gradient flat plate is illustrated [1].
- As can be seen, turbulence intensity has a strong effect on triggering the transition to turbulence.
- Large turbulence intensity values correlates to smaller momentum thickness Reynolds number or shorter distances to onset the transition to turbulence.
- The correlations shown in the figure, imply that the momentum thickness Reynolds number need to be computed somehow in the CFD simulation.



Momentum thickness Reynolds number at start and end of transition for a zero-pressure gradient flat plate [1].

Transition models – Review of the $\gamma - Re_{\theta}$ model

- In the figure, the Abu-Ghannam and Shaw transition criterion is illustrated [1].
- In these correlations, the critical Reynolds Re_{θ} is a function of turbulence intensity τ and pressure gradient λ_{θ} .
- As can be seen, turbulence intensity and pressure gradient parameter add a dependency to transition to turbulence when measured in function of the momentum thickness Reynolds number.
- This criterion is based on a large number of experiments.
- These correlations contain all physics, no need to model individual effects.



$$Re_{\theta t} = f(Tu, \lambda) \quad \text{Correlation for the transition Reynolds number}$$

$$Re_{\theta c} = f(\tilde{Re}_{\theta t}) \quad \text{Correlation for the critical Reynolds number}$$

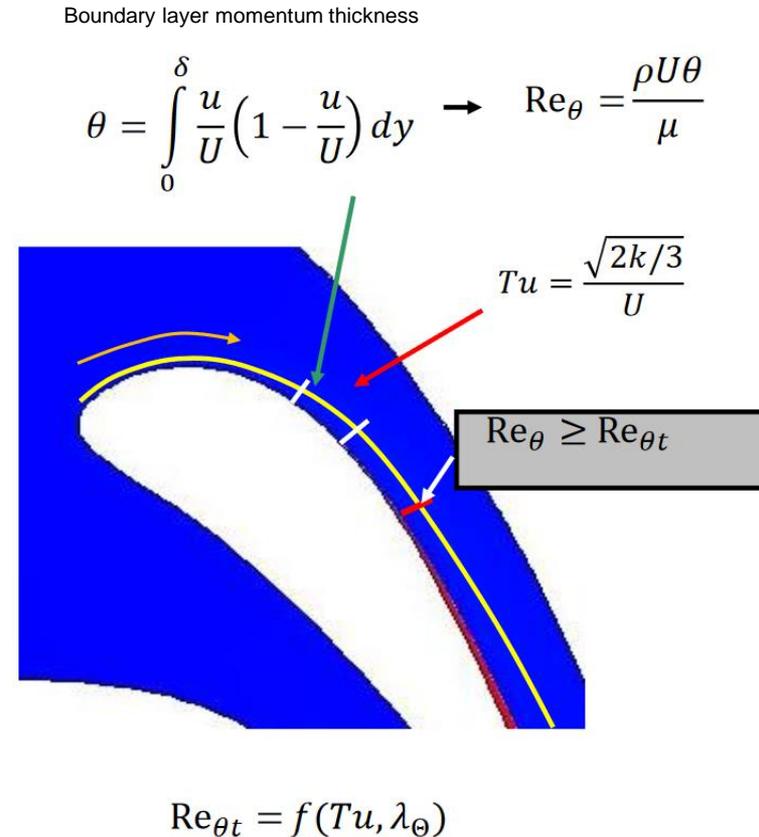
$$F_{length} = f(\tilde{Re}_{\theta t}) \quad \text{Correlation for the length of the transition region}$$

Transition models – Review of the $\gamma - Re_\theta$ model

- These correlations-based methods (based on the momentum thickness Reynolds number), basically work in the following way:
 - Transition is triggered as soon as a criterion is met, for example,

Transition criterion $\rightarrow Re_\theta \geq Re_{\theta t} \leftarrow$ Empirical correlation

- Therefore, Re_θ must be computed in the boundary layer.
- To compute Re_θ , the velocity at the edge of the boundary layer and the integral of the momentum thickness are needed.
- The empirical correlations $Re_{\theta t}$ are based on Tu and the pressure gradient parameter λ_θ outside of the boundary layer (global quantities).
- But the method requires the computation of local quantities that are not readily available in the solution.
- Namely, boundary layer momentum thickness and the velocity at the edge of the boundary layer.
- Therefore, we need an additional transport equation to get information from outside the boundary layer into the boundary layer (through diffusion).



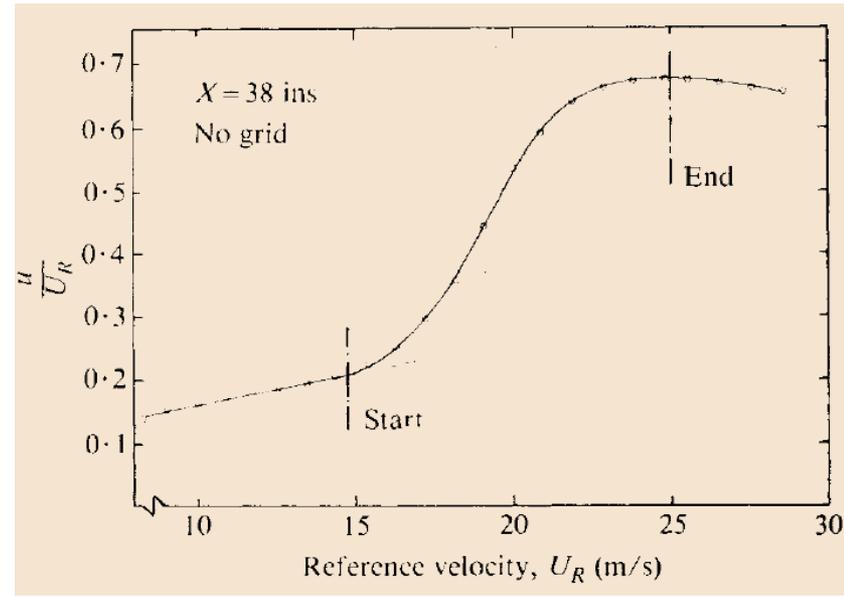
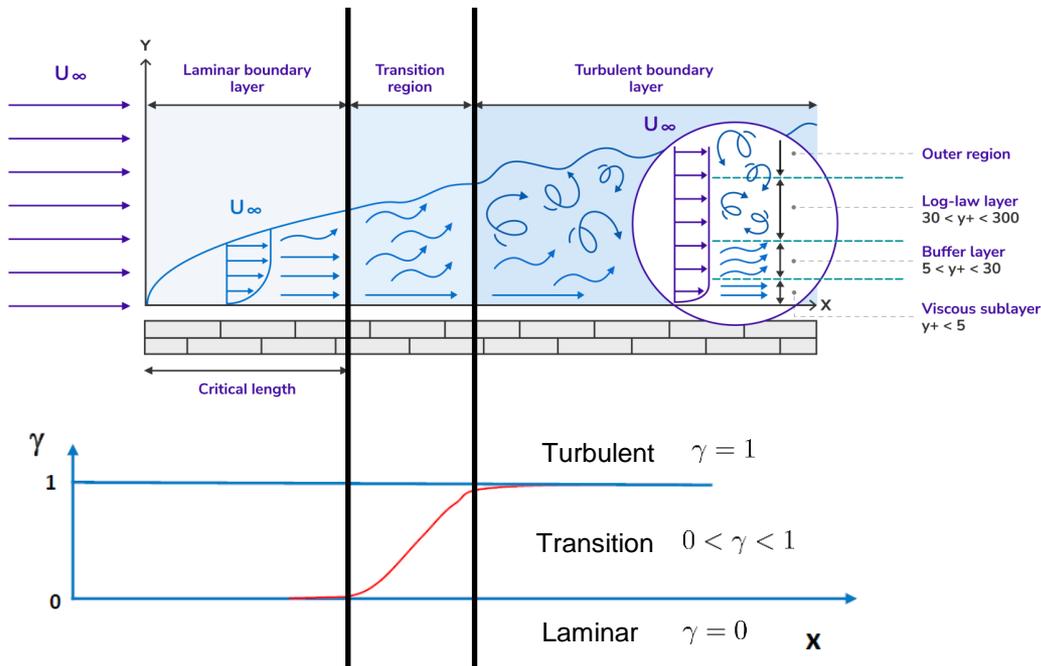
F. Menter. RANS Transition Modelling Using Transport Equations I. ERCOFTAC Course on Transition Modelling III. May 2015. Copyright on the images is held by the contributors. Apart from Fair Use, permission must be sought for any other purpose.

Transition models – Review of the $\gamma - Re_\theta$ model

- When the transition to turbulence is triggered, according to,

Transition criterion $\rightarrow Re_\theta \geq Re_{\theta t} \leftarrow$ Empirical correlation – Onset of the transition

- A ramp function is used to compute the length of the transition region.
- This ramp function is based on correlations.



Transition detection in experiments [1].

Transition models – Review of the $\gamma - Re_{\theta}$ model

- As we stated previously, transition to turbulence is elusive and difficult to solve.
- It requires very fine meshes and specialized turbulence models that solve for additional transport equations.
- Let us briefly address the $\gamma - Re_{\theta}$.
- This model is also known as local correction-based transition model or LCTM [1,2].
- And it is based on the $k - \omega$ SST model [3,4].
- It solves two additional equations to model the transition to turbulence.
- One equation for the intermittency γ and one equation for the momentum thickness Reynolds number $Re_{\theta t}$.
- Then, the model couples the intermittency and the momentum thickness Reynolds number with the $k - \omega$ SST model, plus additional corrections.
- The intermittency equation is a local formulated transport equation which is used to trigger transition (fraction of turbulent flow against fraction of laminar flow).
- The momentum thickness equation is used to pass information about freestream conditions into the boundary layer.

[1] R. Langtry, F. Menter. Transition Modeling for General CFD Applications in Aeronautics. 2005.

[2] F. Menter, R. Langtry, S. Likki, Y. Suzen, P. Huang, S. Volker. A Correlation-Based Transition Model Using Local Variables Part I – Model Formulation. 2006.

[3] F. Menter. Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications. 1994.

[4] F. Menter, M. Kuntz, R. Langtry. Ten Years of Industrial Experience with the SST Turbulence Model. 2003.

Transition models – Review of the $\gamma - Re_{\theta}$ model

- The intermittency transport equation is given as follows,

$$\frac{\partial \rho \gamma}{\partial t} + \frac{\partial (\rho \bar{u}_j \gamma)}{\partial x_j} = P_{\gamma 1} - E_{\gamma 1} + P_{\gamma 2} - E_{\gamma 2} + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\gamma} \right) \frac{\partial \gamma}{\partial x_j} \right]$$

- Where,

$$P_{\gamma 1} = C_{a1} F_{length} \rho S (\gamma F_{onset})^{c_{\gamma 3}} \quad E_{\gamma 1} = C_{e1} P_{\gamma 1} \gamma$$

$$P_{\gamma 2} = C_{a2} \rho \Omega \gamma F_{turb} \quad E_{\gamma 2} = C_{e2} P_{\gamma 2} \gamma$$

- In the previous relations, F_{length} is an empirical correlation that controls the length of the transition region, F_{onset} controls the onset of the transition when $Re_{\theta} \geq Re_{\theta t}$, and S is the strain rate magnitude.
- As you can see, there are many additional coefficients and auxiliary equations to close this equation.
- The interested reader should refer to the original references [1, 2] for a complete description of the model.

[1] R. Langtry, F. Menter. Transition Modeling for General CFD Applications in Aeronautics. 2005.

[2] F. Menter, R. Langtry, S. Likki, Y. Suzen, P. Huang, S. Volker. A Correlation-Based Transition Model Using Local Variables Part I – Model Formulation. 2006.

