Turbulence and CFD models: Theory and applications

Roadmap to Lecture 6

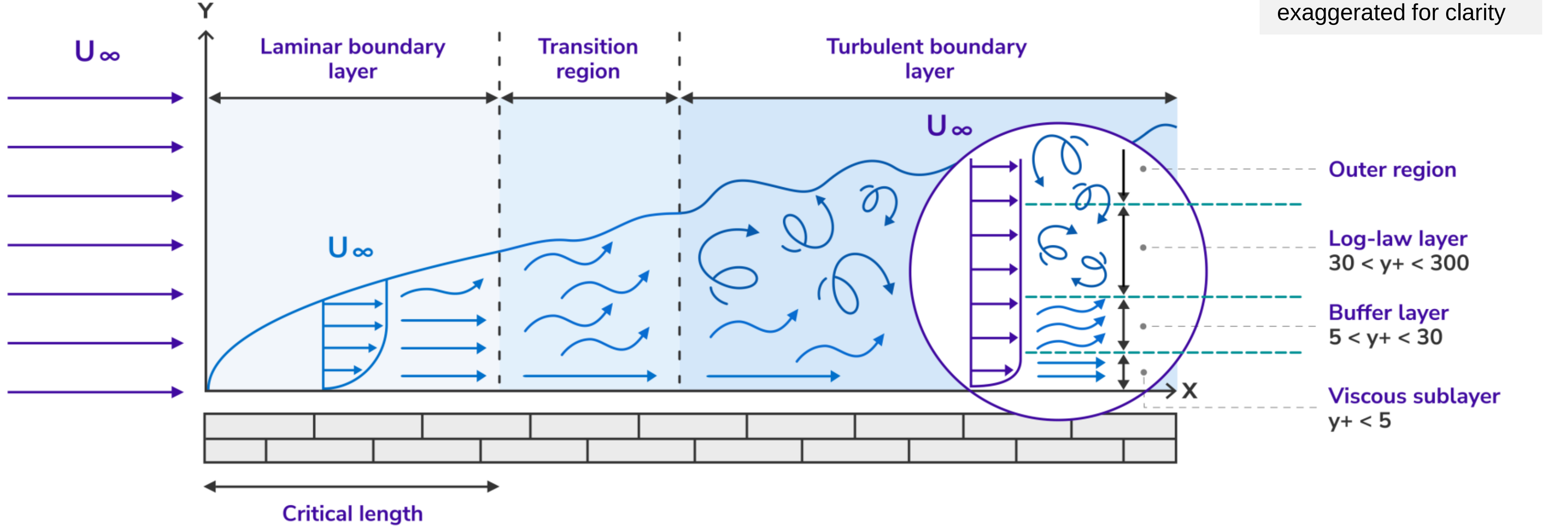
Part 4

1. **Near wall treatment**

Near wall treatment

Turbulence near the wall – Boundary layer

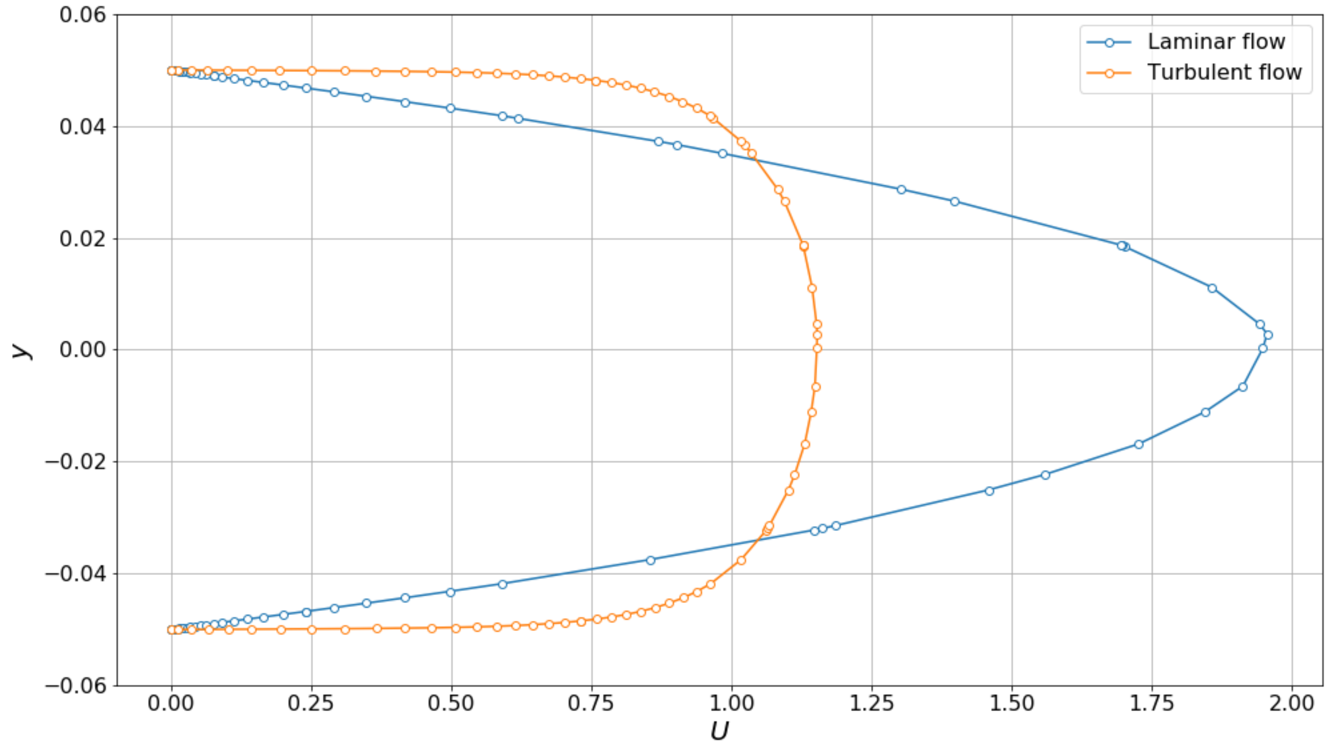
Note: The scales are exaggerated for clarity



- Walls are the main source of turbulence generation in most engineering applications.
- The presence of walls imply the existence of boundary layers.
- In the boundary layer, large gradients exist (velocity, temperature, and so on).
- To properly resolve these gradients, we need to use very fine meshes close to the walls.

Near wall treatment

- Comparison of laminar and turbulent velocity profiles in a pipe.
- As it can be observed, close to the walls the velocity gradient is larger in the turbulent case.
- Therefore, fine meshes are required in order to properly resolve the steep gradients (velocity, temperature, etc.) close to the walls.



Near wall treatment

- To resolve the flow close to the walls two approaches can be used:
  - **Wall resolving approach.**
  - **Wall modeling approach.**

- In the **wall resolving approach**, the equations are integrated down to the viscous sublayer.
  - This approach allows for the accurate computation of steep gradients of velocity and transported quantities near the walls.
  - However, it is computationally expensive as it requires very fine meshes close to the walls.

Near wall treatment

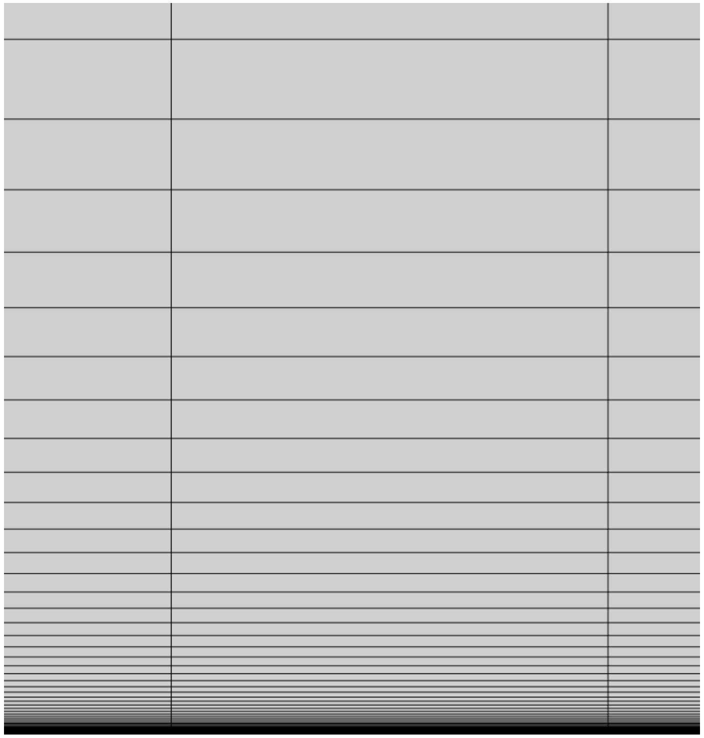
- To resolve the flow close to the walls two approaches can be used:
  - **Wall resolving approach.**
  - **Wall modeling approach.**

- In the **wall modeling approach**, the equations are solved a distance away from the wall, in the log-law region.
  - In this approach, we apply boundary conditions based on log-law relations some distance away from the wall, so we do not need to resolve the viscous sublayer.
  - The reduction of the computational overhead, because we are using a coarser mesh, is a compelling reason to support the use of wall functions in CFD.
  - However, under many flow conditions, such as, strong adverse pressure gradient, separated and impinging flows, strongly anisotropic flows, and so on; their physical basis is uncertain, and their accuracy is questionable.

Near wall treatment

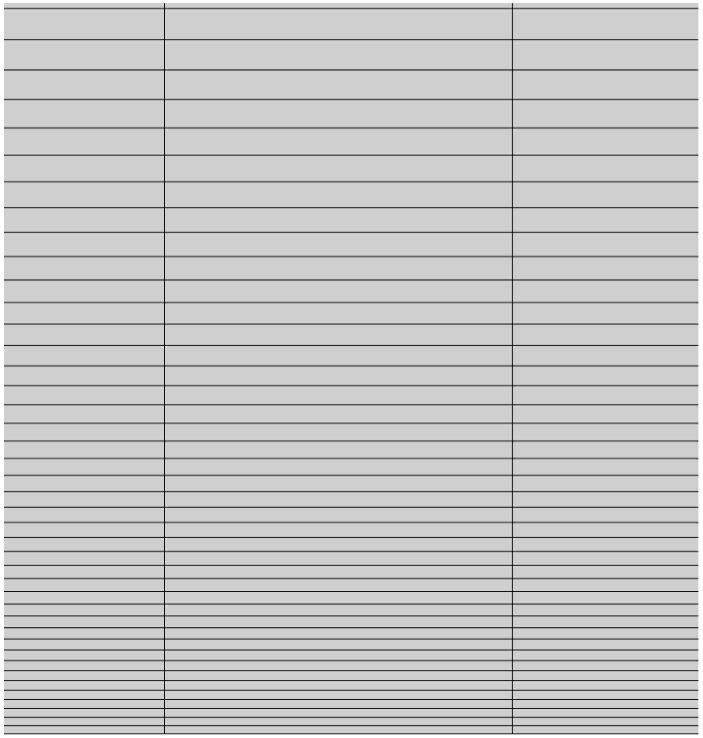
Wall resolving mesh

- Wall resolving meshes resolve the boundary layer down to the viscous sublayer.
- These meshes allow for the accurate computation of steep gradients of velocity and transported quantities near the walls.
- The only drawback is that you will require a lot of cells close to the walls.



Wall modeling mesh

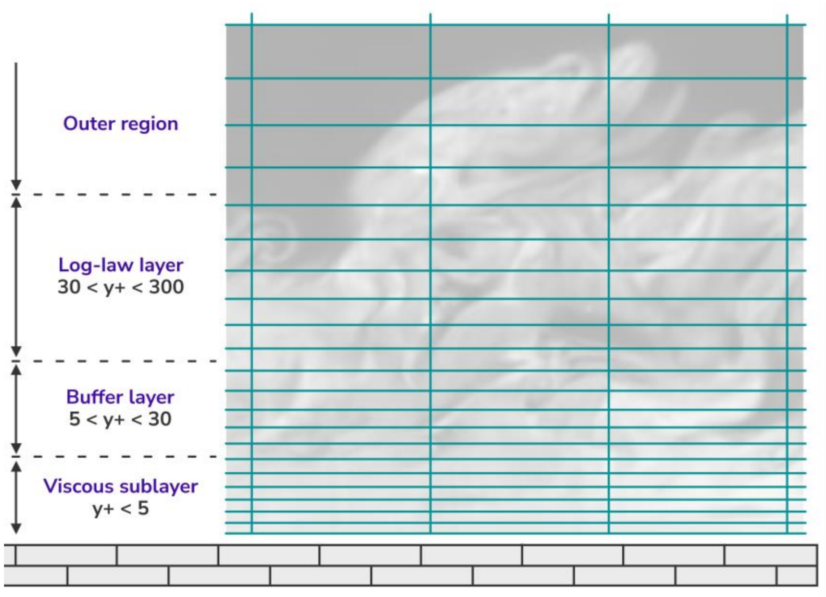
- In wall modeling meshes we apply boundary conditions some distance away from the wall, so we do not need to resolve the viscous sublayer.
- The goal is to use coarser meshes without losing accuracy.
- In the cell next to the wall, the field quantities and wall shear stresses are approximated using correlations (*e.g.*, log-law layer).

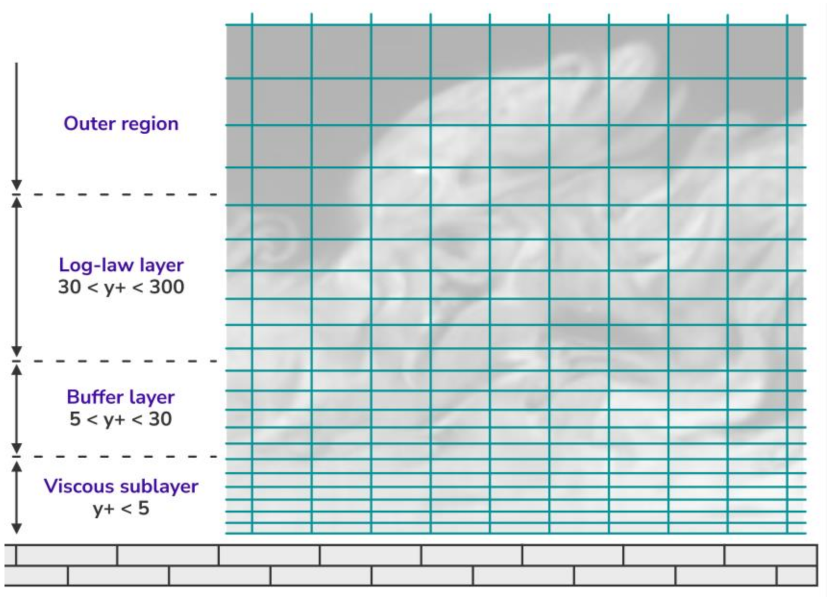


Near wall treatment

- Resolving the streamwise and spanwise directions is also important in turbulence modeling, for example,
  - when dealing with transition to turbulence,
  - when conducting LES simulations,
  - when conducting DNS simulations.