Transition models – Review of the $\gamma - Re_\theta$ model

- The intermittency transport equation is given as follows,

$$\frac{\partial \rho \gamma}{\partial t} + \frac{\partial (\rho \bar{u}_j \gamma)}{\partial x_j} = P_{\gamma 1} - E_{\gamma 1} + P_{\gamma 2} - E_{\gamma 2} + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\gamma} \right) \frac{\partial \gamma}{\partial x_j} \right]$$

- This equation is entirely based on dimensional arguments.
- If the intermittency value is equal to 0, the flow is laminar.
- And when the intermittency is equal to 1, the flow is fully turbulent.
- All values in between 0 and 1 correspond to transition.
- The term F_{onset} triggers the transition onset.
- However, F_{onset} requires as input the critical Reynolds number $Re_{\theta c}$ from the correlation.
- Therefore, another equation needs to be solved for the critical Reynolds number.

$$F_{onset} = \max [F_{onset2} - F_{onset3}, 0]$$

$$F_{onset1} = \frac{Re_v}{2.193 Re_{\theta c}}$$

$$F_{onset2} = \min \left[\max (F_{onset1}, F_{onset1}^4), 2.0 \right]$$

$$F_{onset3} = \max \left[1 - \left(\frac{Re_T}{25} \right)^3, 0 \right]$$

Transition models – Review of the $\gamma - Re_{\theta}$ model

- The momentum thickness Reynolds number transport equation is given as follows,

$$\frac{\partial \rho \tilde{Re}_{\theta t}}{\partial t} + \frac{\partial (\rho \bar{u}_j \tilde{Re}_{\theta t})}{\partial x_j} = P_{\theta t} + \frac{\partial}{\partial x_j} \left[\sigma_{\theta t} (\mu + \mu_t) \frac{\partial \tilde{Re}_{\theta t}}{\partial x_j} \right]$$

- Where,

$$P_{\theta t} = c_{\theta t} \frac{\rho}{t} \left(Re_{\theta t} - \tilde{Re}_{\theta t} \right) (1.0 - F_{\theta t}) \quad \leftarrow \begin{array}{l} 1 \text{ for laminar flow} \\ 0 \text{ in freestream} \end{array} \quad t = \frac{500\mu}{\rho U^2}$$

- The equation for the transition Reynolds number $\tilde{Re}_{\theta t}$ provides the critical Reynolds number for the intermittency equation.
- Since the correlations are based on freestream conditions, the production term is only active outside the boundary layer.
- This behavior is enabled by means of the blending function $F_{\theta t}$.
- The diffusion term transfers the information into the boundary layer.
- Again, there are many additional coefficients and auxiliary equations to close this equation.
- The interested reader should refer to the original references for a complete description of the model.

Transition models – Review of the $\gamma - Re_\theta$ model

- The coupling of the intermittency equation to the turbulence model is achieved by modifying the production and dissipation terms of the $k - \omega$ SST turbulence model.
- In the $k - \omega$ SST the transport equation of the turbulence kinetic energy is written as follows,

$$\frac{\partial \rho k}{\partial t} + \frac{\partial (\rho k u_i)}{\partial x_i} = P_k^* - D_k^* + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_j} \right]$$

- Where,

$$P_k^* = \gamma_{eff} P_k$$

$$D_k^* = \min [\max (\gamma_{eff}, 0.1), 1.0] D_k$$

- P_k and D_k are the production and dissipation terms of the original formulation of the $k - \omega$ SST turbulence model.

Transition models – Review of the $\gamma - Re_{\theta}$ model

- This model uses quite complex correlations, which often are proprietary (industrial secrets).
- Hereafter, we show the correlations found in the public domain.
- Which likely are not the same implemented in Fluent (the Fluent documentation do not show the correlations).

$$F_{length} = \begin{cases} 39.8189 - 0.011927\tilde{Re}_{\theta t} - 0.0001325\tilde{Re}_{\theta t}^2, & \tilde{Re}_{\theta t} < 400 \\ 263.404 - 1.2393\tilde{Re}_{\theta t} + 0.001945\tilde{Re}_{\theta t}^2 - 0.000001016\tilde{Re}_{\theta t}^3, & 400 \leq \tilde{Re}_{\theta t} < 596 \\ 0.5 - 0.0003(\tilde{Re}_{\theta t} - 596), & 596 \leq \tilde{Re}_{\theta t} < 1200 \\ 0.3188, & 1200 \leq \tilde{Re}_{\theta t} \end{cases}$$

$$Re_{\theta c} = \begin{cases} \tilde{Re}_{\theta t} + \left(-3.9603 + 0.01206\tilde{Re}_{\theta t} - 0.000868\tilde{Re}_{\theta t}^2 + 0.000000696\tilde{Re}_{\theta t}^3 - 0.000000000174\tilde{Re}_{\theta t}^4\right), & \tilde{Re}_{\theta t} \leq 1870 \\ \tilde{Re}_{\theta t} - \left(593.11 + 0.482(\tilde{Re}_{\theta t} - 1870)\right), & \tilde{Re}_{\theta t} > 1870 \end{cases}$$

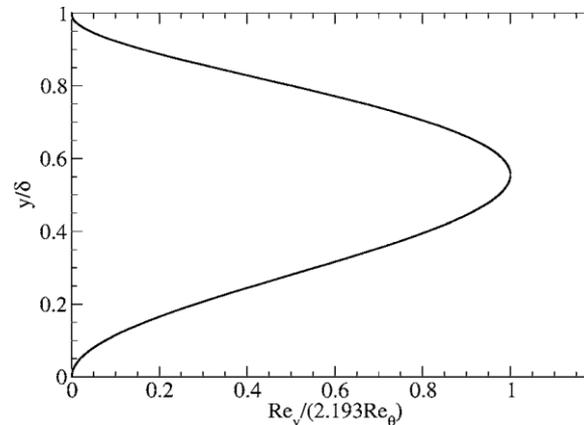
Transition models – Review of the $\gamma - Re_\theta$ model

- Computing the momentum thickness in unstructured CFD solvers is not an easy task because the boundary layer edge is not well defined, and the integration will therefore depend on the implementation of a search algorithm.
- In old times, many simulations were run to match the critical transition criterion with the correlations.
- When the critical point was found, the turbulence model was triggered at that point.
- To avoid this lengthy iterative process, the $\gamma - Re_\theta$ transitional model uses a local formulation to compute the critical transition criterion
- So, instead of using the momentum thickness Reynolds number to trigger the onset of transition, this model is based on the vorticity (or strain-rate) Reynolds number.
- The vorticity Reynolds number Re_v is defined as follows,

$$Re_v = \frac{\rho y^2}{\mu} \left| \frac{\partial u}{\partial y} \right| = \frac{\rho y^2}{\mu} \Omega$$

Transition models – Review of the $\gamma - Re_\theta$ model

- A scaled profile of the vorticity Reynolds number is shown in the figure for a Blasius boundary layer.



- The scaling is achieved by dividing the profile by the corresponding momentum thickness Reynolds number and a constant equal to 2.193.
- The scaling is chosen in order to have a maximum of one inside the boundary layer.
- In other words, the maximum of the profile is proportional to the momentum thickness Reynolds number and can therefore be related to transition correlations as follows:

$$Re_\theta = \frac{Re_v^{\max}}{2.193}$$

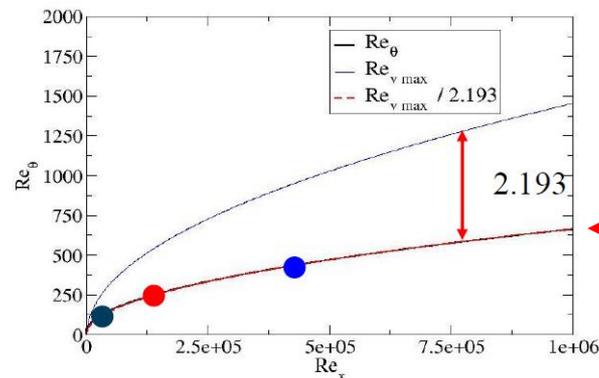
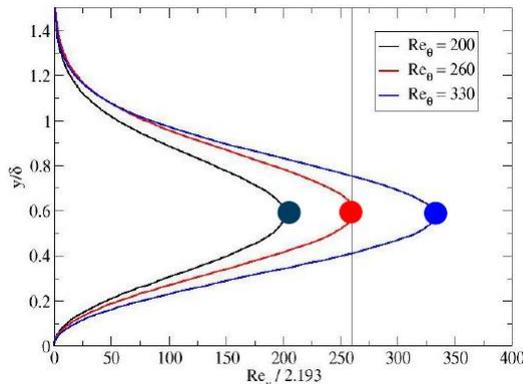
Transition models – Review of the $\gamma - Re_\theta$ model

- Based on the previous observation, a general framework can be built that can serve as a local environment for correlation-based transition models.
- To compute Re_θ locally, this formulation uses the vorticity Reynolds number, which is defined as follows,

$$Re_v = \frac{\rho y^2}{\mu} \left| \frac{\partial u}{\partial y} \right| = \frac{\rho y^2}{\mu} \Omega$$

- Then, Re_θ can be correlated as follows using local quantities,

$$Re_\theta = Re_v^{\max} / 2.193 \quad \leftarrow \text{Another correlation.}$$



Middle of the boundary layer
 $Re_\theta = Re_v^{\max} / 2.193$

Transition models – Review of the $\gamma - Re_{\theta}$ model

- To activate the term F_{onset} (the trigger for the transition onset), this formulation uses the vorticity Reynolds number, as follows,

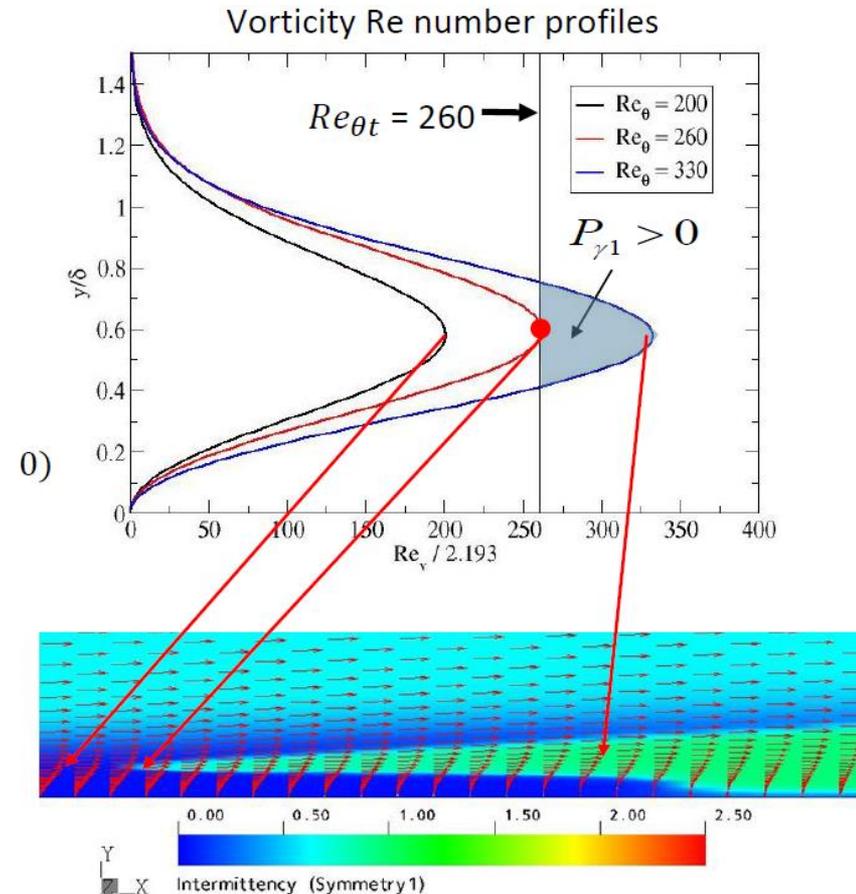
$$Re_v = \frac{\rho y^2}{\mu} \left| \frac{\partial u}{\partial y} \right|$$

$$F_{onset1} \approx \max \left(\frac{Re_v}{2.193 Re_{\theta t}} - 1, 0 \right)$$

$Re_{\theta t} = f(T_u, \lambda_{\theta t})$

If this term is more than one, it is a transition point

- For example, assume a critical $Re_{\theta t}$, e.g., 260.
- The function F_{onset} is zero upstream of the critical point.
- Once it reaches the critical point, the function becomes active in the center of the boundary layer, and it can be used to trigger transition (blue zone close to the walls).