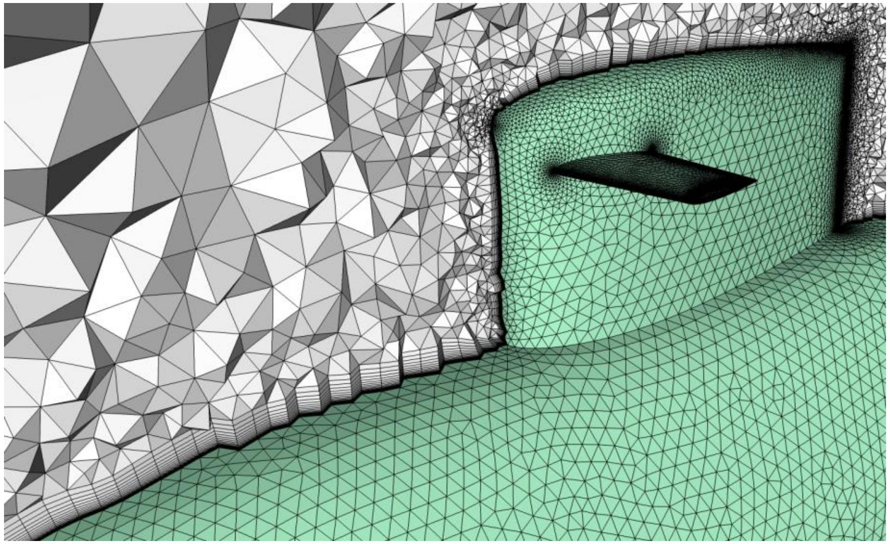
- By looking at the figures below, it is easy to understand that the mesh in the right figure will resolve better the streamwise coherent structures.

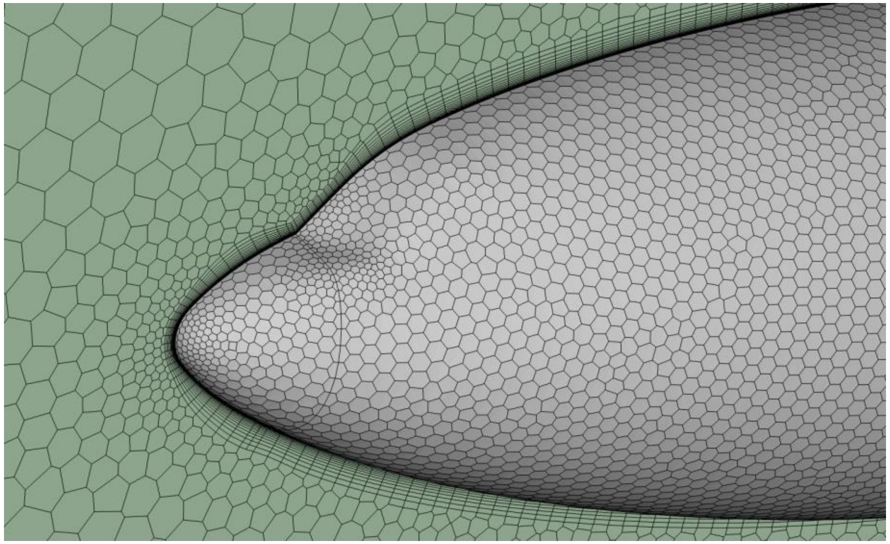




Near wall treatment

- The easiest way to resolve the steep gradients near the walls is by resolving the viscous sublayer.
  - To resolve the viscous sublayer, we need to cluster a lot of cells in the region where y^+ is less than 5.
  - Usually, we need to cluster more than 10 layers in order to accurately resolve the profiles of velocity and the transported quantities.
  - This can significantly increase the cell count.
  - And in the case of unsteady simulations, it can have a significant impact in the time-step.
  - In high accuracy unsteady simulations, a CFL number in the order of one is usually required for stability and accuracy reasons.

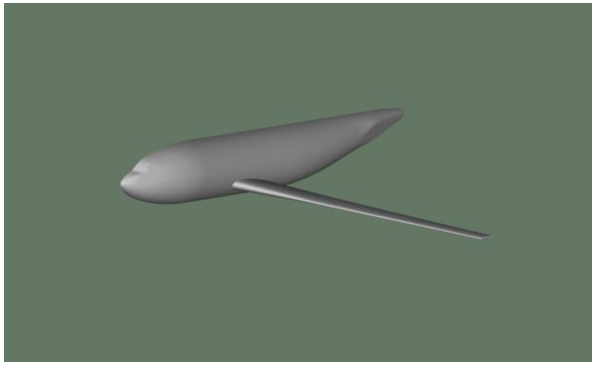


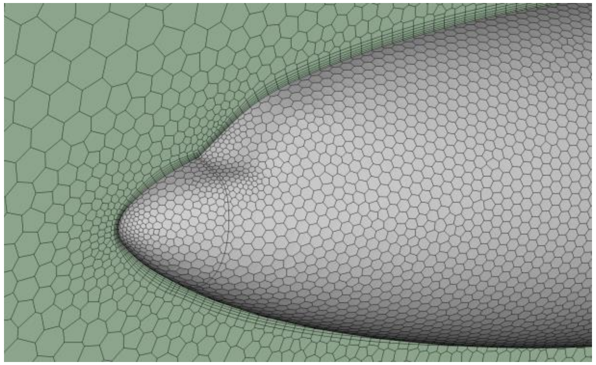


Wall resolving meshes – In both cases, the average y^+ is approximately 1

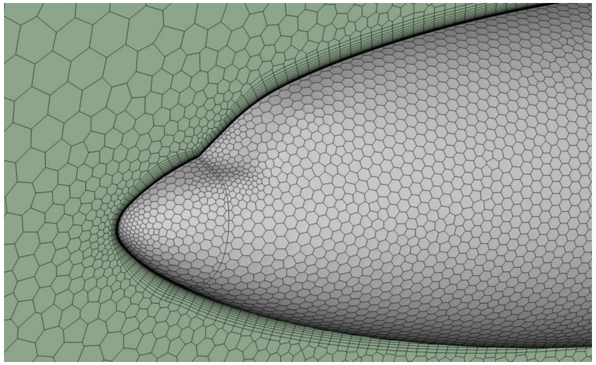
Near wall treatment

- In the wall resolving mesh, we only modified the inflation layers parameters (the prismatic cells close to the walls).
- The parameters of the surface mesh and volume mesh remained the same.
- By taking a mesh resolving approach, we almost doubled the cell count.





Wall modeling mesh
Average y^+ approximately 60

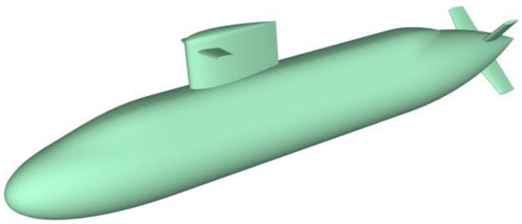


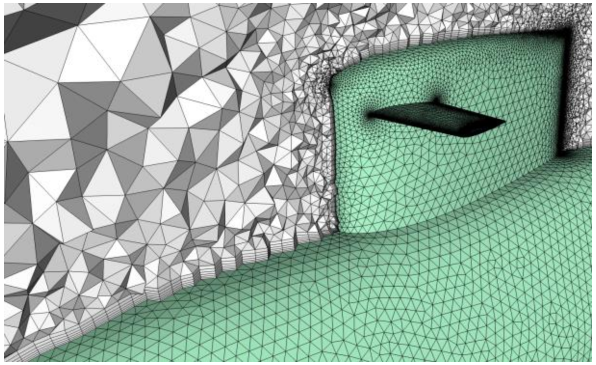
Wall resolving mesh
Average y^+ approximately 1

	Wall modeling mesh	**Wall resolving mesh**
Number of cells	649 619	1 160 135

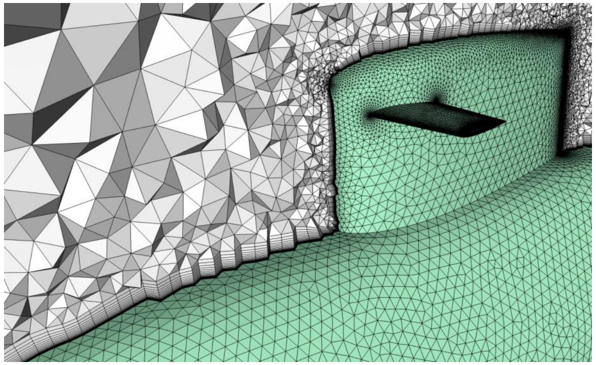
Near wall treatment

- In the wall resolving mesh, we only modified the inflation layers parameters (the prismatic cells close to the walls).
- The parameters of the surface mesh and volume mesh remained the same.
- By taking a mesh resolving approach, we almost doubled the cell count.





Wall modeling mesh
Average y^+ approximately 60

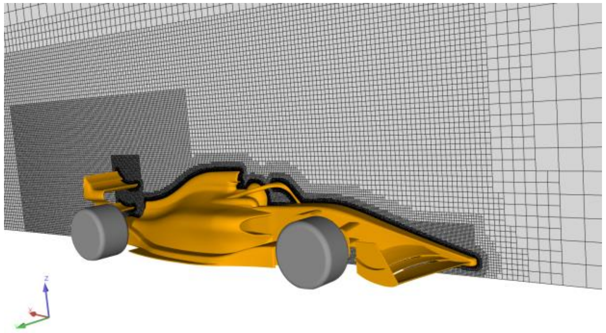


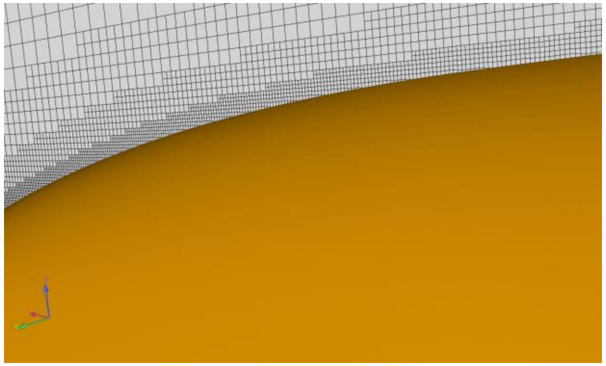
Wall resolving mesh
Average y^+ approximately 1

	Wall modeling mesh	Wall resolving mesh
Number of cells	6 613 049	11 149 266

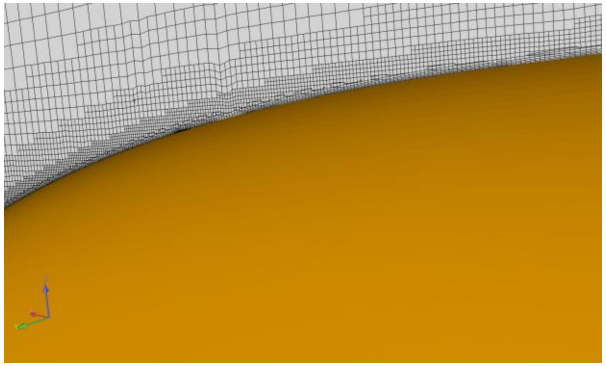
Near wall treatment

- In the wall resolving mesh, we only modified the inflation layers parameters (the prismatic cells close to the walls).
- The parameters of the surface mesh and volume mesh remained the same.
- By taking a mesh resolving approach, we almost doubled the cell count.





Wall modeling mesh
Average y^+ approximately 60

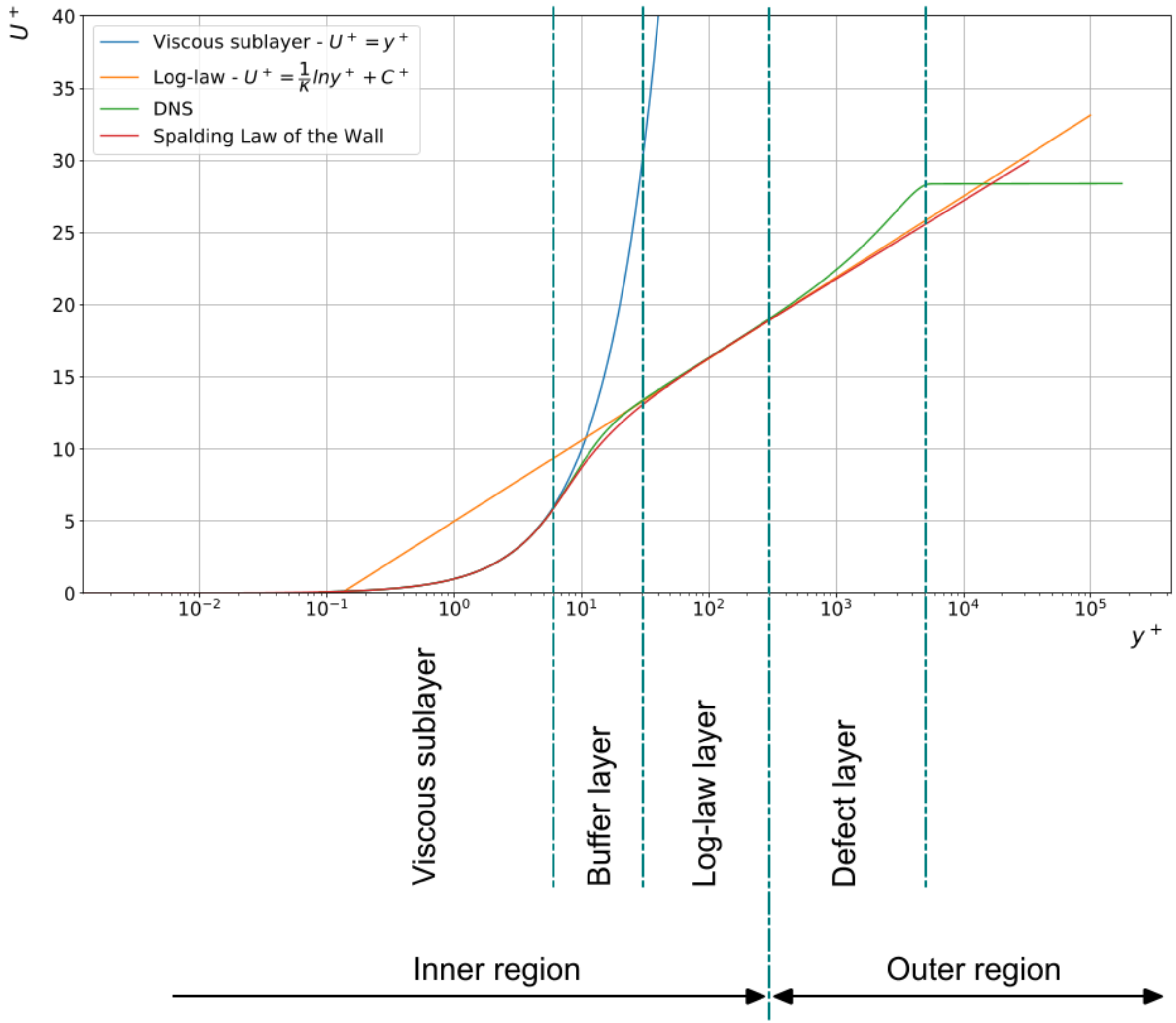


Wall resolving mesh
Average y^+ approximately 1

	Wall modeling mesh	Wall resolving mesh
Number of cells	57 853 037	111 137 673

Near wall treatment

Turbulence near the wall – Relations according to the y^+ value



Viscous sublayer

$$y^+ < 5$$

$$u^+ = y^+$$

Buffer layer

$$5 < y^+ < 30$$

$$u^+ \neq y^+$$

$$u^+ \neq \frac{1}{\kappa} \ln y^+ + C^+$$

Log-law layer

$$30 < y^+ < 300$$

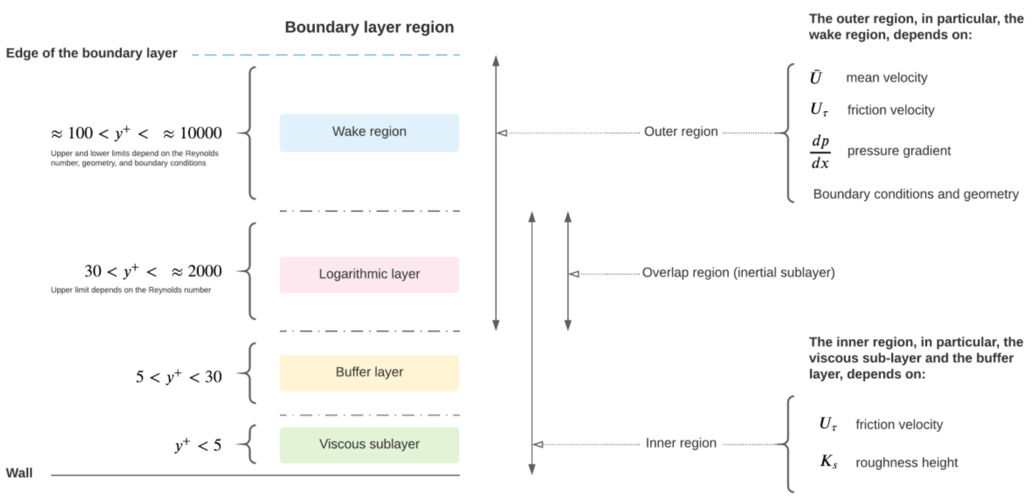
$$u^+ = \frac{1}{\kappa} \ln y^+ + C^+$$

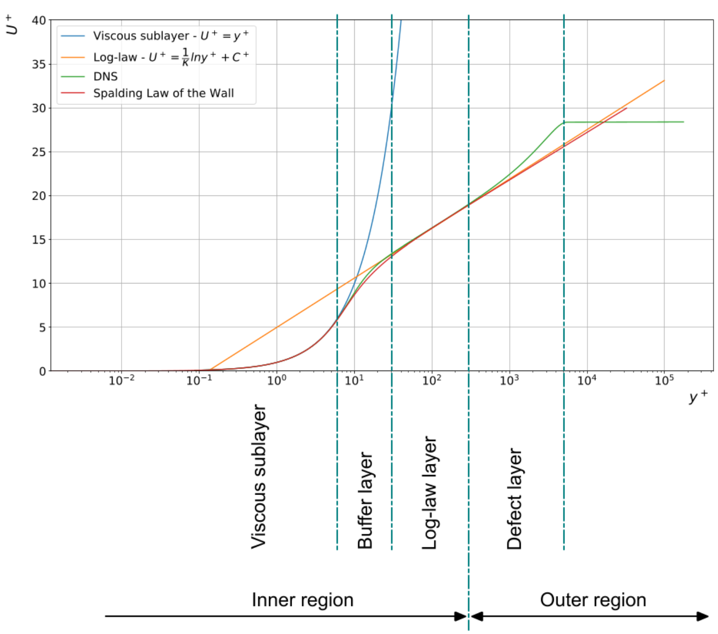
$$\kappa \approx 0.41 \quad C^+ \approx 5.0$$

Note: the range of y^+ values might change from reference to reference but roughly speaking they are all close to these values.

Turbulence near the wall – The boundary layer and the Law of the wall

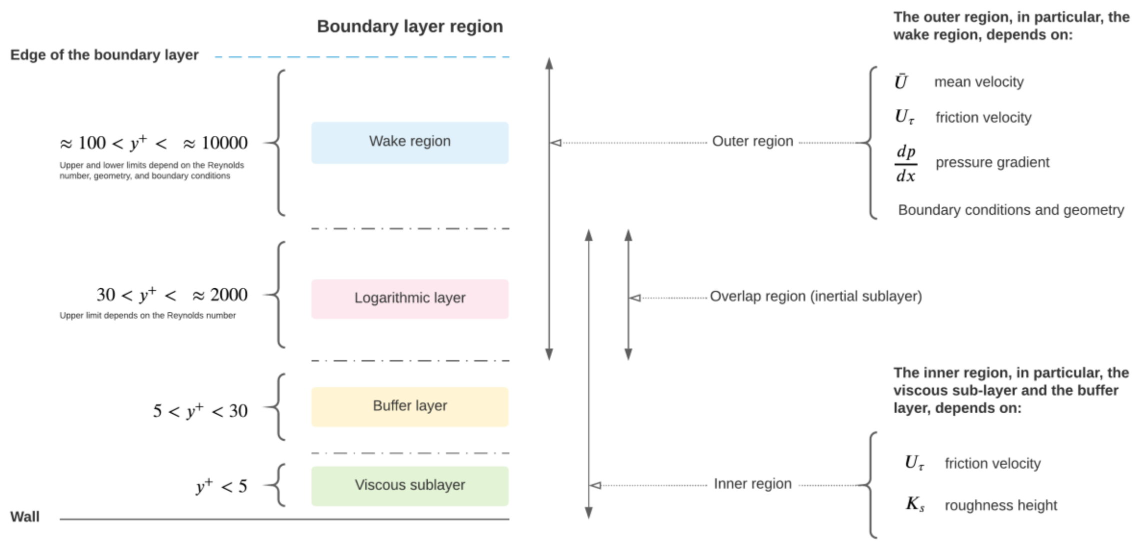
- The velocity profile near the wall can be represented by using the previous non-dimensional quantities and correlations.
- By using non-dimensional quantities, the flow behavior near the wall is independent of the Reynolds number, geometry, or relevant physics (to some extent).
- The correlations take a very predictable behavior close to the walls for a wide variety of flows.
- The outer or mean flow, depends on the geometry, boundary conditions, physics, and so on.





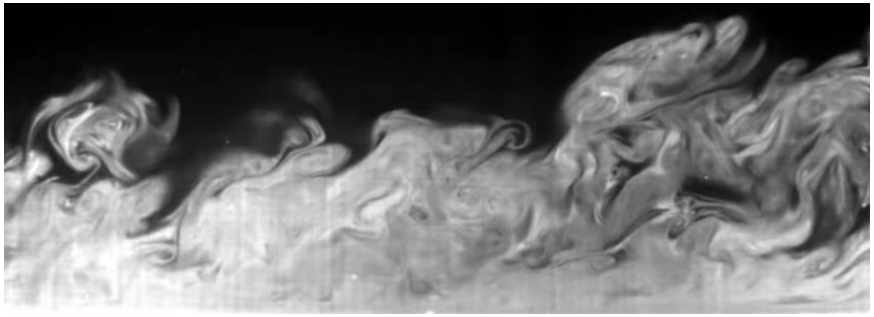
Near wall treatment

Turbulence near the wall – The boundary layer and the Law of the wall

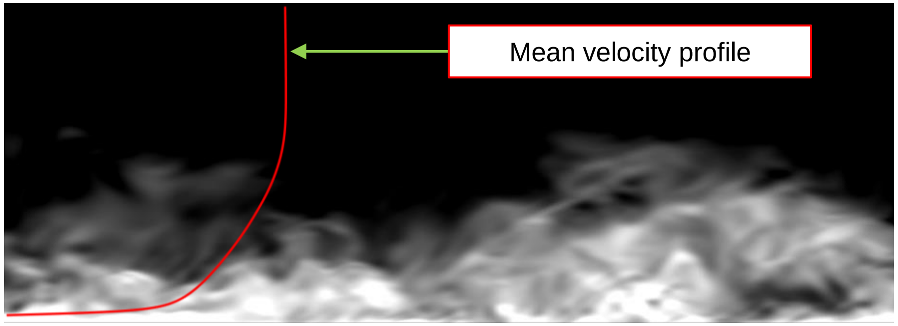


Regions in the turbulent boundary layer.
Adapted from references [1, 2].
The figure is not to scale. The figure does not scale in reference to the images below.

Turbulent boundary layer on a flat plate.



Experimental results [3].

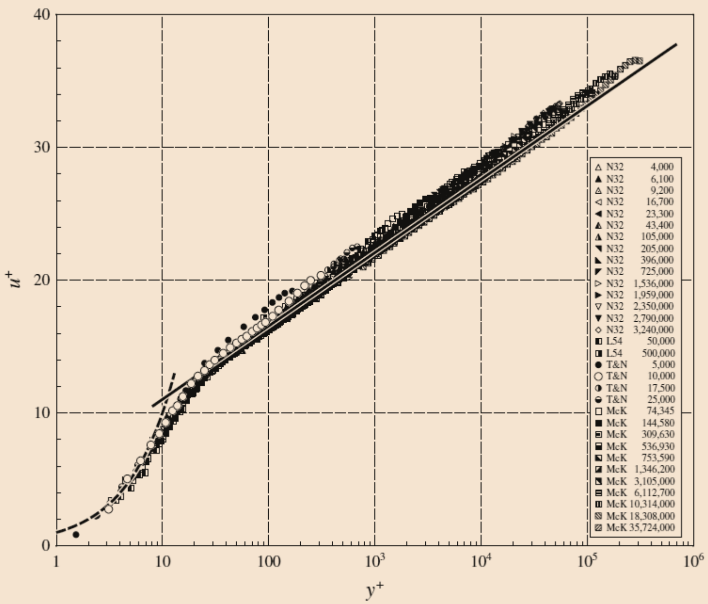


DNS simulation. Instantaneous velocity field [4].

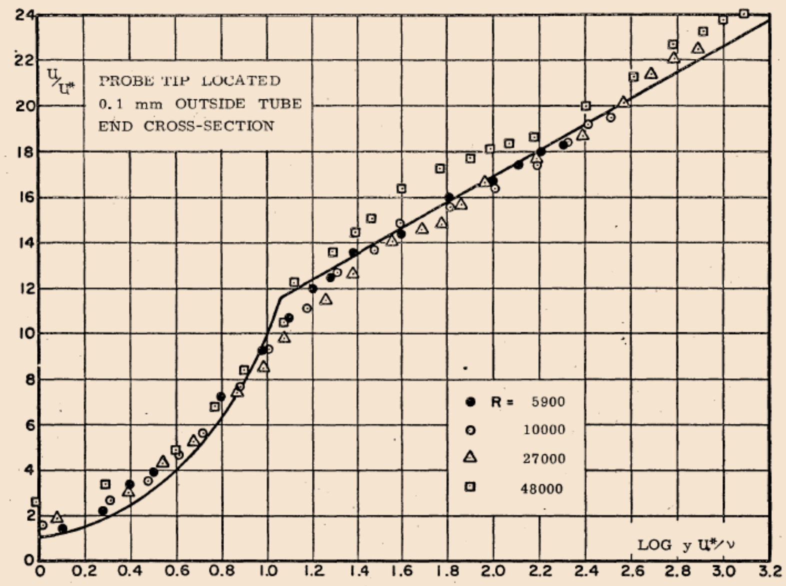
[1] F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer, 2016.
[2] S. Pope. Turbulent Flows, Cambridge University Press, 2000.
[3] Photo credit: https://arxiv.org/abs/1210.3881. Copyright on the images is held by the contributors. Apart from Fair Use, permission must be sought for any other purpose.
[4] J. Lee, T. Zaki. Transitional boundary layer dataset. http://turbulence.pha.jhu.edu/Transition_bl.aspx

Near wall treatment

- The law of the wall, is one of the cornerstones of fluid dynamics and turbulence modeling.
- The logarithmic law, refers to the region of the inner-region of the boundary layer that can be described using a simple analytic function in the form of a logarithmic equation.
- This is one of the most famous empirically determined relationships in turbulent flows near solid boundaries.
- Measurements show that, for both internal and external flows, the streamwise velocity in the flow near the wall varies logarithmically with distance from the surface.



Dimensionless mean velocity profile u^+ as a function of the dimensionless wall distance y^+ for turbulent pipe flow with Reynolds numbers between 4000 and 3600000 [1].

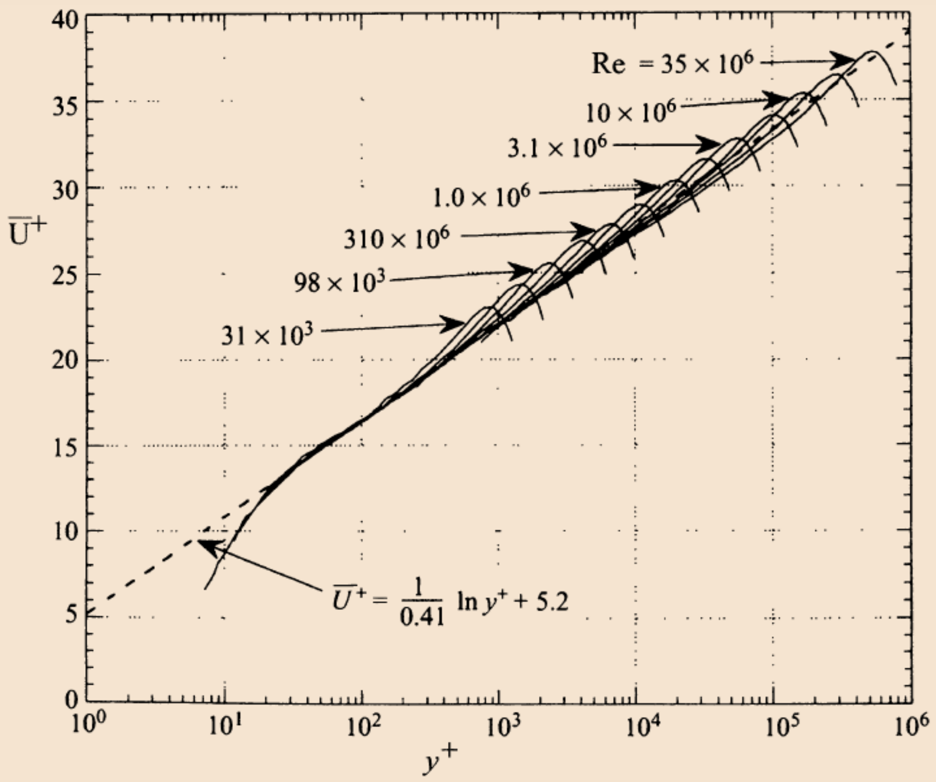


Nondimensional velocity profile. Experimental measurements [2].

[1] F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer. 2016.
[2] E. Lindgren. Experimental study on turbulent pipe flows of distilled water. Oklahoma State University, AD621071, Technical Report No. 2, 1965.