Transition models – Review of the $\gamma - Re_{\theta}$ model

- This model has many empirical correlations, closure coefficients, auxiliary relations, limiter functions, and blending functions that we did not cover.
- The interested reader should refer to references [1,2,3] for a complete description of the model.
- Also, for a complete description of the $k - \omega$ SST, the interested reader should refer to references [4,5].
- It is worth mentioning that the correlations used are often proprietary and are calibrated for very specific experiments.
- Use transition models only when you are sure that the effect of transition is important and relevant to the flow that you are solving, as these models requires very fine meshes and additional equations.
- These models have the highest level of complexity.

References:

- [1] R. Langtry, F. Menter. Transition Modeling for General CFD Applications in Aeronautics. 2005.
- [2] F. Menter, R. Langtry, S. Likki, Y. Suzen, P. Huang, S. Volker. A Correlation-Based Transition Model Using Local Variables Part I – Model Formulation. 2006.
- [3] ANSYS Fluent Theory Guide, 2020R1
- [4] F. Menter. Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications. 1994.
- [5] F. Menter, M. Kuntz, R. Langtry. Ten Years of Industrial Experience with the SST Turbulence Model. 2003.

Simplifying the $\gamma - Re_\theta$ model **The intermittency transition model γ**

Transition models – Review of the $\gamma - Re_\theta$ model

Simplifying the $\gamma - Re_\theta$ model – The intermittency transition model γ

- The intermittency γ transition model can be considered as a variant of $\gamma - Re_\theta$ model
- The idea of simplifying the $\gamma - Re_\theta$ model is to avoid solving the extra equation for $\tilde{Re}_{\theta t}$.
- This model reduces the computational effort by solving one transport equation instead of two.
- The transition modeling is achieved by using a local approximation (subscript L in our notation) of the turbulence intensity and the pressure gradient parameter in the center of the boundary layer.
- Another advantage of this model is that by eliminating the transport equation for $\tilde{Re}_{\theta t}$, the new formulation is Galilean invariant.
- In the $\gamma - Re_\theta$ model, the turbulence intensity depends on the velocity; therefore, this formulation is not Galilean invariant.
- A Galilean invariant turbulence model, is a model that is valid in different reference frames.
- That is, it can be use with moving bodies or multiple reference frames.
- This is a deficiency of some turbulence models, in particular, transition models that depend on the global turbulence intensity.

Transition models – Review of the $\gamma - Re_\theta$ model

Simplifying the $\gamma - Re_\theta$ model – The intermittency transition model γ

- In the intermittency transition model, the following intermittency γ transport equation is solved,

$$\frac{\partial \rho \gamma}{\partial t} + \frac{\partial (\rho \bar{u}_j \gamma)}{\partial x_j} = P_\gamma - E_\gamma + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\gamma} \right) \frac{\partial \gamma}{\partial x_j} \right]$$

- With the following source terms,

$$P_\gamma = F_{length} \rho S_\gamma F_{onset} (1 - \gamma)$$

$$E_\gamma = c_{a2} \rho \Omega_\gamma F_{turb} (c_{e2} \gamma - 1)$$

- Where the terms S_γ and Ω_γ are the strain rate magnitude and vorticity magnitude, respectively.

Transition models – Review of the $\gamma - Re_\theta$ model

Simplifying the $\gamma - Re_\theta$ model – The intermittency transition model γ

- This model uses the following auxiliary relationships to control the onset of the transition,

$$F_{onset} = \max(F_{onset2} - F_{onset3}, 0)$$

$$F_{turb} = \exp\left[\frac{R_T^4}{2}\right]$$

$$F_{onset1} = \frac{Re_v}{2.2Re_{\theta c}}$$

$$R_T = \frac{\rho k}{\mu \omega}$$

$$F_{onset2} = \min(F_{onset1}, 2.0)$$

$$F_{onset3} = \max\left(1 - \left(\frac{R_T}{3.5}\right)^3, 0\right)$$

$$R_V = \frac{\rho d_w^2 S}{\mu}$$

Transition models – Review of the $\gamma - Re_\theta$ model

Simplifying the $\gamma - Re_\theta$ model – The intermittency transition model γ

- In this model, the following correlation is used to trigger transition,

$$Re_{\theta_c}(T_{UL}, \lambda_{\theta L}) = C_{TU1} + C_{TU2} \exp[-C_{TU3} T_{UL} F_{PG}(\lambda_{\theta L})]$$

Local quantities – Turbulence intensity and pressure gradient parameter

- The local formulation of the turbulence intensity is expressed as follows,

$$T_{UL} = \min \left(100 \frac{\sqrt{2k/3}}{\omega d_w}, 100 \right)$$

Local formulation of turbulence intensity
This formulation avoids the use of the velocity;
therefore, is Galilean invariant.

- And the local pressure gradient parameter is computed as follows,

$$\lambda_{\theta L} = \min [\max (\lambda_{\theta L}, -10), 10]$$

$$\lambda_{\theta L} = -0.1111 \frac{dV}{dy} \frac{d_w^2}{v} + 0.1875$$

Transition models – Review of the $\gamma - Re_\theta$ model

Simplifying the $\gamma - Re_\theta$ model – The intermittency transition model γ

- This model uses fairly simple correlations, at least much simpler than those for the $\gamma - Re_\theta$ transition model.

$$F_{PG}(\lambda_{\theta L}) = \begin{cases} \min(1 + C_{PG1}\lambda_{\theta L}, C_{PG1}^{lim}), & \lambda_{\theta L} \geq 0 \\ \min(1 + C_{PG2}\lambda_{\theta L} + C_{PG3}\min[\lambda_{\theta L} + 0.068, 0] C_{PG2}^{lim}), & \lambda_{\theta L} < 0 \end{cases}$$

$$Re_{\theta c}(T_{UL}, \lambda_{\theta L}) = C_{TU1} + C_{TU2}\exp[-C_{TU3}T_{UL}F_{PG}(\lambda_{\theta L})]$$

- The resulting formulation is local and Galilean invariant and requires only one equation.
- The closure coefficients of this model have carefully calibrated but can be fine tuned by the user.
- A detailed description of this model can be found in the Ansys Fluent Help System [1].

Transition models – Review of the $\gamma - Re_\theta$ model

Simplifying the $\gamma - Re_\theta$ model – The intermittency transition model γ

- Model dependence – Turbulence intensity and correlation parameters.

$\gamma - Re_\theta$ model

$$T_U = \frac{100}{U} \sqrt{\frac{2}{3}k}$$

$$Re_{\theta_c}(T_U, \lambda_\theta)$$

γ Intermittency model

$$T_{UL} = \min \left(100 \frac{\sqrt{2k/3}}{\omega d_w}, 100 \right)$$

$$Re_{\theta_c}(T_{UL}, \lambda_{\theta L})$$

On the Galilean invariance of turbulence models and laminar separation bubbles mesh requirements

Transition models – Review of the $\gamma - Re_{\theta}$ model

On the Galilean invariant of the turbulence models

- The fact that the model is not Galilean invariant is only a limitation.
- It means that it cannot be used with moving bodies or moving reference frames.
- Another strong argument that we must understand the theory and the physics when dealing with turbulence modeling and CFD.

Flow Turbulence Combust (2006) 77: 277–303
DOI 10.1007/s10494-006-9047-1

Transition Modelling for General Purpose CFD Codes

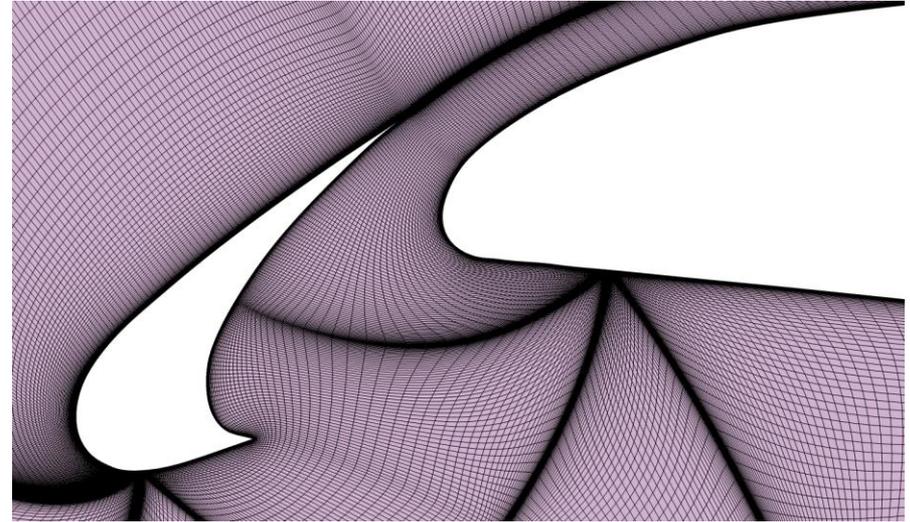
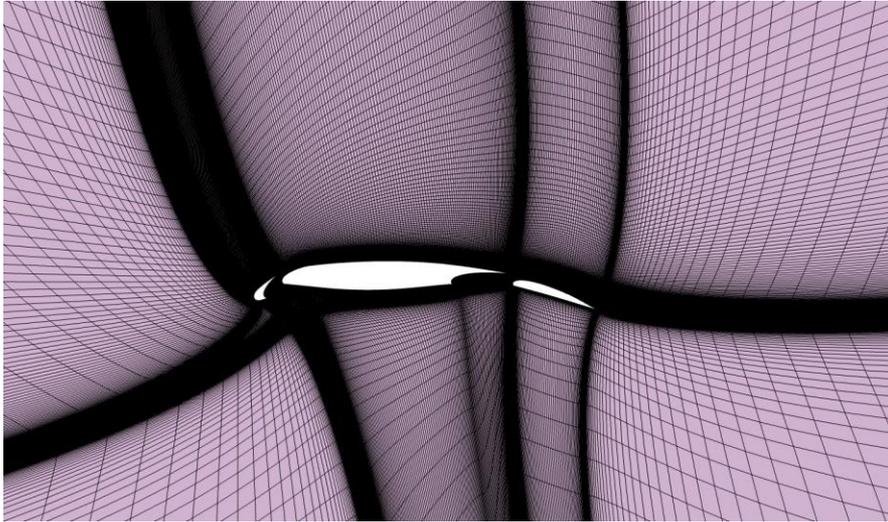
F. R. Menter · R. Langtry · S. Völker

The present authors have recently developed a correlation-based transition model, built on transport equations, using only local variables. The concept is termed LCTM – Local Correlation-based Transition Model. The central idea behind this concept has been described in a model by Menter et al. [15]. The major numerical and modelling deficiencies associated with that prototype model have been eliminated by Menter et al. [16] and a wide range of turbomachinery-related flow problems has been computed by Langtry et al. [6]. The model has since been extended to aerodynamic flows [7] and is now run within the software package CFX-5, as well as the GE in-house code Tacoma on numerous industrial applications. The model satisfies all requirements given above, except for the last one – coordinate independence. This is a consequence of the fact that transition correlations are based on non-Galilean invariant parameters, like the turbulence intensity Ti . As boundary layer transition is always relative to walls, this is only an issue if multiple moving walls exist in a single computational domain. Efforts are underway for eliminating this restriction.

The present transition model accounts for transition due to free-stream turbulence intensity, pressure gradients and separation. It is fully CFD-compatible and does not negatively affect the convergence of the solver. Current limitations of the model are that crossflow instability is not included in the correlations and that the transition correlations are formulated non-Galilean invariant. Both limitations are currently being investigated and can be removed in principle. Another area for future investigations is the important effect of wake induced transition, as it occurs on blades of axial turbines/compressors affected by the wake of upstream blade rows. The helicopter test case shows that wake-induced transition can be handled by the model. The increased FSTI caused by the wake, affects the transition onset correlation, leading to earlier transition. Tests for unsteady turbomachinery cases are required for further validation of this functionality.

Transition models – Review of the $\gamma - Re_{\theta}$ model

Laminar separation bubbles and transition to turbulence – Mesh resolution requirements



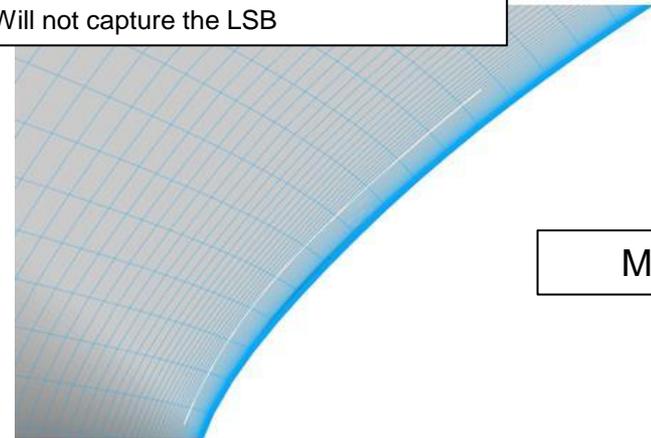
- Trying to predict transition to turbulence in CFD requires very fine meshes and well calibrated models.
- Many traditional turbulence models will fail in predicting transition to turbulence.
- However, some advanced turbulence models (such as nonlinear eddy viscosity models) can also predict transition to turbulence.
- However, it is always recommended to use transitional turbulence models if you are dealing with this physics.
- In many applications, transition to turbulence is preceded by laminar separation bubbles (LSB), which are laminar recirculation areas that separate from the wall and reattach in a very short distance and are very sensitive to disturbances.
- After the LSB, the flow becomes turbulent.

Transition models – Review of the $\gamma - Re_{\theta}$ model

Laminar separation bubbles and transition to turbulence – Mesh resolution requirements

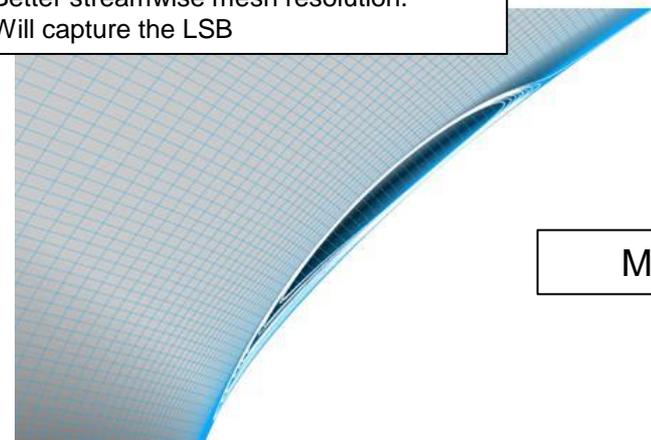
- In many engineering applications laminar separation bubbles (LSB) trigger transition to turbulence.
- The physics of LSB is very complex and needs to be resolved with fine meshes.
- This complex physics (separation and reattachment) needs to be resolved well in the streamwise direction.
- And in 3D cases, also needs to be resolved in the spanwise direction.
- Generally speaking, about 20 cells are required in the streamwise direction in order to resolve the LSB.
- For airfoils/blades operating at low Reynolds numbers, this requirement is easy to achieve as usually the bubbles are about 20% of the chord.
- For airfoils/blades operating at high Reynolds numbers, LSB can be about 1% of the chord.
- By the way, the location of the LSB is not known a-priori, so the meshing/analysis is an iterative task.

Streamwise mesh resolution too coarse.
Will not capture the LSB



Mesh 1

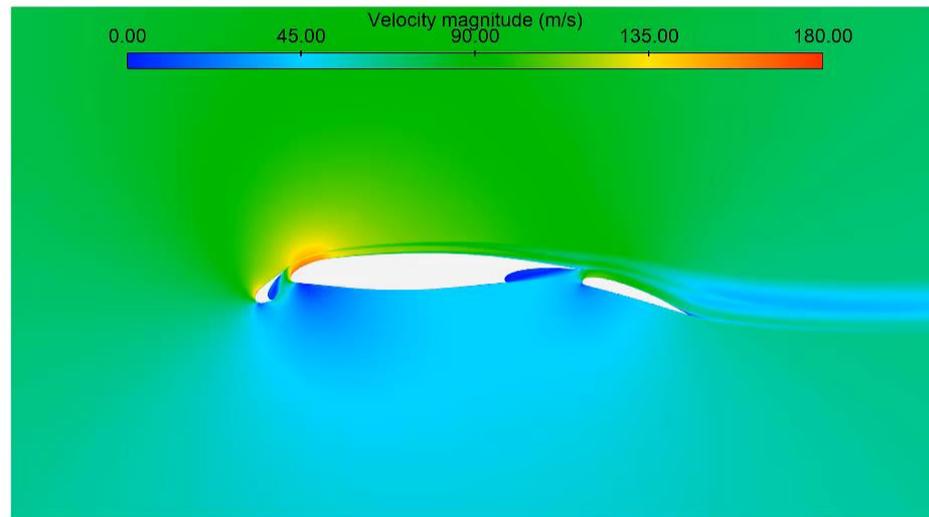
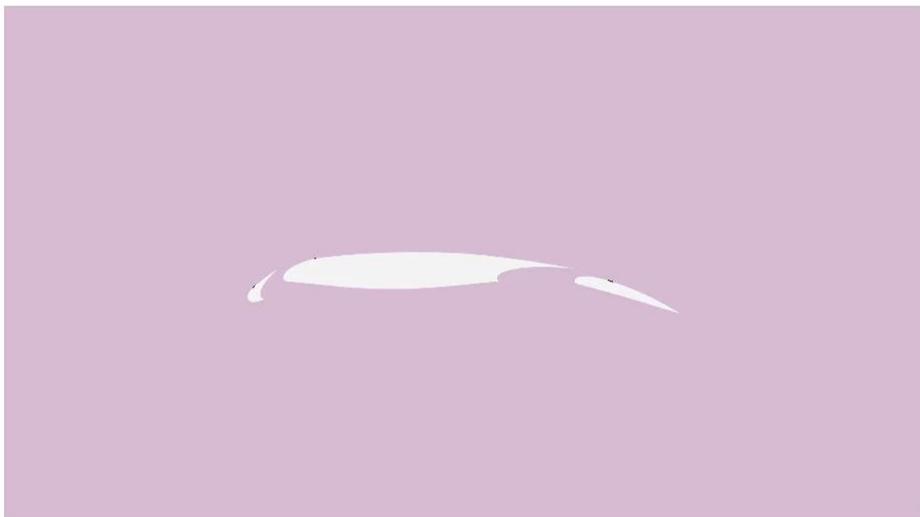
Better streamwise mesh resolution.
Will capture the LSB



Mesh 2

Transition models – Review of the $\gamma - Re_{\theta}$ model

Laminar separation bubbles and transition to turbulence – Mesh resolution requirements



Laminar separation bubbles

<http://www.wolfdynamics.com/images/airfoil.mp4>

- Transitional flow over a three-element airfoil.

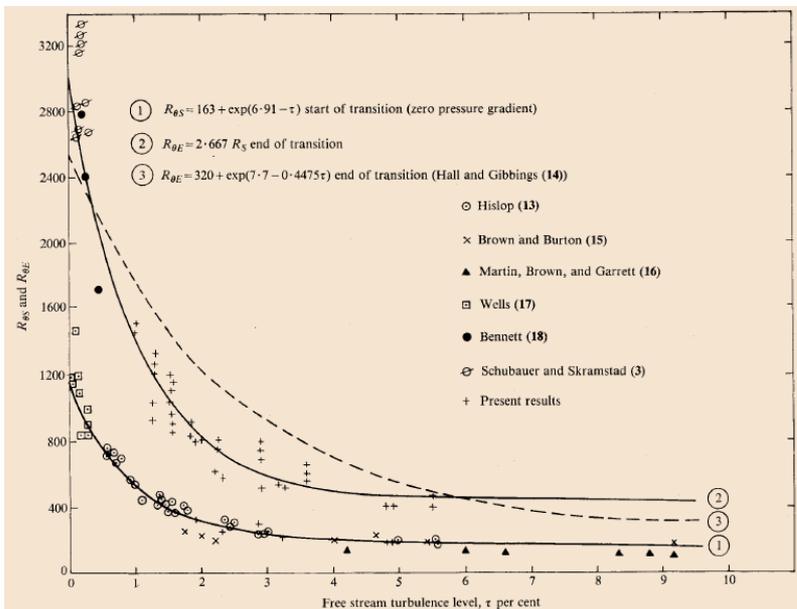
Final remarks

Transition models – Review of the $\gamma - Re_{\theta}$ model

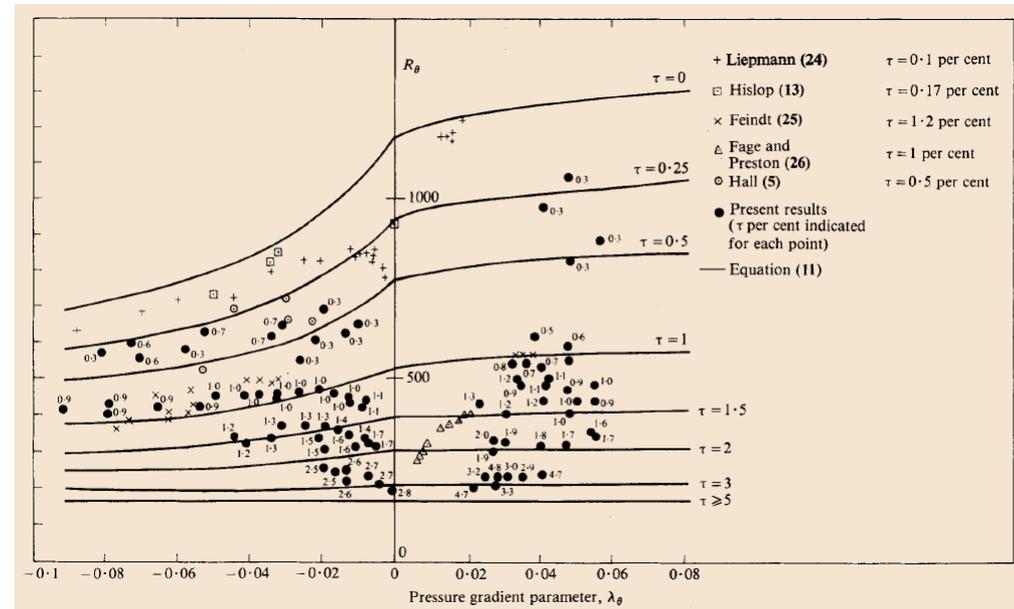
- Transition models are very sensitive to the inlet turbulence intensity Tu value.
- If local formulations are used, the location of the transition point and its extension strongly depends on the local turbulence intensity Tu.
- For external aerodynamics, high inlet values for Tu and eddy viscosity ratio (EVR) are required to allow for decay.
- Correlations are available to compute this decay.
- It is recommended to initialize the intermittency to 1 in the whole domain.
- It is strongly recommended to use production limiters together with transition to turbulence models.
- Good quality meshes are required.
- The use of hexahedral meshes is also strongly recommended.
- Low expansion ratios normal to the walls are recommended (in the order of 1.1).

Transition models – Review of the $\gamma - Re_{\theta}$ model

- The initial value of Re_{θ} can be estimated using the model correlations.
- Remember, these correlations are often based on the turbulence intensity and pressure gradient.
- Therefore, these quantities must be defined in order to get and estimate of Re_{θ} .
- No need to mention that the values used need to be physically realistic.