Near wall treatment

- In the literature, you will find that wall functions are valid for $30 < y^+ < 300$.
- This is the most common range that you will find in the literature but have in mind that different authors might define different values.
- The range $30 < y^+ < 300$ is fine for most applications.
- It is a subject of discussion the upper limit of the log-law layer.
- Most of the times you will find in the literature a y^+ upper limit of 300.
- In reality, this upper limit depends on the Reynolds number, as shown in the figure.
- For high Reynolds number, the overlap region is large. The upper limit can be as high as 2000 or more.
- Whereas, for lower Reynolds number, the overlap region is shorter.
- And this imposes a limit on the usability of wall functions for lower Reynolds numbers, as it becomes very difficult to cluster enough computational cells in the log-law region to resolve the profiles.
- If the y^+ upper limit is below or close to 100, it is better to use a wall resolving approach.
- The value of the lower limit is generally accepted to be equal to 30.

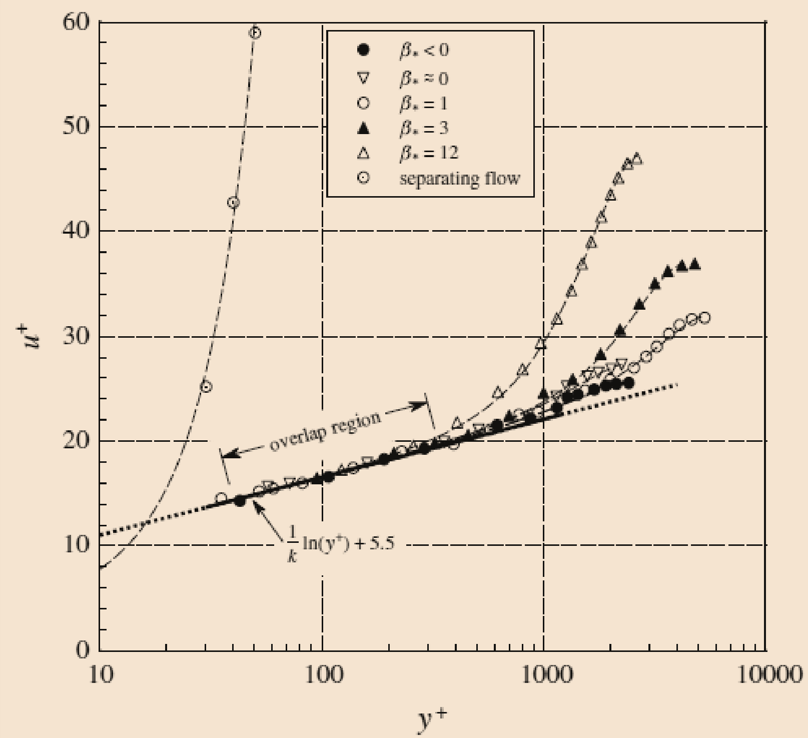


U+ vs. y+ plot for high-Reynolds number experiments [1].

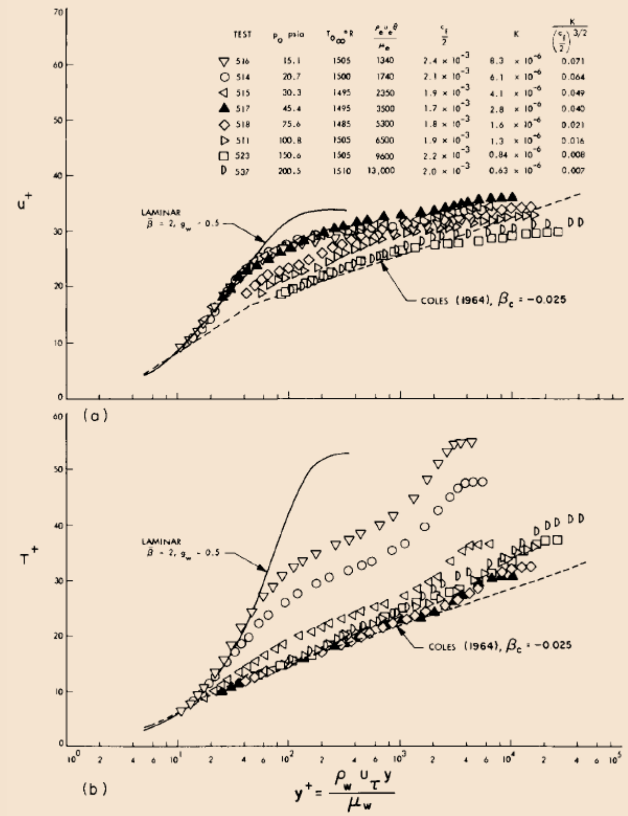
[1] M. Zagarola, A. Smits. Mean flow scaling in turbulent pipe flow. J. Fluid Mech. 373. 1998.

Near wall treatment

- As shown in the figures, the pressure gradient has an influence on the velocity profile.
- Therefore, if we are planning to use wall functions, it is strongly recommended to use wall functions that use corrections to take into account this effect.



Dimensionless mean velocity profile u^+ as a function of the dimensionless wall distance y^+ for various values of the pressure gradient , expressed in terms of the Clauser parameter β_* [1].

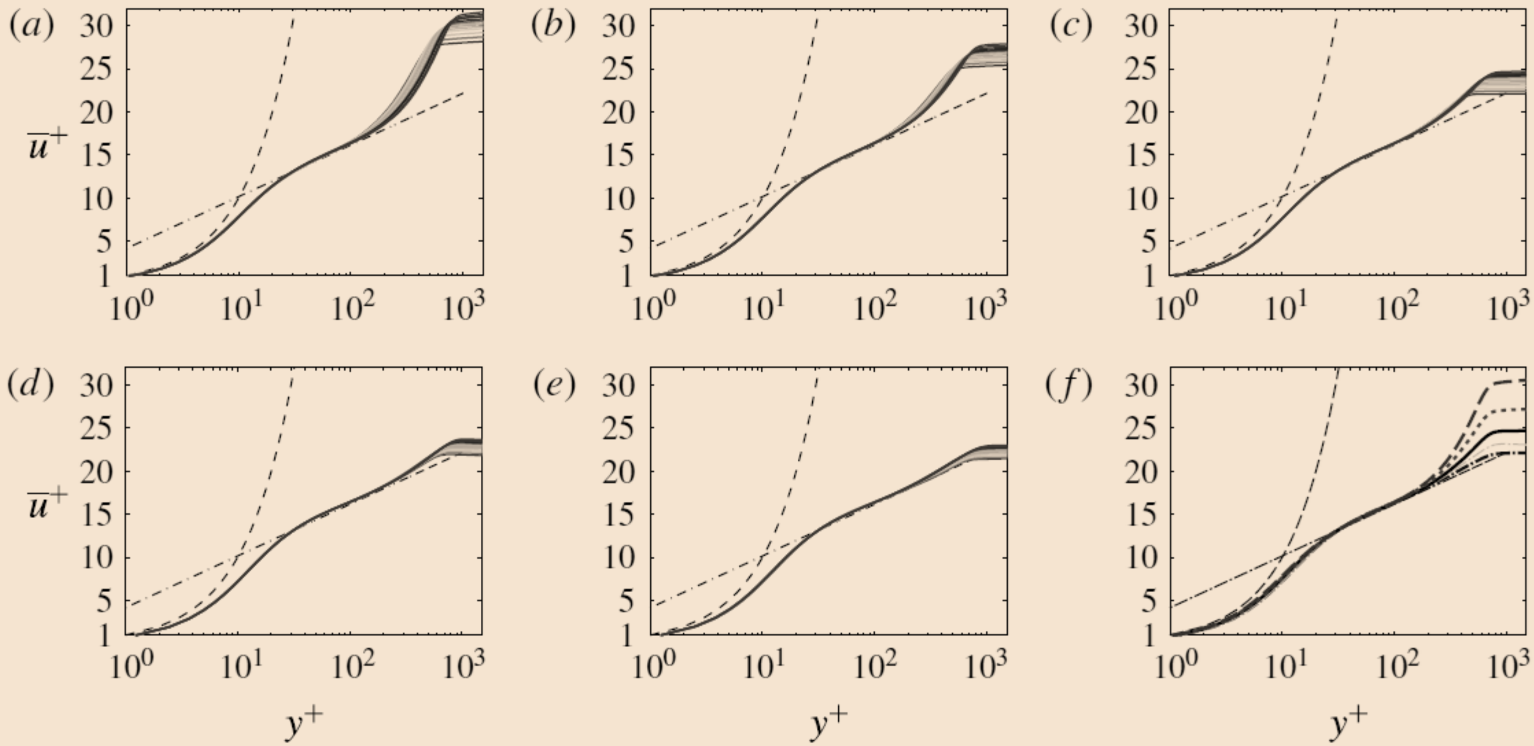


Effect of strong flow acceleration on the (a) velocity profiles and (b) temperature profiles measured in a nozzle. [2].

[1] F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer. 2016.
[2] T. Cebeci, A. M. O. Smith. Analysis of Turbulent Boundary Layers. Academic Press, 1974.

Near wall treatment

- As shown in the figures, the pressure gradient has an influence on the velocity profile.
- Therefore, if we are planning to use wall functions, it is strongly recommended to use wall functions that use corrections to take into account this effect.



Mean velocity profiles in wall units u+. Thirteen profiles between xmin and xmax of the domain. (a) APGs (strong adverse pressure gradient). (b) APGw (weak adverse pressure gradient). (c) ZPG (zero pressure gradient). (d) FPGw (weak favorable pressure gradient). (e) FPGs (strong favorable pressure gradient). (f) Comparison of the five cases at similar friction Reynolds number [1].

[1] E. Cohen, X. Gloerfelt. Influence of pressure gradients on wall pressure beneath a turbulent boundary layer. Journal of Fluid Mechanics, volume 838, pp. 715-758, 2018.

Near wall treatment

- Before addressing how to compute the flow close to the walls, let us summarize all the non-dimensional variables near the walls.

Wall shear stresses

$$U_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

Shear velocity

Velocity parallel to the wall

$$u^+ = \frac{U}{U_\tau}$$

Non-dimensional near the wall velocity

Distance normal to the wall

$$y^+ = \frac{yU_\tau}{\nu}$$

Non-dimensional distance from the wall

- Close to the walls we only know the wall shear stress, viscosity, and distance,

$$U = f\left(\tau_w, \rho, \mu, y\right)$$

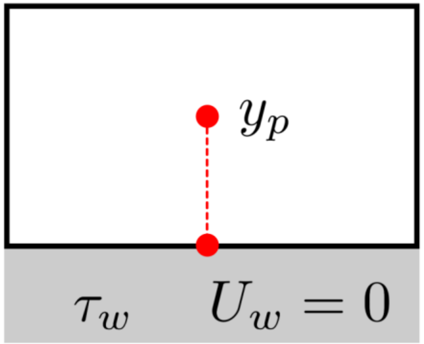
- Therefore, we use these quantities to create the non-dimensional groups.

Near wall treatment

- If we are dealing with globally laminar flows, we can compute the wall shear stress as follows,

In the viscous sublayer or with laminar flows we use the molecular viscosity

$$\tau_w = \mu \frac{\partial U}{\partial y} = \mu \frac{U_p - 0}{y_p} = \mu \frac{U_p}{y_p}$$



- In our notation, the subscript p indicates values at the cell center and the subscripts w indicates values at the walls
- Remember, some solvers use cell-centered quantities, and some solvers use node-centered quantities.
- From now on, we are going to assume that all quantities are computed at the cell center.
- Sometimes in this approach, damping functions are added to gain robustness.

Near wall treatment

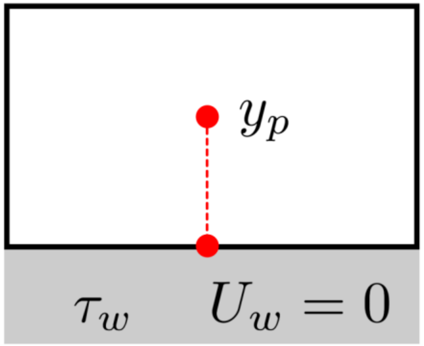
- If the same way, if we are dealing with turbulent flows, and the mesh fine enough to resolve the viscous sublayer, we can compute the wall shear stress in the same way as for laminar flows.
- After all, we are resolving the viscous sublayer, which is laminar.

In the viscous sublayer or with laminar flows we use the molecular viscosity

$$\tau_w = \mu \frac{\partial U}{\partial y} = \mu \frac{U_p - 0}{y_p} = \mu \frac{U_p}{y_p}$$

Recall that in turbulent flows we use the effective viscosity

$$\mu_{eff} = \mu_{molecular} + \mu_{turbulent}$$



- In our notation, the subscript p indicates values at the cell center and the subscripts w indicates values at the walls
- Remember, some solvers use cell-centered quantities, and some solvers use node-centered quantities.
- From now on, we are going to assume that all quantities are computed at the cell center.
- Sometimes in this approach, damping functions are added to gain robustness.

Near wall treatment

- We just described how to compute the wall shear stresses in laminar flows and in turbulent flows using the wall resolving approach.
- As you can see, this approach is straight forward to implement.
- Let us address how to compute the wall shear stresses using the wall modeling approach.
- If we are dealing with turbulent flows and if we are using a coarse mesh such that $y^+ > 30$, using the previous relations to compute the wall shear stresses is not accurate anymore.
- We are missing a lot of gradient information if we use the previous approach, namely,

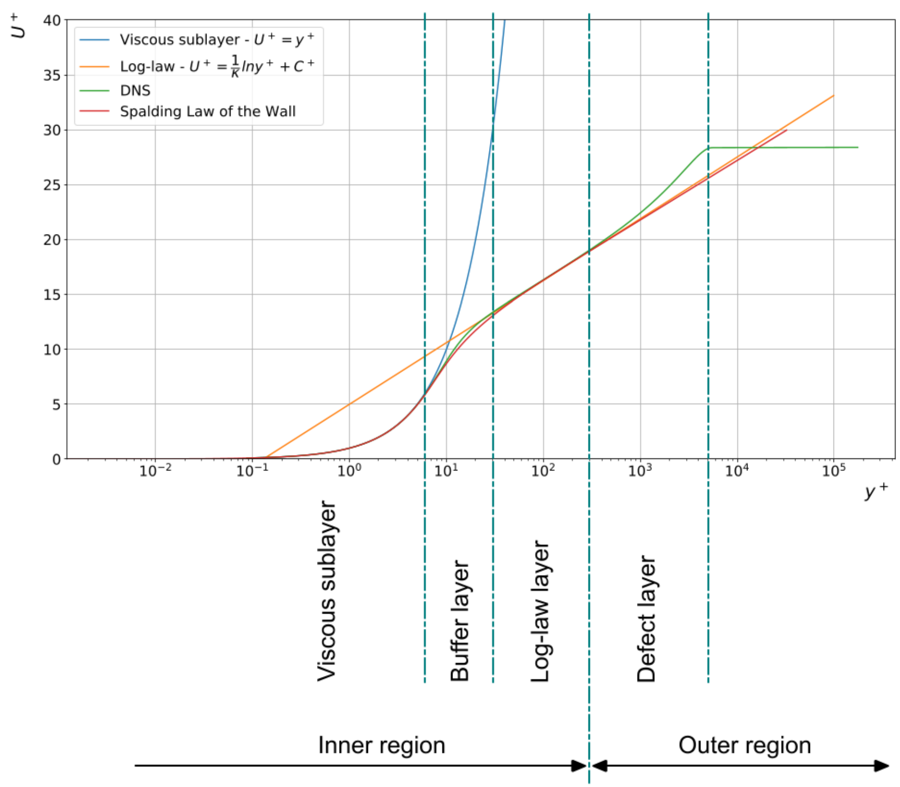
$$\tau_w = \mu \frac{\partial U}{\partial y} = \mu \frac{U_p - 0}{y_p} = \mu \frac{U_p}{y_p}$$

- This is not accurate if $y^+ > 10$.
- This value roughly corresponds to the intersection of the viscous sublayer law and the log-law

- In the wall modeling approach, we need to somehow correct this computation.
- We do this by constructing a bridge between the wall values and the log-law correlations.
- The question is?
  - How do we transfer information from the empirical correlations to the walls and to the flow?
  - How do we compute the wall shear stresses?

Near wall treatment

- If the first cell center is in log-law layer, we cannot use the viscous sublayer relationship because it is too inaccurate. Therefore, we need to use wall functions.
- By using wall functions, we can use empirical correlations to bridge wall conditions to the log-law layer.
- The correlations provide a link between U and U_τ (or τ_w).



Log-law layer

$$30 < y^+ < 300$$

$$u^+ = \frac{1}{\kappa} \ln y^+ + C^+$$

$$\kappa \approx 0.41 \qquad C^+ \approx 5.0$$

Viscous sublayer

$$y^+ < 5$$

$$u^+ = y^+$$

Buffer layer

None of the previous correlations apply

Note: the range of y^+ values might change from reference to reference but roughly speaking they are all close to these values.

Near wall treatment

- By using wall functions, we bridge the wall conditions and cell centered values with the log-law empirical correlations.
- The wall functions reduce the computational effort significantly because we do not need to resolve the viscous sublayer.
- Let us explain the standard wall functions using the method proposed by Launder and Spalding [1], which is probably the most widely used method.
- In this approach,

$$u^* = \begin{cases} y^* & \text{In the viscous sublayer} \\ \frac{1}{\kappa}\ln\left(Ey^*\right) & \text{In the log-law layer} \end{cases}$$

In the viscous sublayer ← This equation is not used in the wall function approach that we will discuss, we just wrote it for completeness.

In the log-law layer ← In the wall function formulation that we will discuss [1], we solve this equation which is valid when y+ > 30.
If the y+ value is less than 30 the results deteriorate.

- Notice that we are using u* and y* instead of u^+ and y^+.
- Also, the log-law layer correlation is slightly different from what we have seen so far.
- Let us address these two issues.

[1] B. E. Launder, D. B. Spalding. The Numerical Computation of Turbulent Flows. Computer Methods in Applied Mechanics and Engineering. 1974.

Near wall treatment

- The idea of introducing the new quantity u*, is to avoid the singularity that occurs when the wall shear stress is equal to zero in u^+ (*i.e.*, in a separation point).
- Recall that the shear velocity is equal to,

$$U_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

- In a separation point, the wall shear stress is equal to zero, therefore,

$$U_\tau = 0 \quad \text{if} \quad \tau_w = 0$$

- If the shear velocity is equal to zero, then we have a singularity when computing u^+, and this might pose numerical problems.

$$u^+ = \frac{U}{U_\tau}$$

[1] B. E. Launder, D. B. Spalding. The Numerical Computation of Turbulent Flows. Computer Methods in Applied Mechanics and Engineering. 1974.

Near wall treatment

- The new quantities u* and y+ are defined as follows [1],

$$U_\tau^* = C_\mu^{1/4} k_p^{1/2} \qquad y^* = \frac{C_\mu^{1/4} k_p^{1/2} y_p}{\nu} \qquad u^* = \frac{1}{\kappa} \ln \left(E y^* \right)$$

- Notice that these new quantities do not depend anymore on the shear velocity.
- Therefore, in the case of wall shear stress equal to zero, we do not risk anymore a singularity when computing u^+.
- It is worth noting that y^+ is equal to y^* in the ideal situation of equilibrium conditions.
  - That is, production equal to dissipation.
  - Or roughly speaking, the ratio of production to dissipation is approximately 1.
  - And this situation does not occur in the buffer layer or in the presence of massive separation.

[1] B. E. Launder, D. B. Spalding. The Numerical Computation of Turbulent Flows. Computer Methods in Applied Mechanics and Engineering. 1974.

Near wall treatment

- All the relations of the standard wall functions formulation of Launder and Spalding [1], can be summarized as follows,

$$u^* = \frac{U_p C_\mu^{1/4} k_p^{1/2}}{\tau_w / \rho}$$

The only unknown quantity is the wall shear stress

$$P_k \approx \tau_w \frac{\partial \overline{u}}{\partial y} = \tau_w \frac{\tau_w}{\kappa \rho C_\mu^{1/4} k_p^{1/2} y_p}$$

The TKE production term is also modified in order in the wall function formulation

$$\epsilon_p = \frac{C_\mu^{3/4} k_p^{3/2}}{\kappa y_p}$$

The equation of turbulence dissipation rate is not solved in the cell next to the wall.
Instead, its value is computed using this relation.

- Recall that the subscript p indicates values at the cell center and the subscripts w indicates values at the walls

[1] B. E. Launder, D. B. Spalding. The Numerical Computation of Turbulent Flows. Computer Methods in Applied Mechanics and Engineering. 1974.

Near wall treatment

- The boundary condition for TKE at the walls is,

$$\frac{\partial k}{\partial n} = 0$$

- In this formulation, the wall adjacent cells value is obtained from the TKE equation.
- And recall that,

$$u^* = \frac{1}{\kappa} \ln \left(E y^* \right) \qquad y^* = \frac{\rho C_\mu^{1/4} k_p^{1/2} y_p}{\mu} \qquad \underbrace{\kappa = 0.4187 \qquad E = 9.793}$$

These are the values used in Ansys Fluent

- These relations apply only to the cells adjacent to the walls.
- Note that, as shown here, the wall boundary conditions for the solution variables, including mean velocity, temperature, species concentration, turbulent kinetic energy and turbulent dissipation rate, are all taken care of by the wall functions.
- Therefore, you do not need to be concerned about the boundary conditions at the walls. Except maybe for the turbulent dissipation rate.

[1] B. E. Launder, D. B. Spalding. The Numerical Computation of Turbulent Flows. Computer Methods in Applied Mechanics and Engineering. 1974.

Near wall treatment

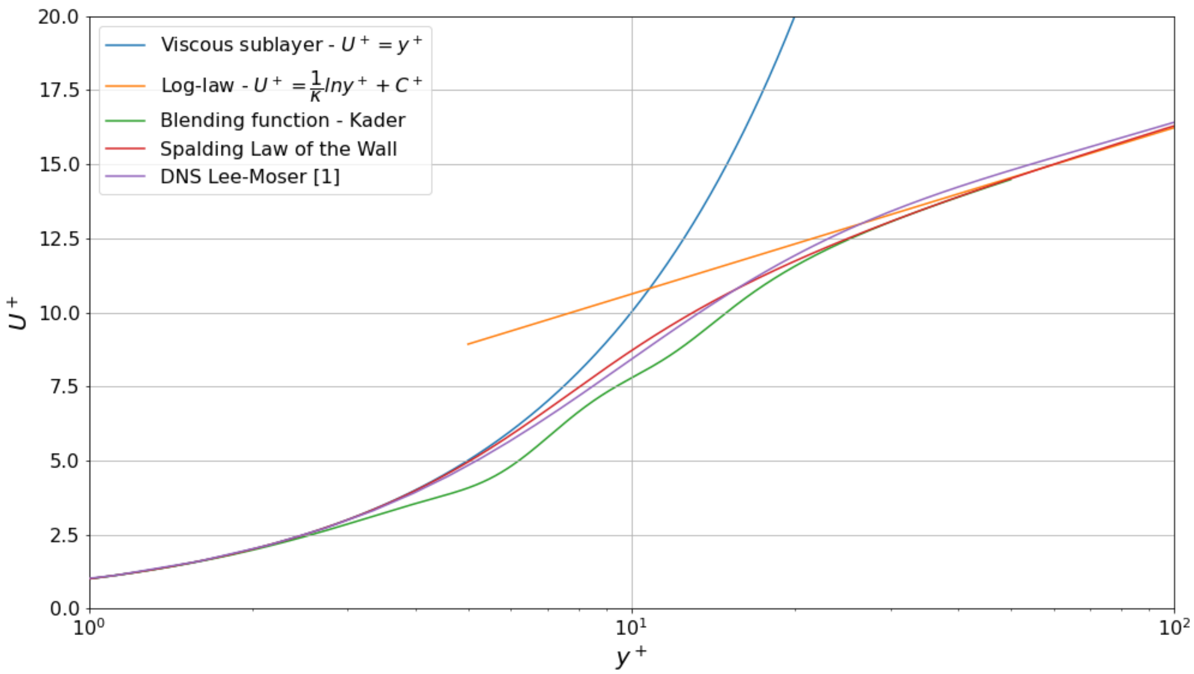
- Let us study another approach to deal with the wall function treatment.
- This approach is known as automatic wall treatment or scalable wall functions.
- It consist in simply adding a conditional clause,

$$u^* = \begin{cases} y^* & y^* < 11.225 \\ \dfrac{1}{\kappa}\ln\left(Ey^*\right) & y^* > 11.225 \end{cases}$$

- The value of 11.225 (which is the one used in Ansys Fluent), comes from the intersection of the two correlations.
- This value might change depending on the constant used and the model implementation.
- If you use the relations that we defined for the law of the wall in the previous lectures, you will find a value of approximately 10.8, of course, we used different values for the constants.

Near wall treatment

- Note that the automatic wall treatment (or scalable wall functions) does not take into account the blending between the viscous sublayer and the log-law region.
- The blending between the laminar and turbulent behaviors has been observed in physical and numerical experiments.
- And from the TKE and Reynolds stresses budget, we know that the buffer layer is very energetic.
- Therefore, small differences in the blending within the buffer region can have a strong influence in the global behavior.
- This is main reason why in CFD we avoid placing the first cell center next to the wall in the buffer layer.
- As can be seen, the automatic wall treatment might not be the best approach if the overlap region is narrow or if the first cell center is in the buffer region. We will study later a better approach.



[1] M. Lee, R. Moser, Direct numerical simulation of turbulent channel flow up to Re_tau = 5200, 2015, Journal of Fluid Mechanics, vol. 774, pp. 395-415.

Near wall treatment

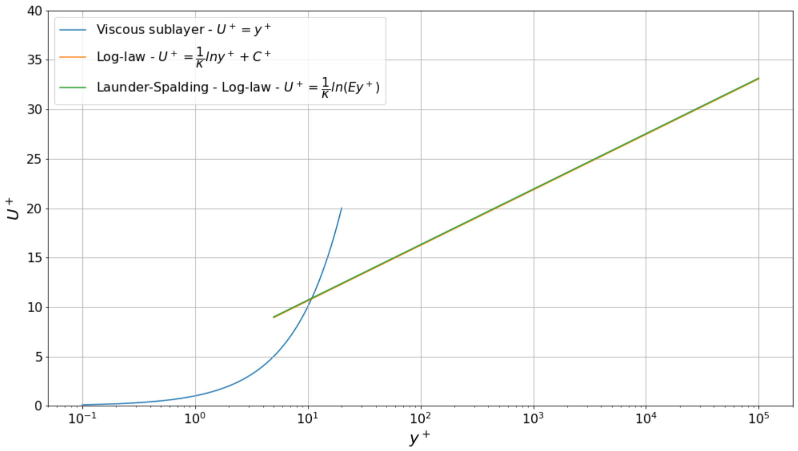
- In the standard wall functions formulation of Launder and Spalding [1], the correlation for the log-law layer is given as follows,

$$u^{*} = \frac{1}{\kappa} \ln \left(E y^{*} \right)$$

- Whereas the traditional correlation is given as follows (apply a logarithmic rule and you will get similar formulations),

$$u^{+} = \frac{1}{\kappa} \ln \left(y^{+} \right) + C^{+}$$

- These two correlations are approximately the same, as shown in the figure.
- Any difference is due to the values of the constants used.



[1] B. E. Launder, D. B. Spalding. The Numerical Computation of Turbulent Flows. Computer Methods in Applied Mechanics and Engineering. 1974.

Near wall treatment

- We just presented the wall functions for the momentum and turbulence variables.
- Similarly, wall functions can be derived for temperature, species, and so on.
- We will briefly address temperature wall functions in Lecture 9.
- The approach we just presented, is also known as a log-law based approach.
  - Specifically, we presented the Launder-Spalding methodology [1].
  - Which is probably the most popular approach.
  - However, this does not mean that it is the most robust approach.
- Generally speaking, the results obtained when using wall functions deteriorate with mesh refinement.
- It is strongly recommended not to place the first cell center in the buffer region.
- Remember, you need to assign boundary conditions to the wall functions.
- In Ansys Fluent, the boundary conditions for the wall functions are all taken care by the solver. You do not need to be concerned about the numerical values.
- Finally, this is not the only approach when using wall functions.
- In the literature, you will find many approaches.

[1] B. E. Launder, D. B. Spalding. The Numerical Computation of Turbulent Flows. Computer Methods in Applied Mechanics and Engineering. 1974.

Near wall treatment

- There are many wall functions implementations, just to name a few,
  - Standard wall functions – Launder-Spalding methodology (the approach we just presented).
  - Generalized wall functions.
  - Scalable wall functions.
  - Non-equilibrium wall functions.
  - Two-layer approach.
  - y^+ insensitive wall treatment.
  - Menter-Lechner wall functions.
  - Werner and Wengle wall functions.
  - Craft wall functions.
  - Chien-Launder wall functions.
  - Eddy viscosity-based wall functions.
  - Spalding continuous wall function.
  - Roughness wall functions.
  - And so on.