- Momentum thickness Reynolds number at start and end of transition for a zero-pressure gradient flat plate [1].



- In the figure, the Abu-Ghannam and Shaw transition criterion is illustrated [1].
- In these correlations, the critical Reynolds Re_{θ} is a function of turbulence intensity τ and pressure gradient λ_{θ} .

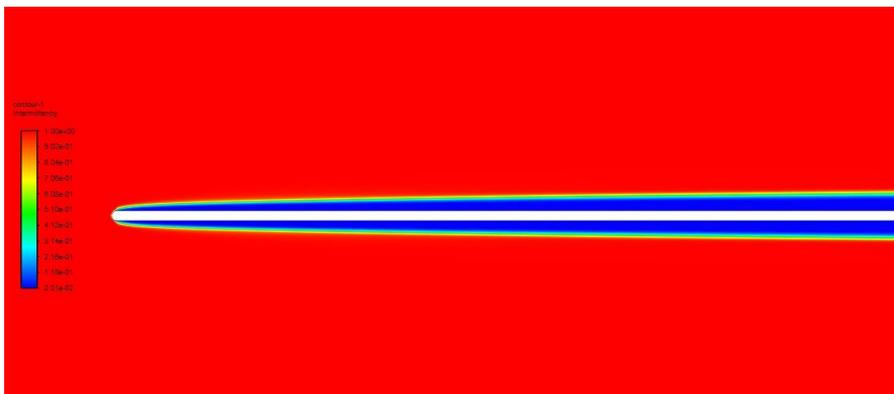
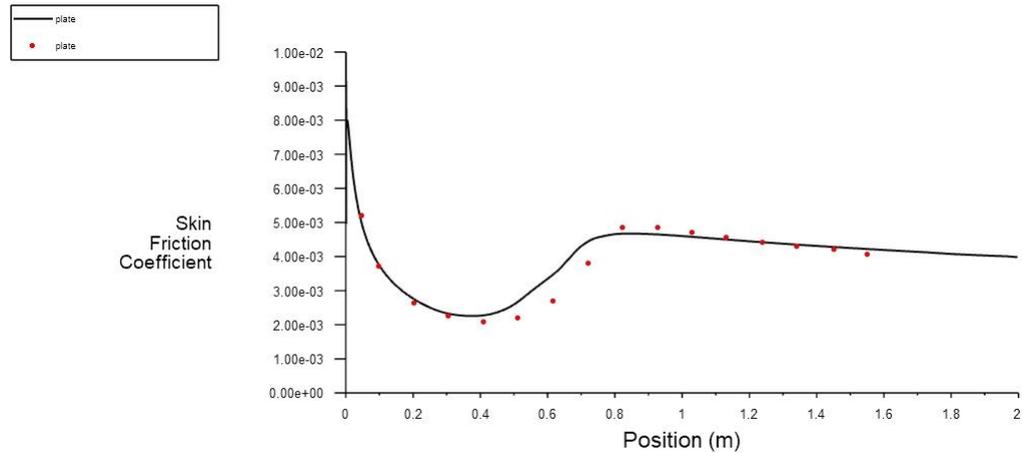
Transition models – Review of the $\gamma - Re_{\theta}$ model

- It is recommended to use low convergence criterion, in the order of $10e-6$ for all variables.
- Intermittency is hard to make converge, do not worry about this behavior.
- Transition to turbulence models are wall resolving and require y^+ values lower than 1.
- However, it is not recommended to use y^+ values lower than 0.01.
- The ideal y^+ value is in the order of 0.1-0.5.
- You also need to cluster enough prismatic layers in the boundary layer in order to solve the profiles of the transported quantities and capture the edge of the boundary layer.
 - Recommended values are between 20 and 30 prismatic layers.
- You also need to use enough cells in the stream-wise direction so you can properly capture the transition region and eventual laminar separation bubbles (LSB).
- It is strongly recommended to follow the standard practices suggested by the model implementation [1].

Some sample applications

Transition models – Review of the $\gamma - Re_\theta$ model

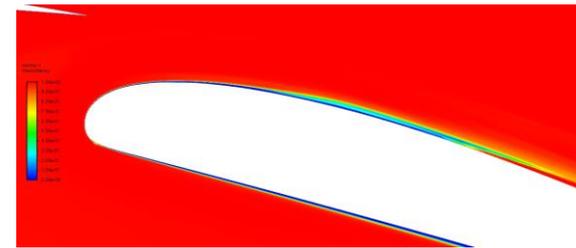
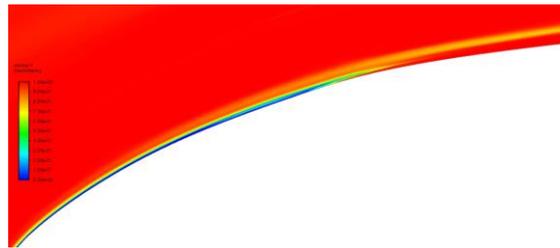
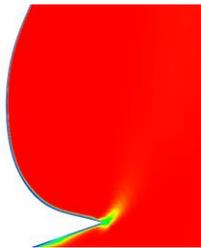
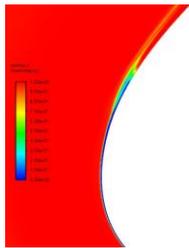
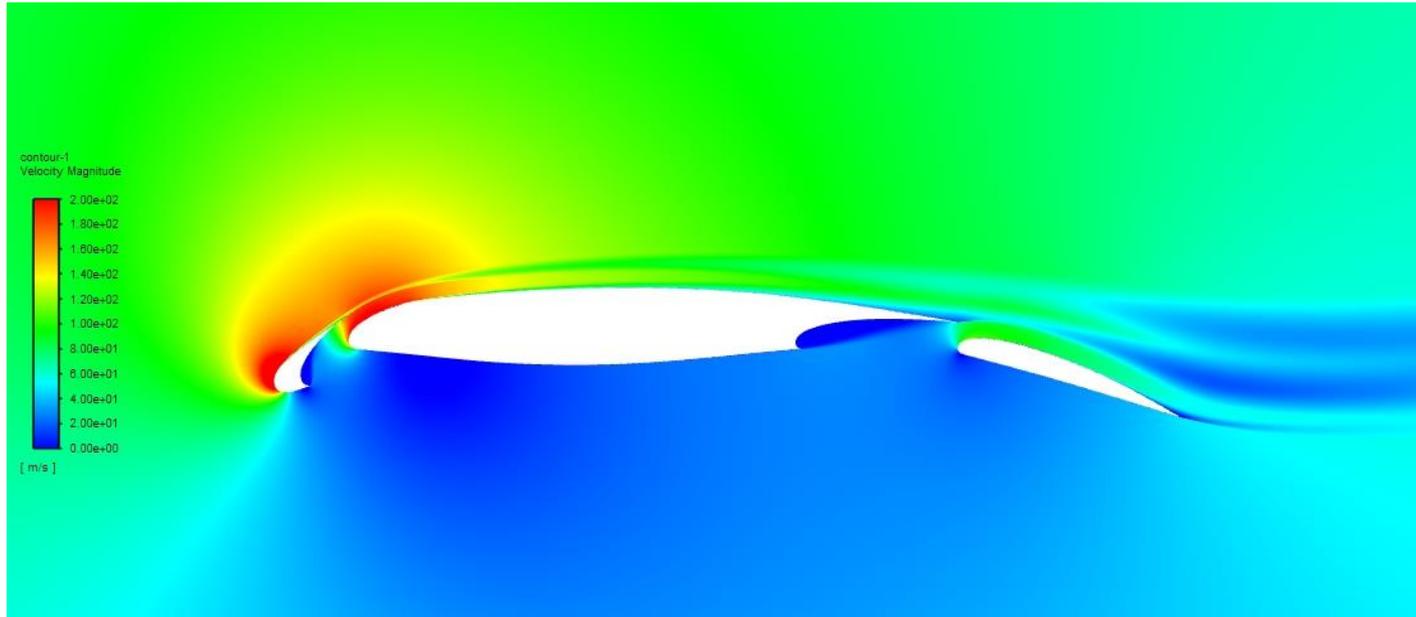
- Laminar to turbulent transition of boundary layer over a flat plate.



The intermittency field in a transitional flat plate.

Transition models – Review of the $\gamma - Re_{\theta}$ model

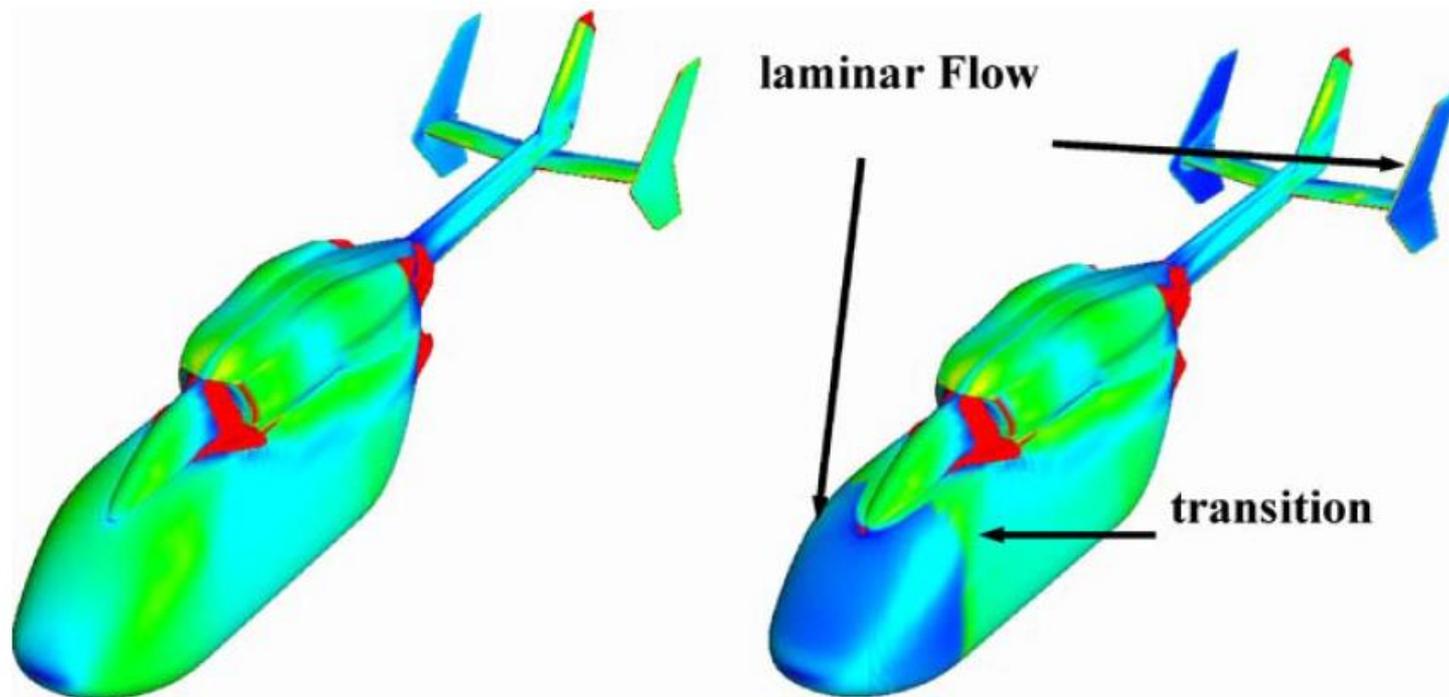
- Transitional flow over a three-element airfoil.



The intermittency field in a three-element airfoil.

Transition models – Review of the $\gamma - Re_{\theta}$ model

- Contour plot of skin friction for a fully turbulent (left) and transitional (right) solution of the Eurocopter cabin.
- The difference in total drag between both simulations is about 5%.