Near wall treatment

- It is also possible to formulate y^+ insensitive wall functions.
- That is, formulations that cover viscous sublayer, buffer region, and log-law region.
- This can be achieved by using a blending function between the viscous sublayer and the log-law layer [1].
- To use this approach, you need to use turbulence models able to deal with wall resolving meshes and wall modeling meshes.
- The $k-\omega$ family of turbulence models are y^+ insensitive.
- Kader [1] proposed the following blending function to obtain a y^+ insensitive formulation,

$$u^+ = e^{\Gamma} u^+_{lam} + e^{1/\Gamma} u^+_{turb}$$

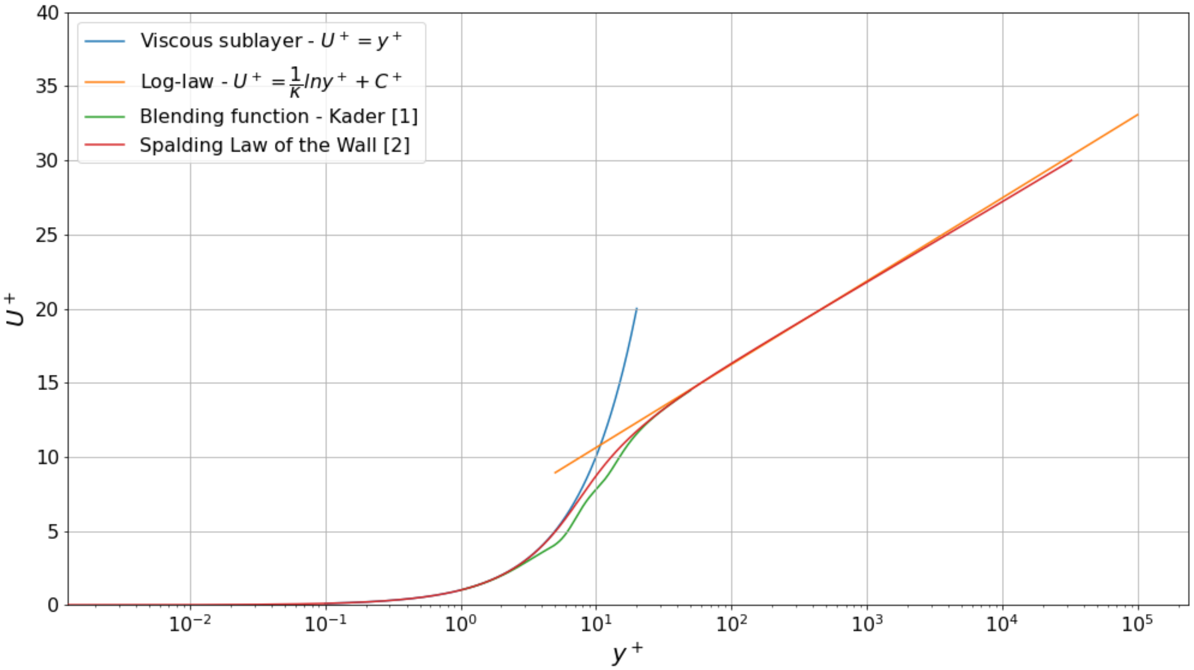
$$\Gamma = -\frac{a\left(y^+\right)^4}{1+by^+} \qquad a = 0.01 \qquad b = 5$$

- This formula guarantees the correct asymptotic behavior for large and small values of y^+ and a reasonable representation of velocity profiles in the cases where y^+ falls inside the buffer region.
- This approach can also be used for temperature and species concentration.

[1] B. Kader. Temperature and Concentration Profiles in Fully Turbulent Boundary Layers. 1981.

Near wall treatment

- Plot of Kader's blending function [1].
- In the plot, the Spalding function [2] is also represented.
- The Spalding function is another alternative to obtain a y^+ insensitive treatment.
- It is essentially a fit of the laminar, buffer and logarithmic regions of the boundary layer.



Kader's blending function,

$$u^+ = e^{\Gamma} u^+_{lam} + e^{1/\Gamma} u^+_{turb}$$

$$u^* = \begin{cases} y^* & y^* < 11.225 \\ \frac{1}{\kappa} \ln\left(E y^*\right) & y^* > 11.225 \end{cases}$$

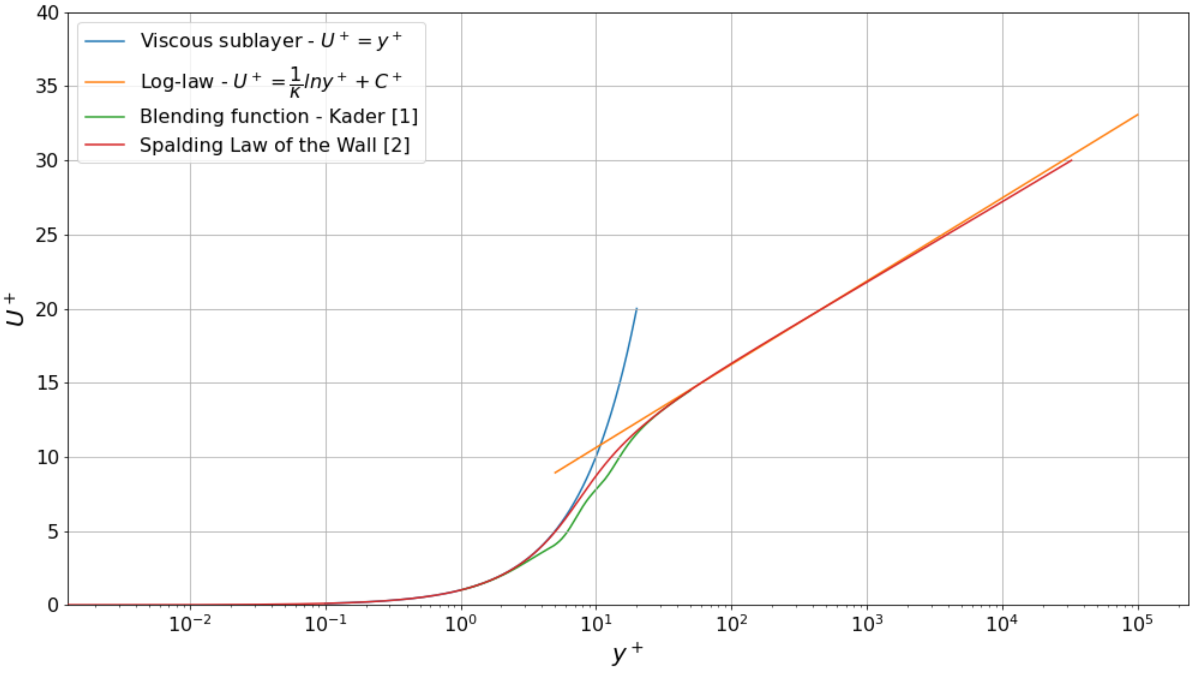
And recall that in equilibrium conditions,

$$u^+ = u^*$$

[1] B. Kader. Temperature and Concentration Profiles in Fully Turbulent Boundary Layers. 1981.
[2] D. Spalding. A single formula for the law of the wall. J. of Applied Mechanics. 1961.

Near wall treatment

- Plot of Kader's blending function [1].
- In the plot, the Spalding function [2] is also represented.
- The Spalding function is another alternative to obtain a y^+ insensitive treatment.
- It is essentially a fit of the laminar, buffer and logarithmic regions of the boundary layer.



Spalding's law,

$$y^+ = u^+ + \frac{1}{E}\left[e^{\kappa u^+} - 1 - \frac{\kappa u^+}{1!} - \frac{(\kappa u^+)^2}{2!} - \frac{(\kappa u^+)^3}{3!} - \frac{(\kappa u^+)^4}{4!}\right]$$

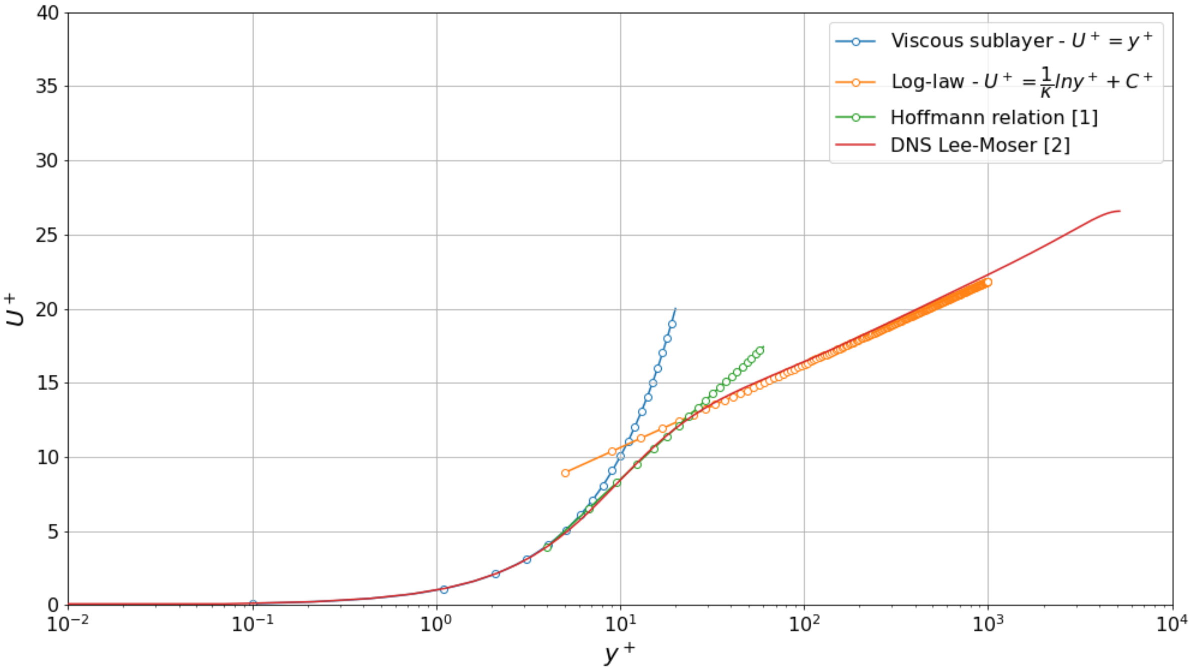
And recall that in equilibrium conditions,

$$u^+ = u^*$$

[1] B. Kader. Temperature and Concentration Profiles in Fully Turbulent Boundary Layers. 1981.
[2] D. Spalding. A single formula for the law of the wall. J. of Applied Mechanics. 1961.

Near wall treatment

- Plot of Hoffmann's blending function [1].
- In the figure, reference DNS data is also plotted [2].
- Note that Hoffmann's blending function is only valid in the buffer region.
- This function does not result in a continuous profile like in Kader's or Spalding's approaches.



Hoffman relation,

$$u^+ = 5.0\ln\left(y^+\right) - 3.05$$

Which is only valid in the buffer region,

$$5 < y^+ < 25 \sim 30$$

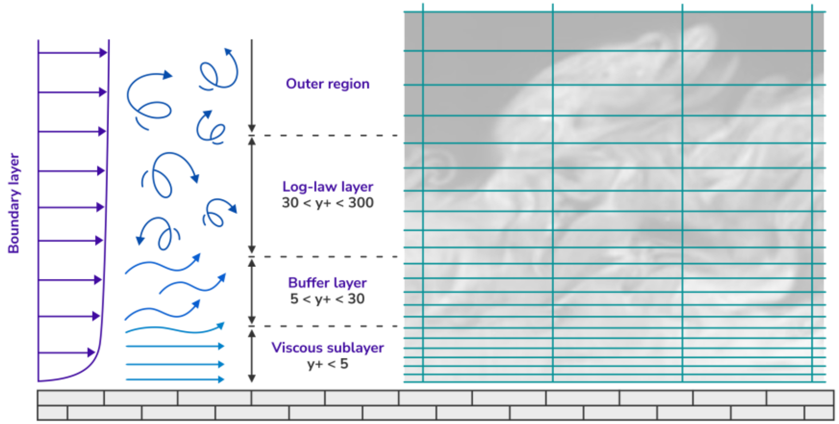
And recall that in equilibrium conditions,

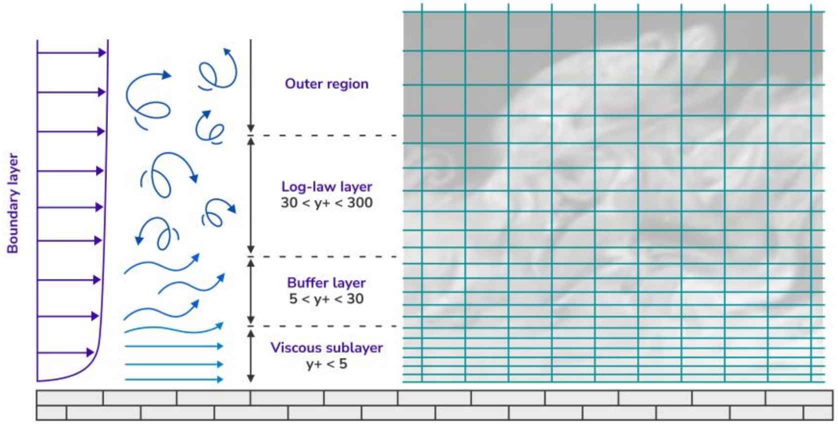
$$u^+ = u^*$$

[1] K. Hoffmann, S. Chiang. Computational Fluid Dynamics Volume III. ESS Books, 2000.
[2] M. Lee, R. Moser, Direct numerical simulation of turbulent channel flow up to Re_tau = 5200, 2015, Journal of Fluid Mechanics, vol. 774, pp. 395-415.

Near-wall treatment and wall functions

- When dealing with wall turbulence, we need to choose a near-wall treatment.
- If you want to resolve the boundary layer, all the way down to the viscous sub-layer, you need very fine meshes close to the wall.
- In terms of y^+, you need to cluster at least 5 to 10 layers at $y^+ < 5$.
    - You need to properly resolve the profiles (U, k, epsilon, Reynolds stresses and so on).
- Usually, this kind of meshes will cluster from 15 to 30 layers (or even more) close to the walls.
- This is the most accurate approach, but it is computationally expensive.

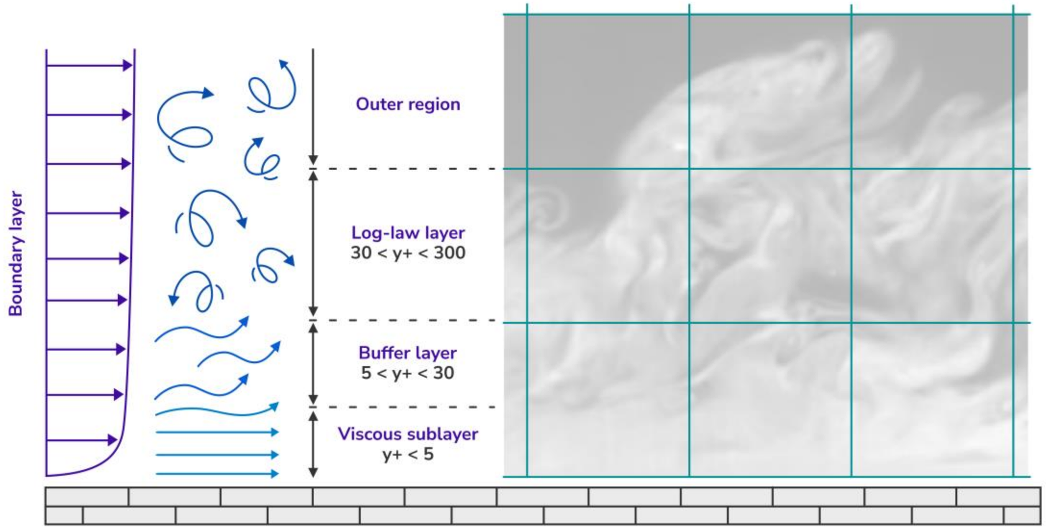




Resolving the streamwise direction is also important, for example, when dealing with transition to turbulence.

Near-wall treatment and wall functions

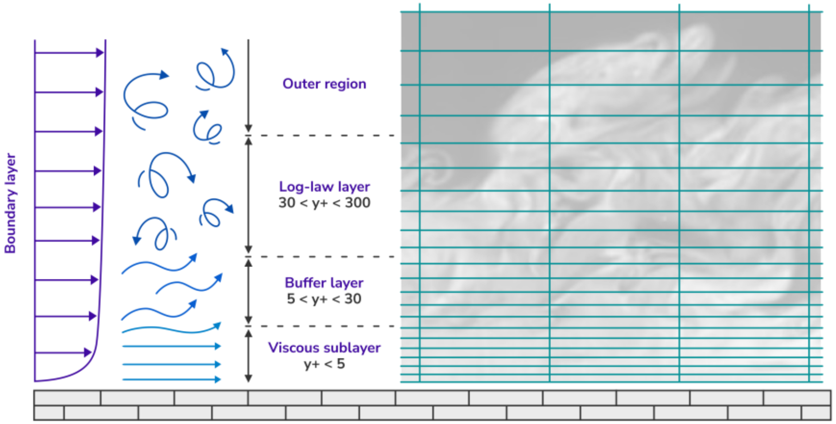
- When dealing with wall turbulence, we need to choose a near-wall treatment.
- If you are not interested in resolving the boundary layer up to the viscous sub-layer, you can use wall functions.
- In terms of y^+, wall functions will model everything below $y^+ < 30$ or the target y^+ value.
- This approach use coarser meshes, but you should be aware of the limitations of the wall functions.
- You will need to cluster at least 5 to 10 layers close to the walls in order to resolve the profiles (U, k, epsilon, Reynolds stresses and so on).
- As a general rule, when using wall functions, the first cell center should be located above y+ > 40-50 and below $y \approx 0.2\delta_{99}$ (boundary layer thickness).

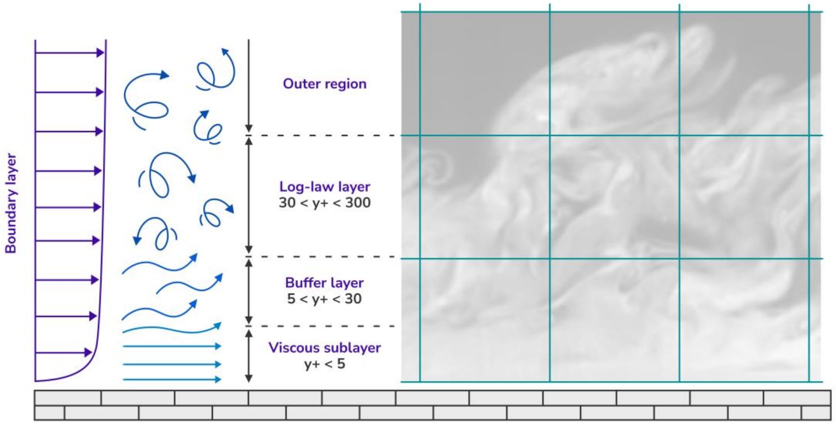


Near wall treatment

Near-wall treatment and wall functions

- When dealing with wall turbulence, we need to choose a near-wall treatment.
- You can also use the y^+ insensitive wall treatment (sometimes known as continuous wall functions or scalable wall functions).
- This kind of wall functions are valid in the whole boundary layer.
- In terms of y^+, you can use this approach for values between $1 < y^+ < 300\text{-}600$ (the upper limit depends on the Reynolds number).
- This approach is very flexible as it is independent of the y^+ value, but is not available in all turbulence models
- Again, you should cluster enough cells close to the walls to resolve the profile of the transported qunatities (at least 8-10 layers).





Insensitive wall treatment will automatically switch between the wall modeling approach or the wall resolving approach according to the y+ value.

Near wall treatment

Final remarks

- If you want good accuracy, use a wall resolving approach.
- This approach is relatively affordable if you are running steady simulations.
- If you have flow separation, have in mind that wall functions are not very accurate.
- Heat transfer and non-equilibrium applications requires high accuracy (wall resolving treatment).
  - This requirement is not compulsory; however, it is strongly recommended.
- Using wall functions is not about putting one single cell in the log-law layer.
  - You need to put enough cells in the log-law region to resolve the profiles of all transported qunatities (velocity, temperature, turbulent quantities and so on).
- When using the wall resolving approach, try to get an average y^+ value close to 1 or lower.
- Values of y^+ lower that 0.1 will not give large improvement.
- Pushing the mesh to values of y^+ below 0.1 can results in low quality meshes for industrial applications.
- Such low values are usually required only for transition to turbulence models.
- It is a common agreement that the upper limit of the viscous sublayer is five.

Final remarks

- When placing the first cell center normal to the wall, you can go as high as 6-7, without loosing too much accuracy.
- But ideally, you should aim for a y^+ value of 1 or 2.
- Yes, one or two y^+ units can make a large difference in the final mesh cell count (about 5%).
- As for the wall modeling approach, in the wall resolving treatment you need to cluster enough cells to resolve the viscous sublayer profiles.
  - Namely, velocity, temperature, turbulence quantities, and so on.
- It is recommended to use at least 15 inflation layers with a low expansion ratio (1.2 or less) to properly resolve the profiles.
- No need to mention it, but hexahedral or prismatic cells are preferred over any other type of cells in the boundary layer region.
- Do not conduct mesh refinement studies with standard wall functions as the solution tends to deteriorate.
  - Generally speaking, standard wall functions perform bad with wall resolving meshes.
- The absolute minimum number of inflation layers when using wall functions is 5-6.

Near wall treatment

Final remarks

- It is recommended to use y^+ insensitive wall functions instead of standard wall functions.
  - However, not all turbulence models support the use of y^+ insensitive wall functions .
- As a general rule, when using wall functions, the first cell center should be located above y+ > 40-50 and below $y \approx 0.2\delta_{99}$ (boundary layer thickness).
- When using wall functions, it is extremely advised to avoid placing the first cell center in the buffer layer, as errors are large in this region.
  - None of the correlation found in literature will give you accurate results in the buffer region.
- Remember, it is very difficult (if not impossible) to have a uniform y^+ value.
- Therefore, you should monitor the average y^+ value at the walls.
- It is also recommended to monitor the maximum and minimum values of y^+ and verify that you do not have very large or very small values.
- But if you have very large values, check that the affected region is not located in critical areas (like the leading edge) or that it does not cover more that 10% of the wall surface.

Near wall treatment

Final remarks

- Generally speaking, wall functions is the approach to use if you are more interested in the mixing in the outer region, rather than the forces on the wall.
- If accurate prediction of forces, heat transfer, and species concentration on the walls are key to your simulation (aerodynamic drag, turbomachinery blade performance, heat transfer, combustion) it is better to use a wall resolving approach.
- By following good standard practices, both approaches can give similar results.
- If you are conducting steady simulations, do use a wall resolving approach.
- On the other hand, if you are conducting unsteady simulations the use of a wall resolving approach can be too time consuming as fine meshes translate into small time-steps for a target CFL number.
    - The choice of the wall resolving approach, or the wall modeling approach depends on the time constrains and computational resources available.
    - But in general, it is always better to use the wall resolving approach.
- Finally, in Ansys Fluent, the wall boundary conditions are all taken care by the solver and the implementation of the wall modeling approach. You do not need to be concerned about the numerical values.

Near wall treatment

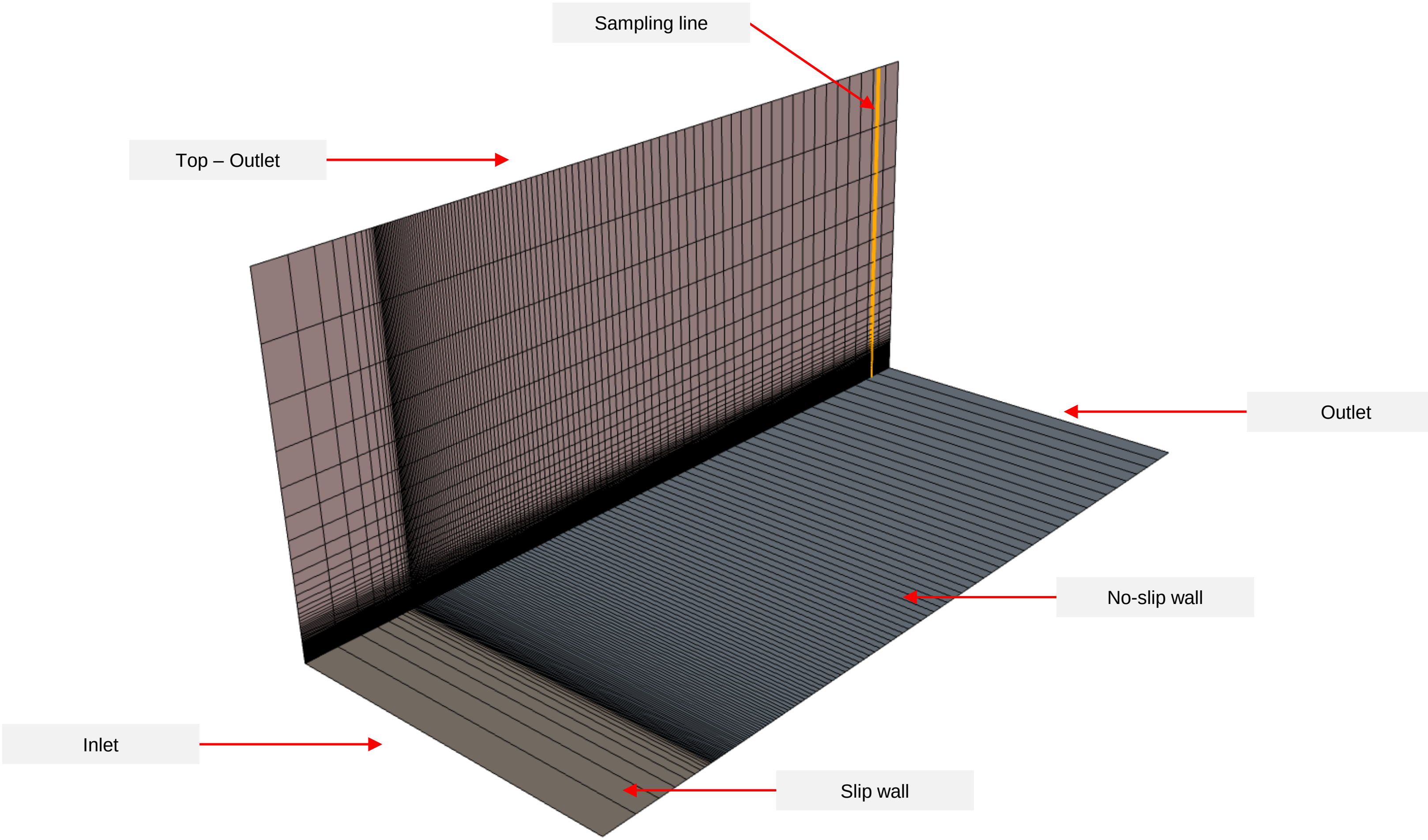
At this point you are invited to revisit
Lecture 3
Turbulence near the wall - Law of the wall

Near wall treatment

Comparison of the wall resolving approach and wall modeling approach for a sample application