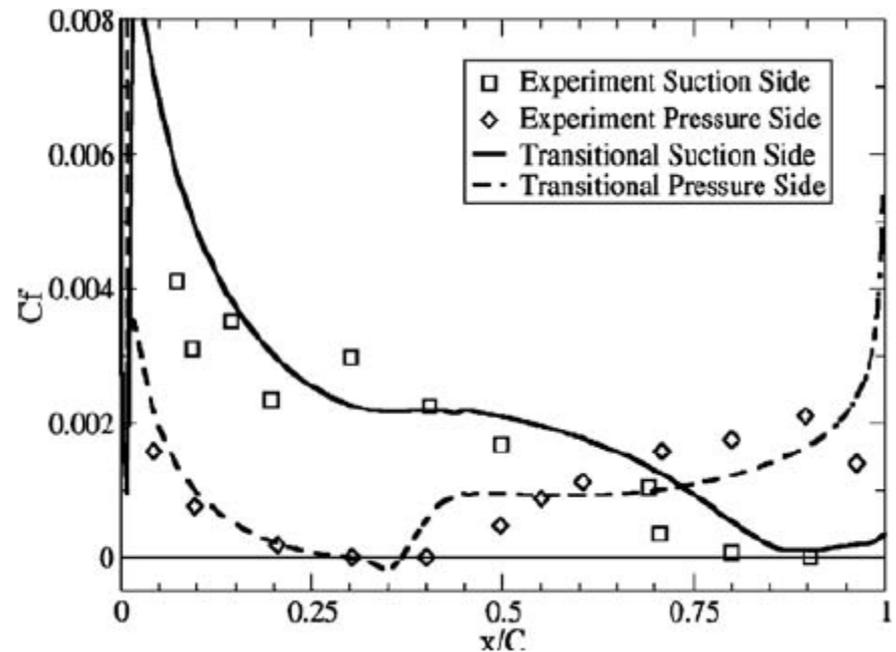
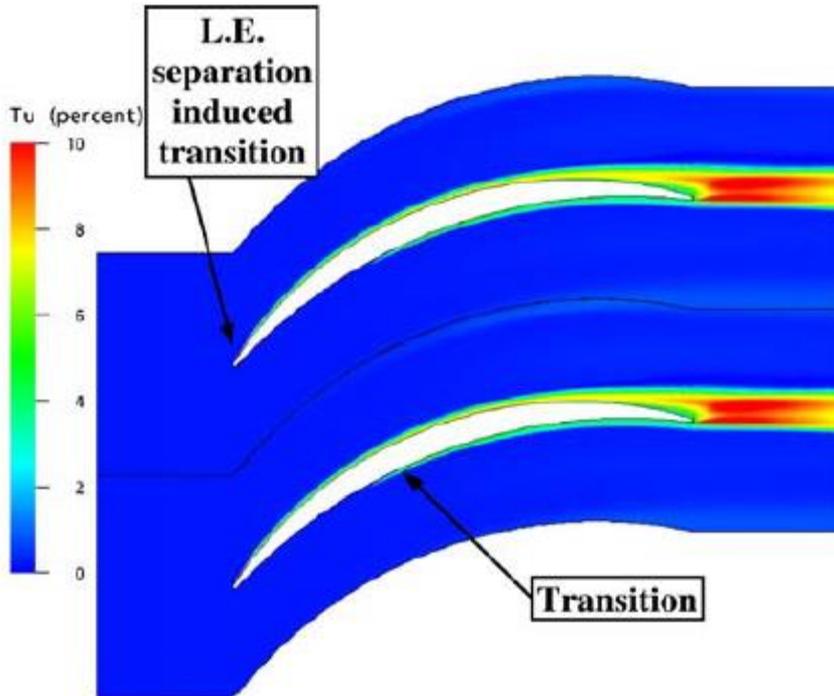
Contours of skin friction. Left: fully turbulent case. Right: transitional case.

References:

R. Langtry, F. Menter, S. Volker. Transition Modeling for General CFD Applications in Aeronautics. Flow turbulence combustion, 77, 2006.

Transition models – Review of the $\gamma - Re_{\theta}$ model

- Turbulence intensity contours (left image) and friction coefficient distribution (right image) against experimental data for the Zierke and Deutsch compressor.

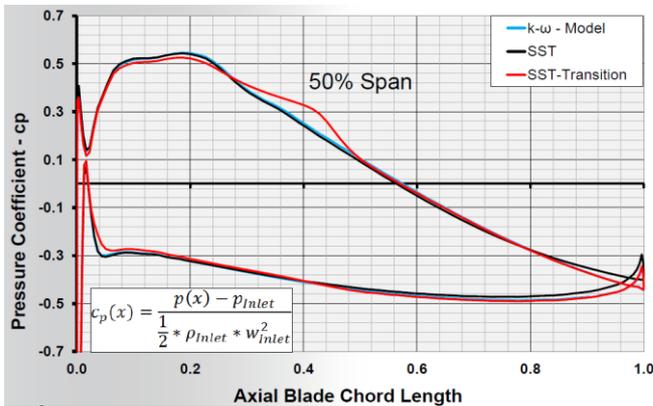
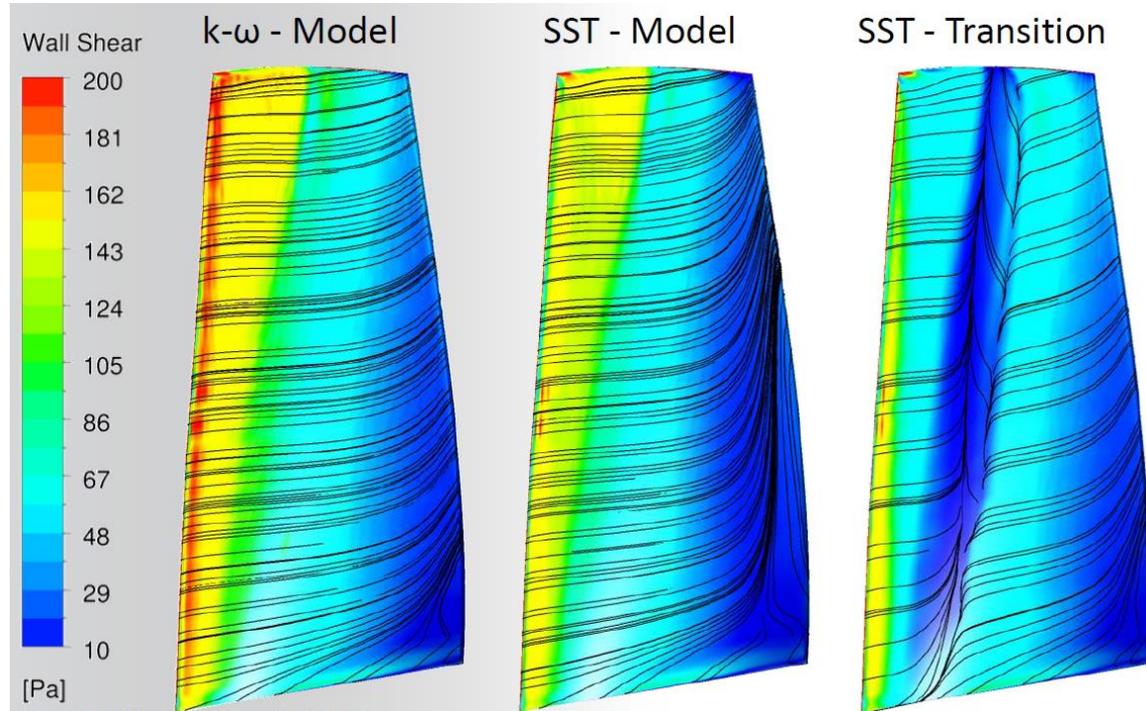
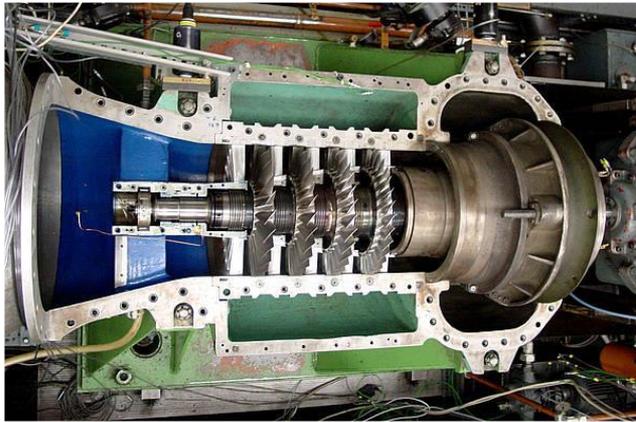


References:

R. Langtry, F. Menter, S. Volker. Transition Modeling for General CFD Applications in Aeronautics. Flow turbulence combustion, 77, 2006.

Transition models – Review of the $\gamma - Re_{\theta}$ model

- Four stage high-speed axial compressor – TFD Hannover.

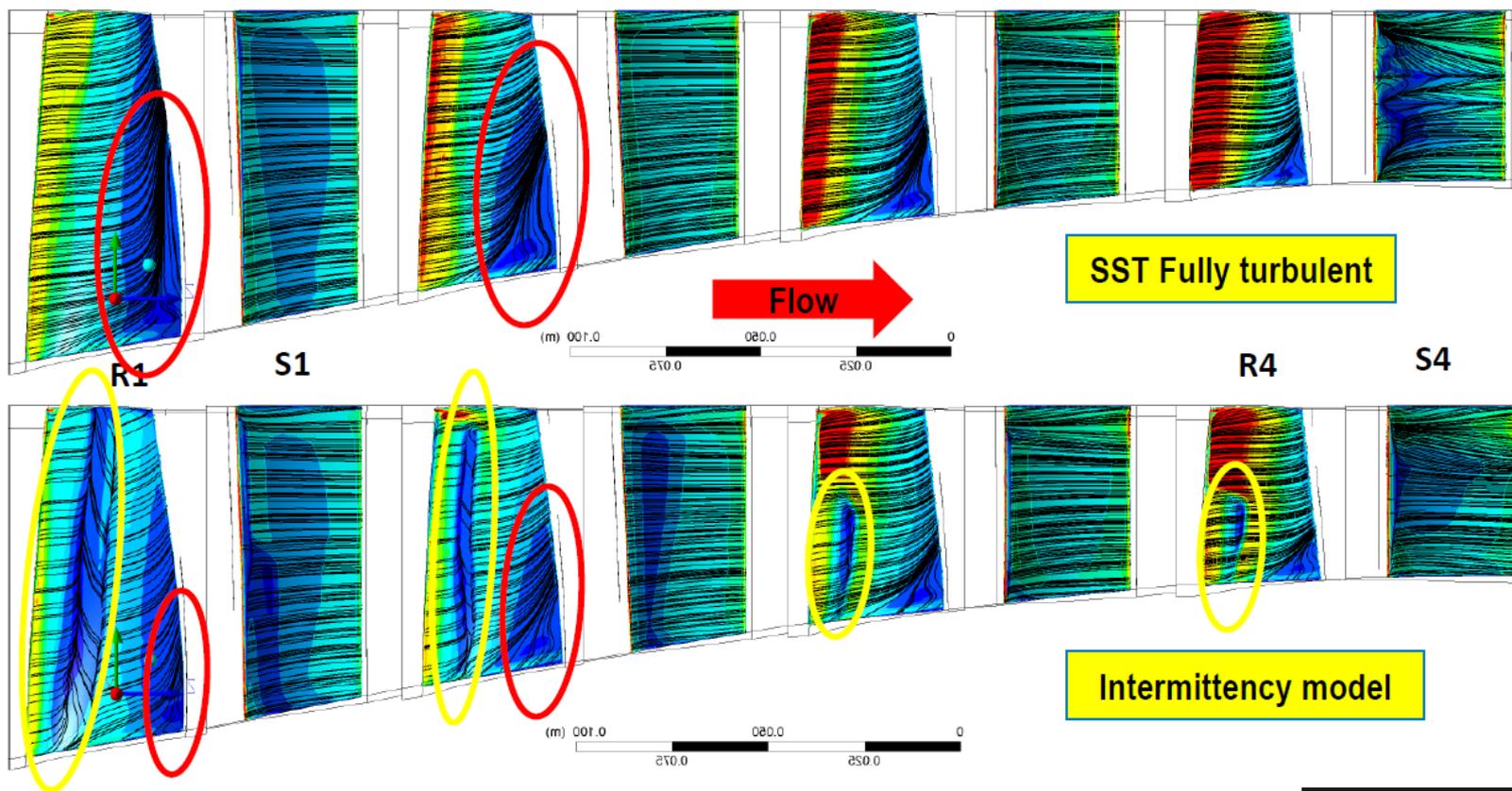


References:

F. Menter. RANS Transition Modelling Using Transport Equations I. ERCOFTAC Course on Transition Modelling III, 2015.

Transition models – Review of the $\gamma - Re_{\theta}$ model

- Four stage high-speed axial compressor – TFD Hannover.



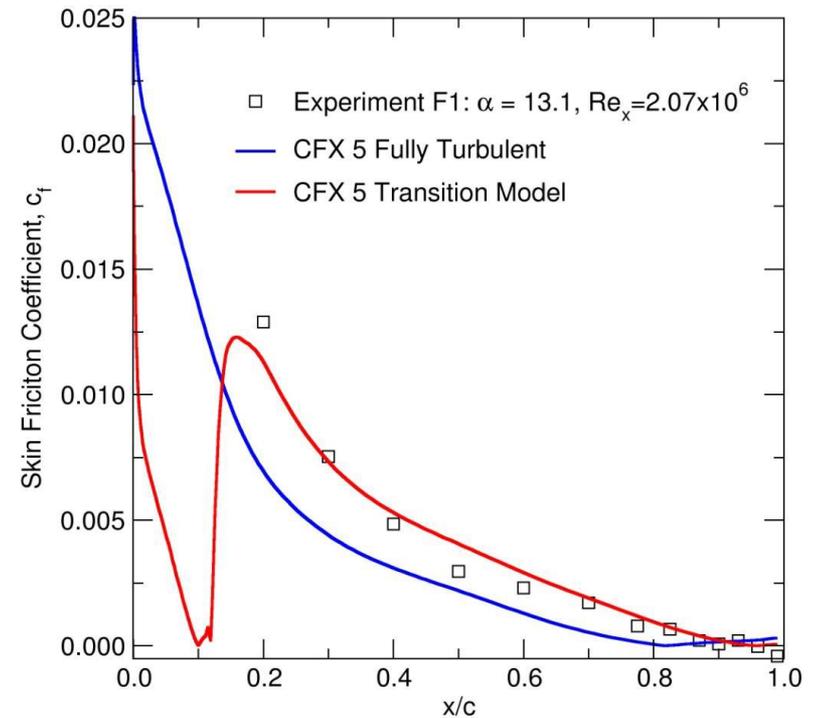
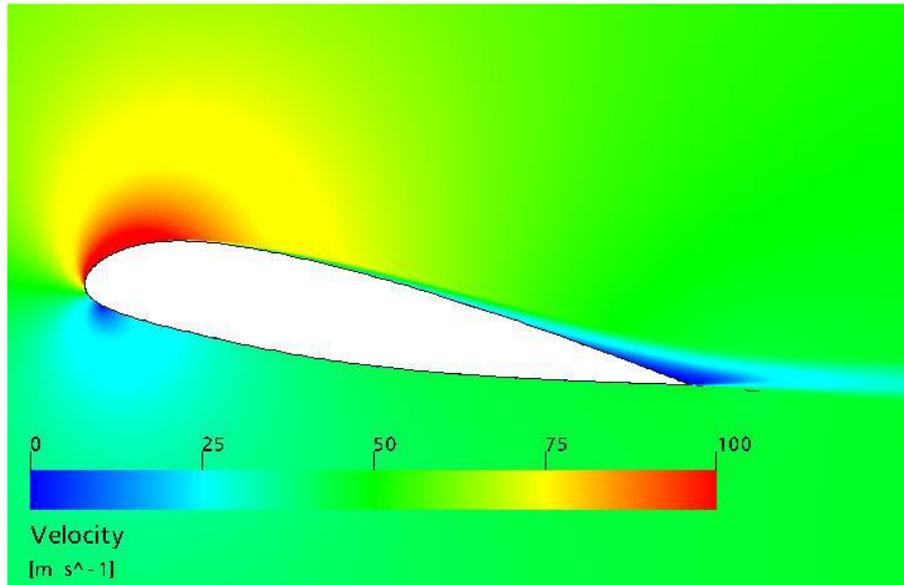
- When transition model is on, the boundary layer is re-energized at the point of separation induced transition and result in smaller turbulent separation zones

References:

F. Menter. RANS Transition Modelling Using Transport Equations I. ERCOFTAC Course on Transition Modelling III, 2015.

Transition models – Review of the $\gamma - Re_{\theta}$ model

- Aerospatiale A Airfoil testcase.

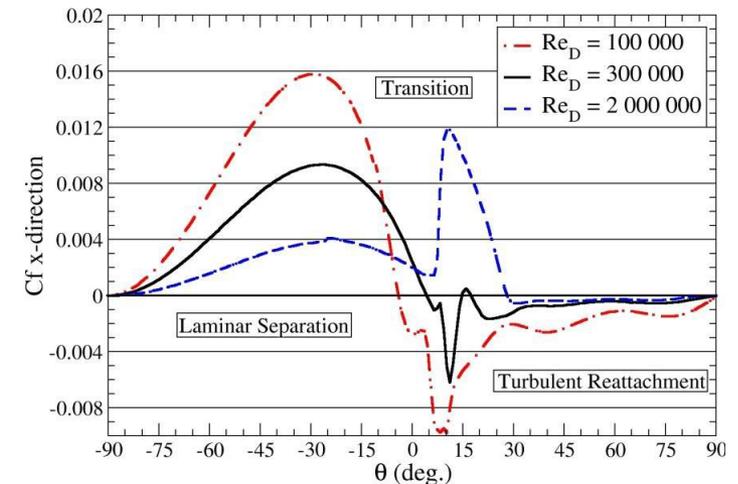
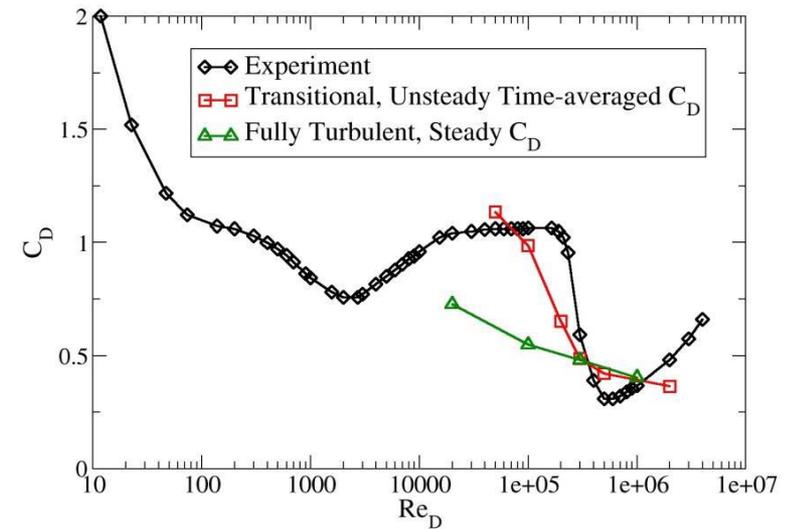
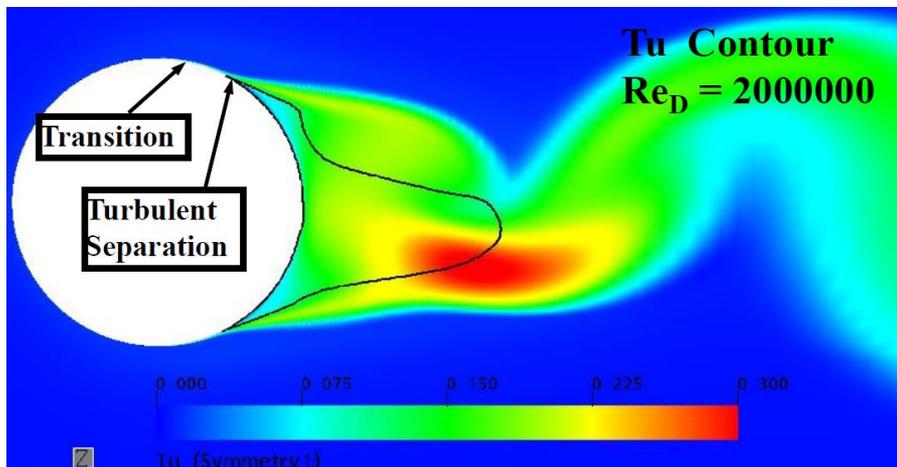
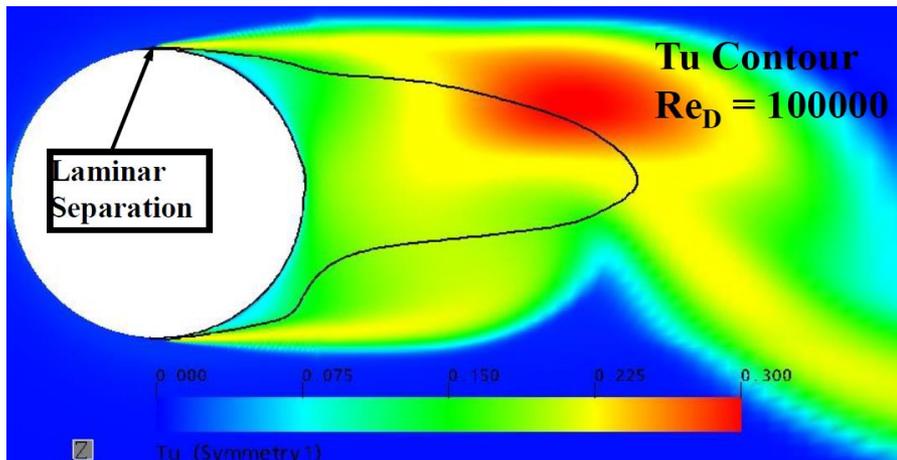


References:

F. Menter. RANS Transition Modelling Using Transport Equations I. ERCOFTAC Course on Transition Modelling III, 2015.

Transition models – Review of the $\gamma - Re_{\theta}$ model

- Flow about a cylinder – Reynolds number in the drag crisis regime

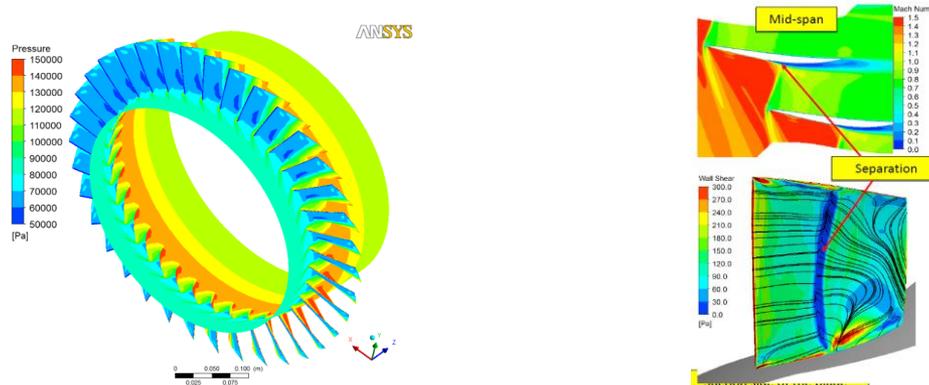


References:

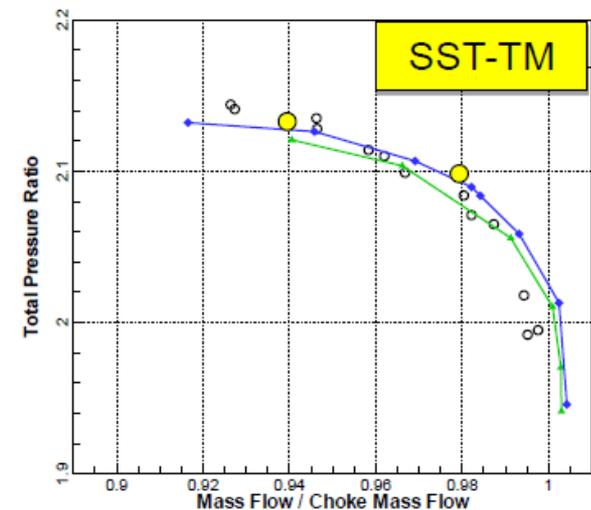
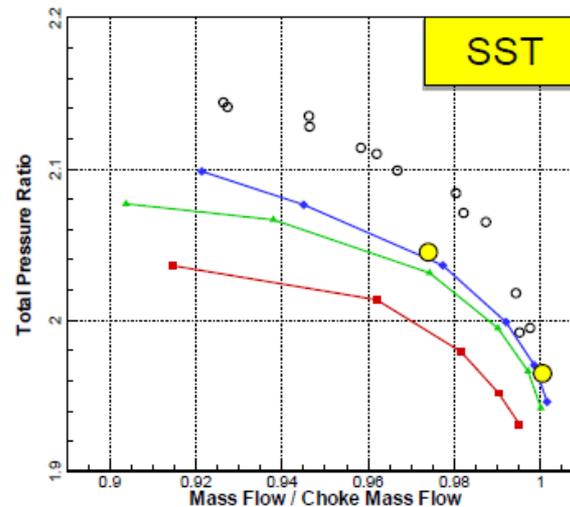
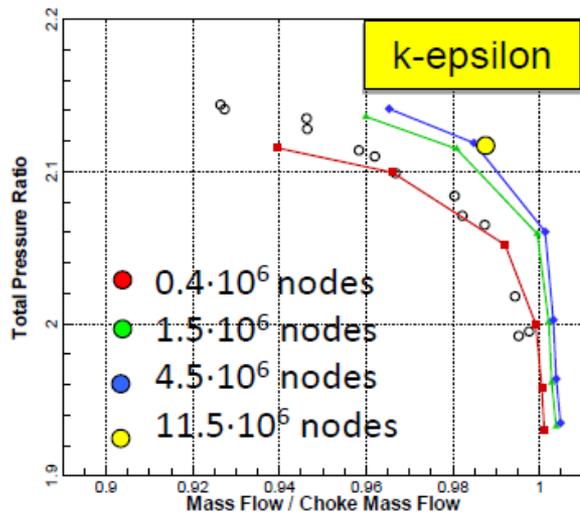
F. Menter. RANS Transition Modelling Using Transport Equations I. ERCOFTAC Course on Transition Modelling III, 2015.

Transition models – Review of the $\gamma - Re_{\theta}$ model

- NASA Rotor 37 test case.



Total Pressure Ratio



References:

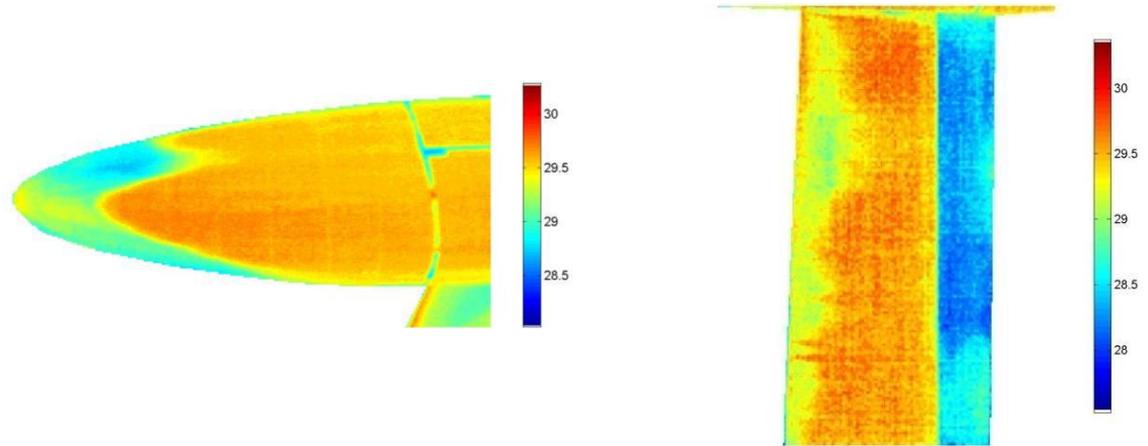
F. Menter. RANS Transition Modelling Using Transport Equations I. ERCOFTAC Course on Transition Modelling III, 2015.

Transition models – Review of the $\gamma - Re_{\theta}$ model

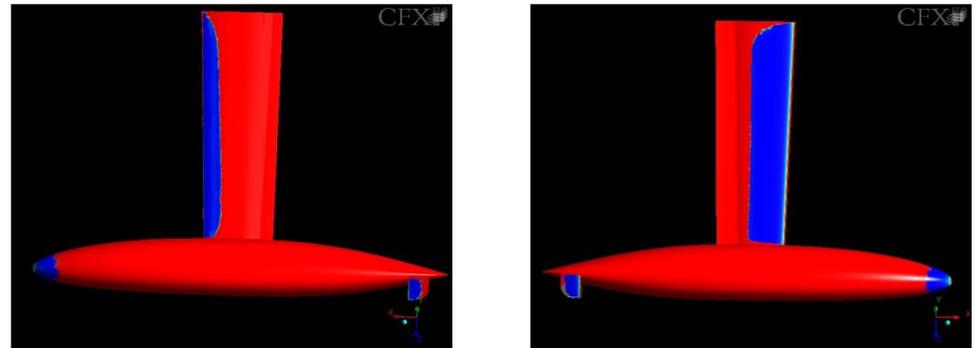
- Sailing application – Team Alinghi.



Wind Tunnel Thermography



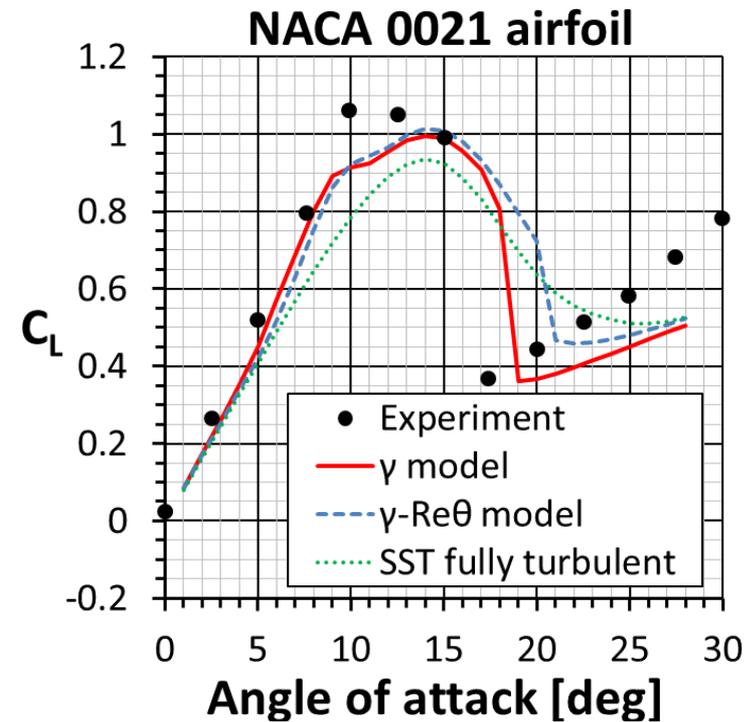
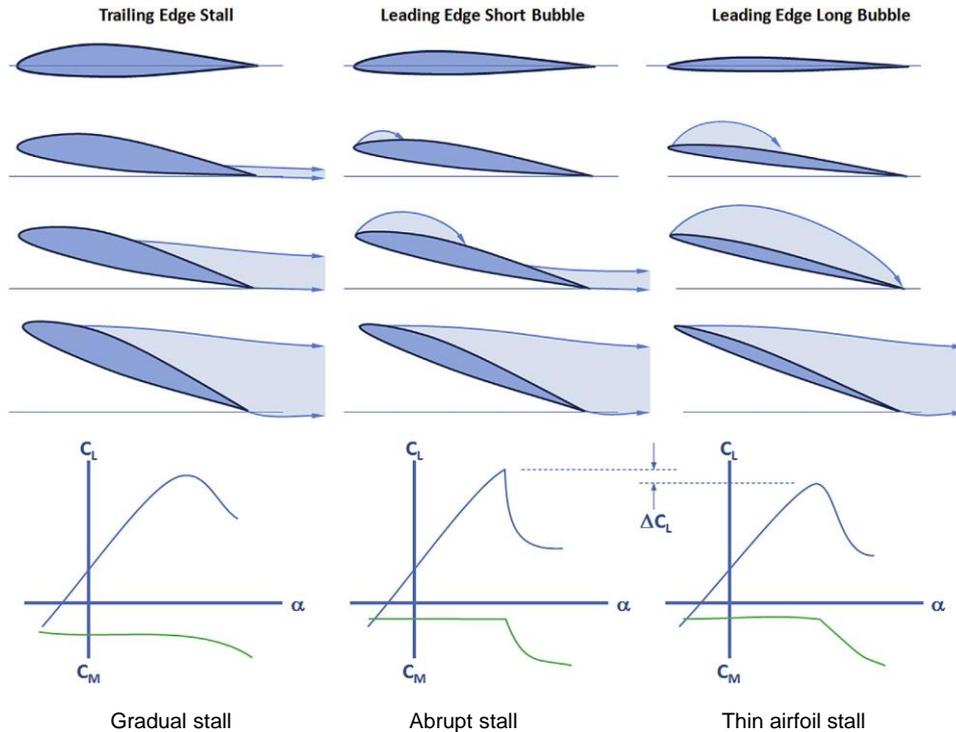
CFD results using a transitional turbulence model



References:

F. Menter. RANS Transition Modelling Using Transport Equations I. ERCOFTAC Course on Transition Modelling III, 2015.

Transition models – Review of the $\gamma - Re_{\theta}$ model



- CFD has been around since the late 1970s, and after all these years is not that easy to compute the flow around 2D airfoils.
- Predicting the maximum lift and stall characteristics is not trivial.
- Recall from aerodynamics that stall is a consequence of the formation of a large separation region located between the leading edge and trailing edge of the airfoil.
- And this large separation region starts from laminar separation bubbles.
- When dealing with these cases, transition models give much better results than traditional turbulence models.
- Note that traditional turbulence models (in this case the kw-SST), captures a smooth behavior in the stall region.

References:

F. Menter. A New Generalized k-omega model. Putting flexibility into Turbulence models (GEKO), Ansys Germany

Transition models – Review of the $\gamma - Re_{\theta}$ model

- In the aerospace industry, all this research in transition to turbulence modeling has conducted to hybrid laminar-flow control technology (HLFC), which is expected to be used in every new large transport aircraft in order to reduce fuel consumption.
- The idea is to create suction (with low power consumption and high reliability) to maintain the flow laminar and delay transition.



- HLFC technology currently is being used in the Boeing 787 and Airbus 350 wide body aircraft.
- Boeing and Airbus do not give much details on how the HLFC system works or how much drag reduction is achieved.
- However, they confirm that the HLFC system uses suction to delay the transition of the airflow boundary layer from laminar flow into turbulent flow.
- This technology is mainly used in the horizontal and vertical stabilizers that produces an unusual large drag.