Near wall treatment

2D Zero pressure gradient flat plate

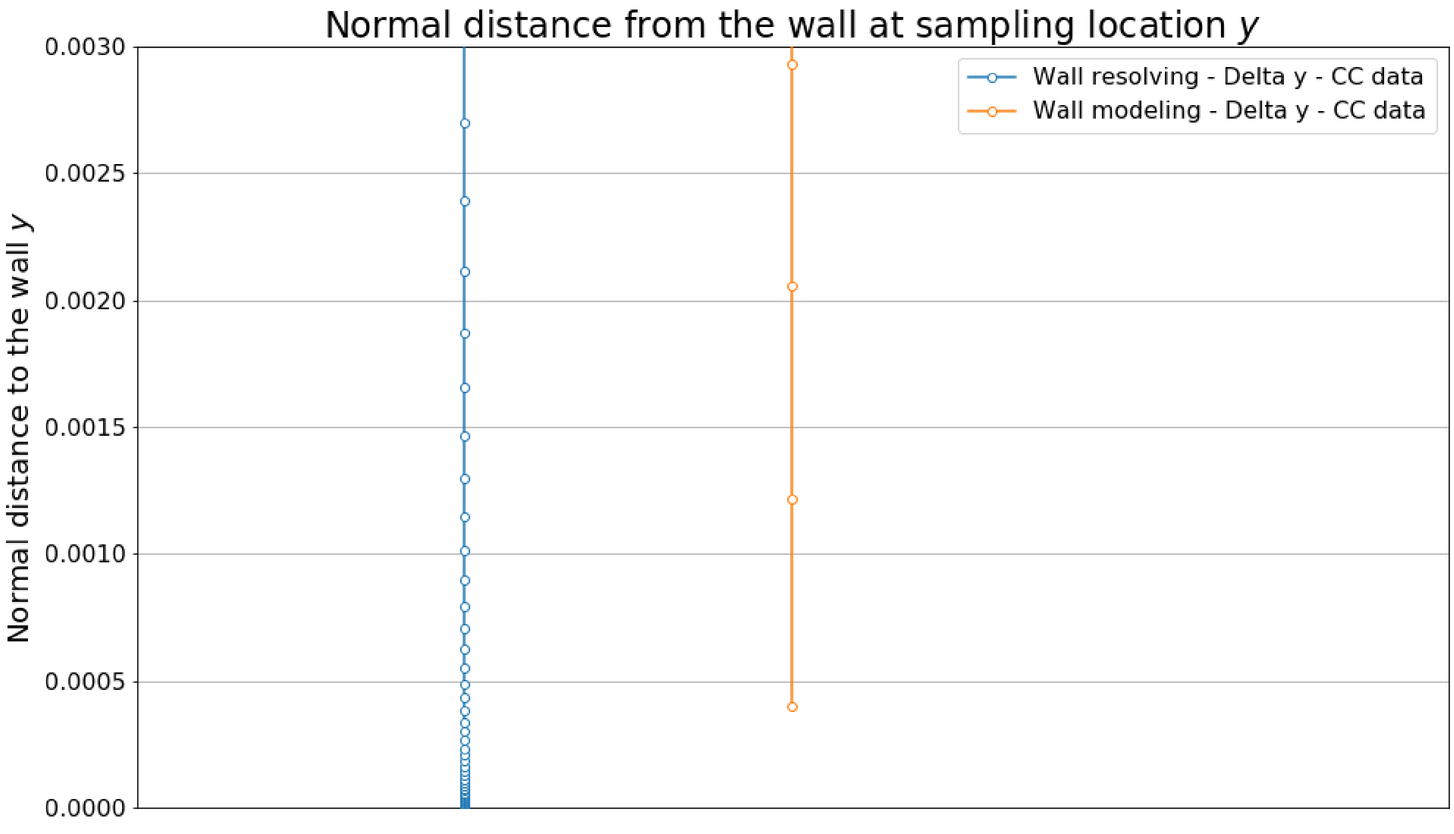


- All the results were obtained using the $k - \omega$ SST turbulence model

Near wall treatment

2D Zero pressure gradient flat plate

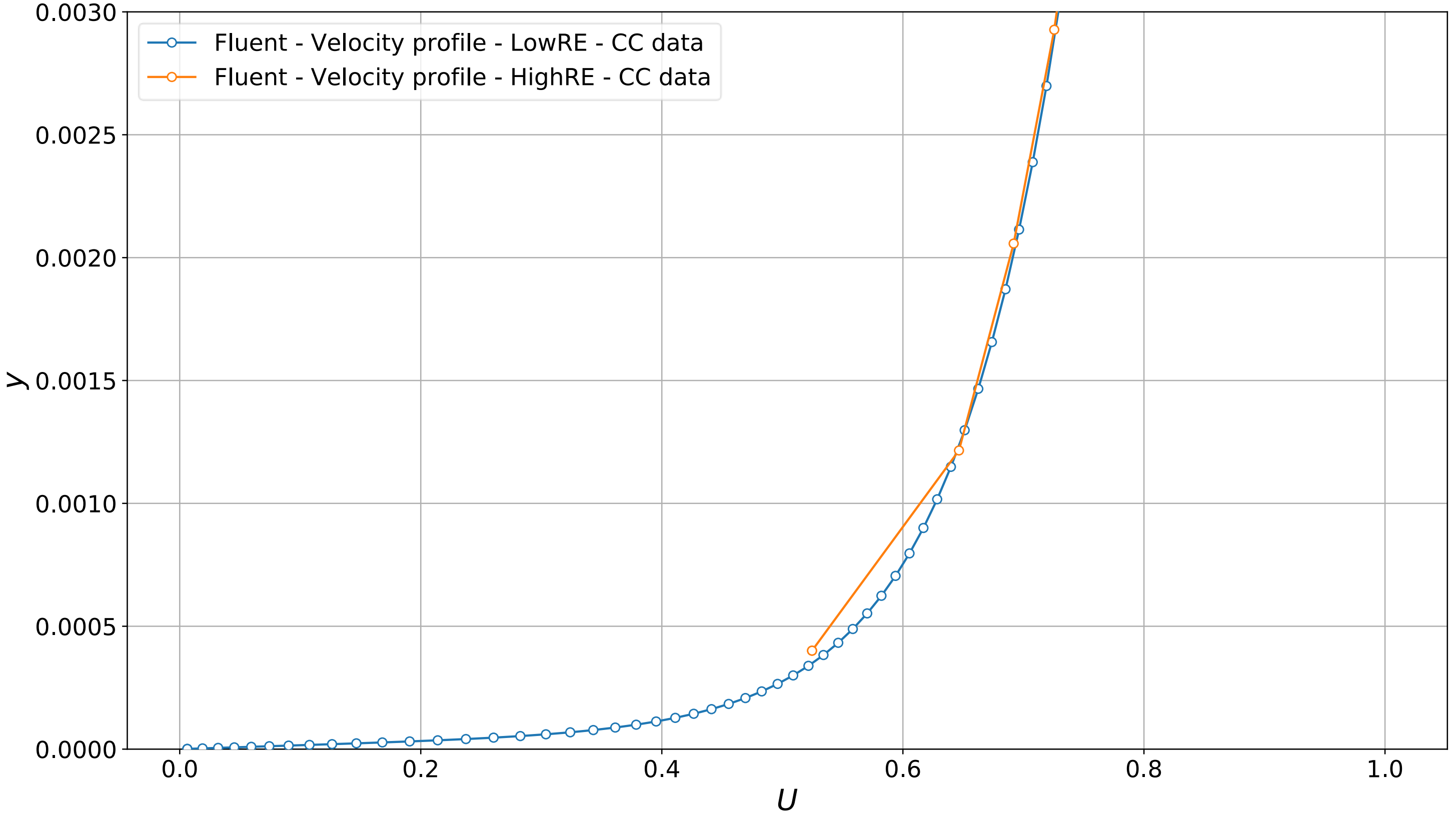
- Distance normal to wall y (m).
- Each circle represents a cell center.



Near wall treatment

2D Zero pressure gradient flat plate

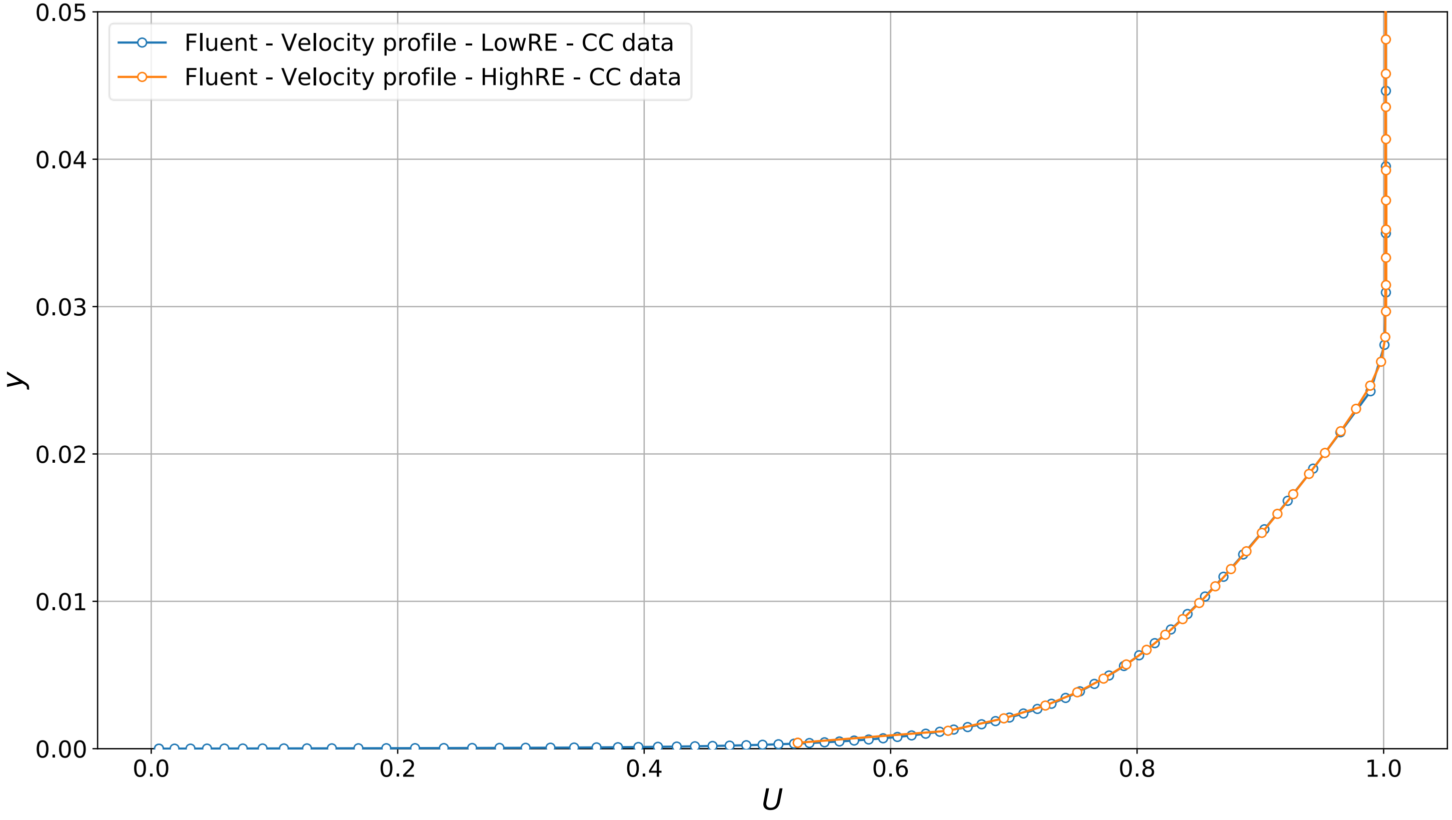
- Velocity profile close to the wall – y vs. U
- Each circle represents a cell center.



Near wall treatment

2D Zero pressure gradient flat plate

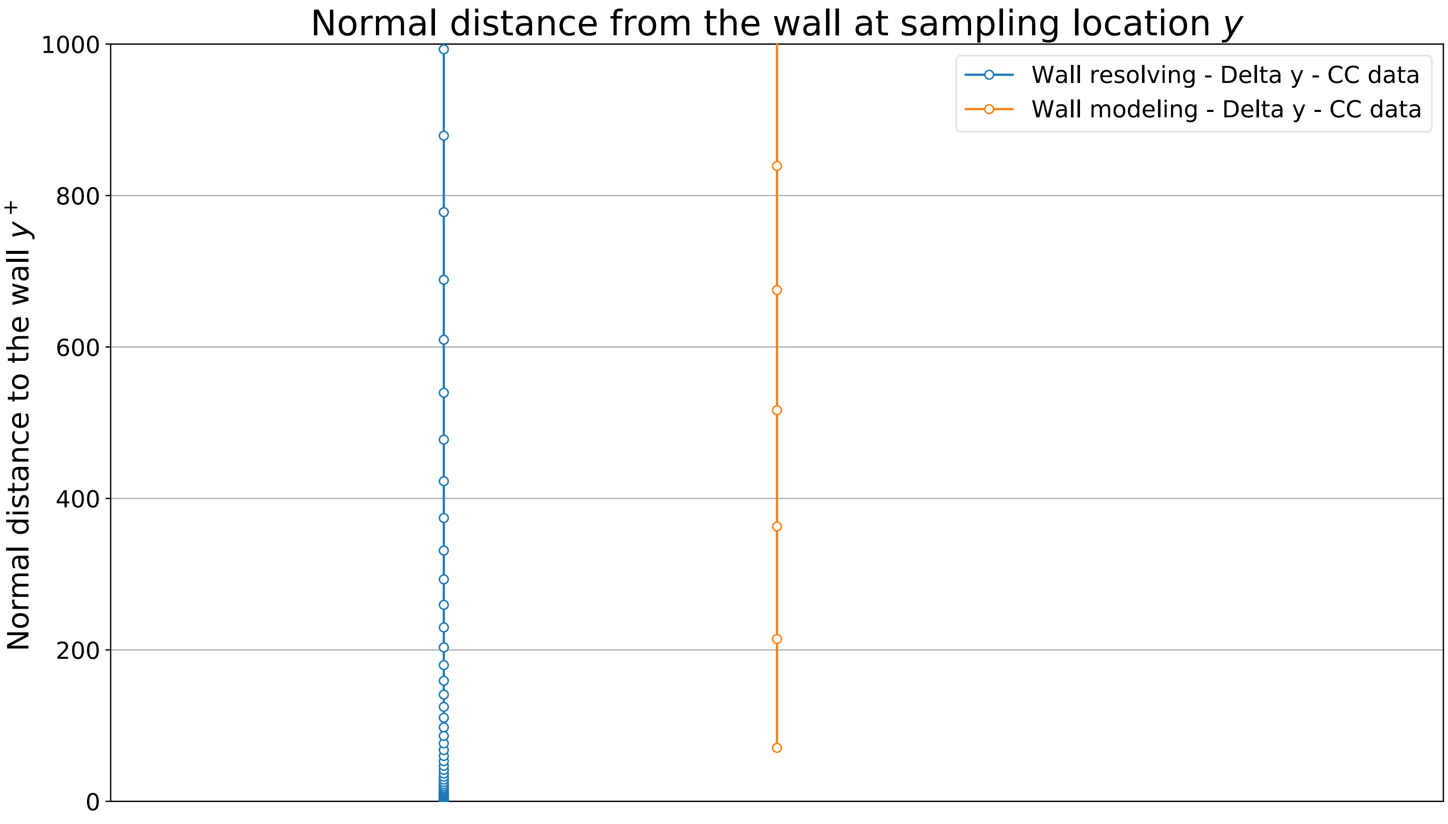
- Velocity profile close to the wall – y vs. U
- Each circle represents a cell center.



Near wall treatment

2D Zero pressure gradient flat plate

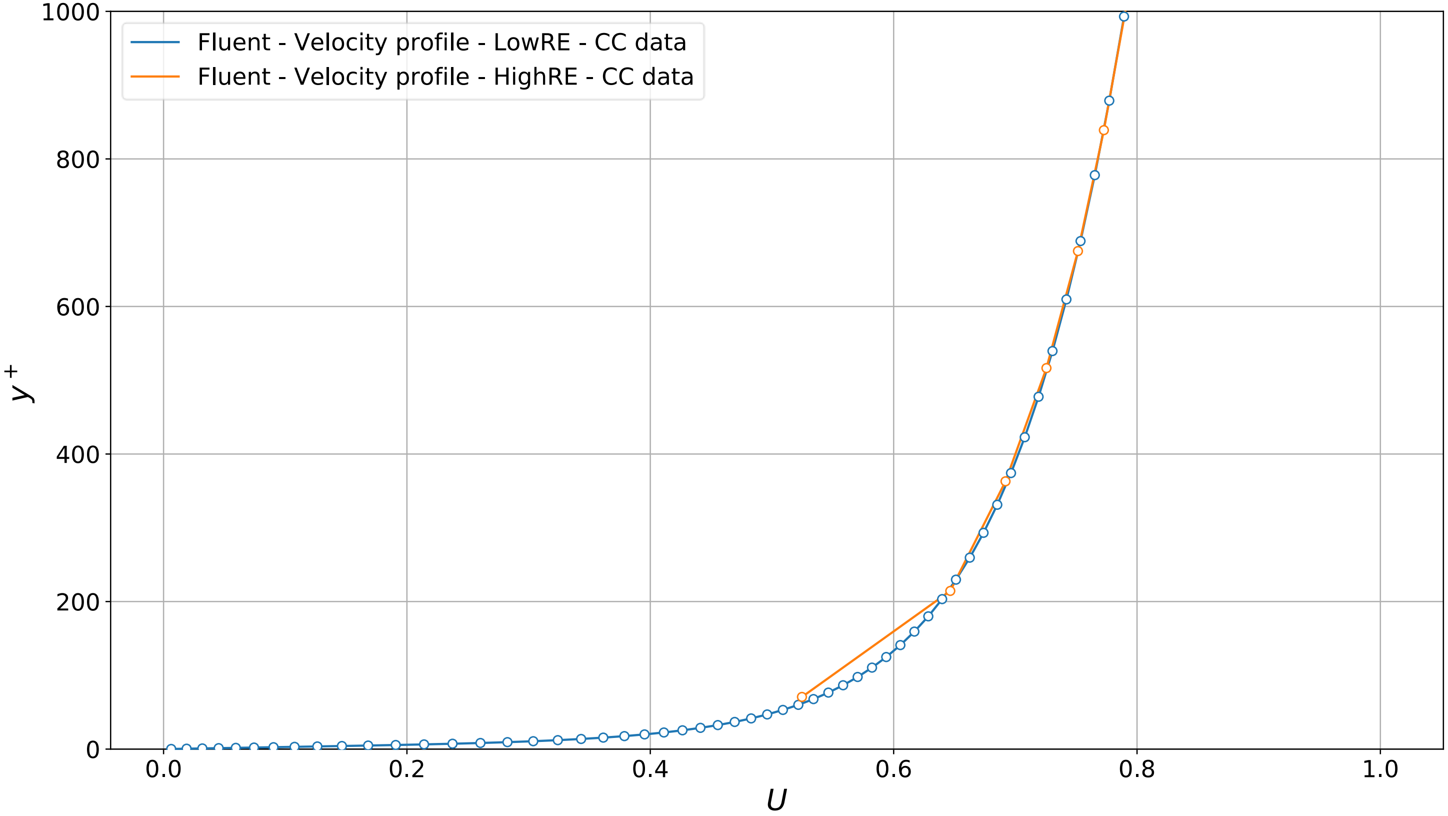
- Non-dimensional distance normal to wall y^+.
- Each circle represents a cell center.



Near wall treatment

2D Zero pressure gradient flat plate

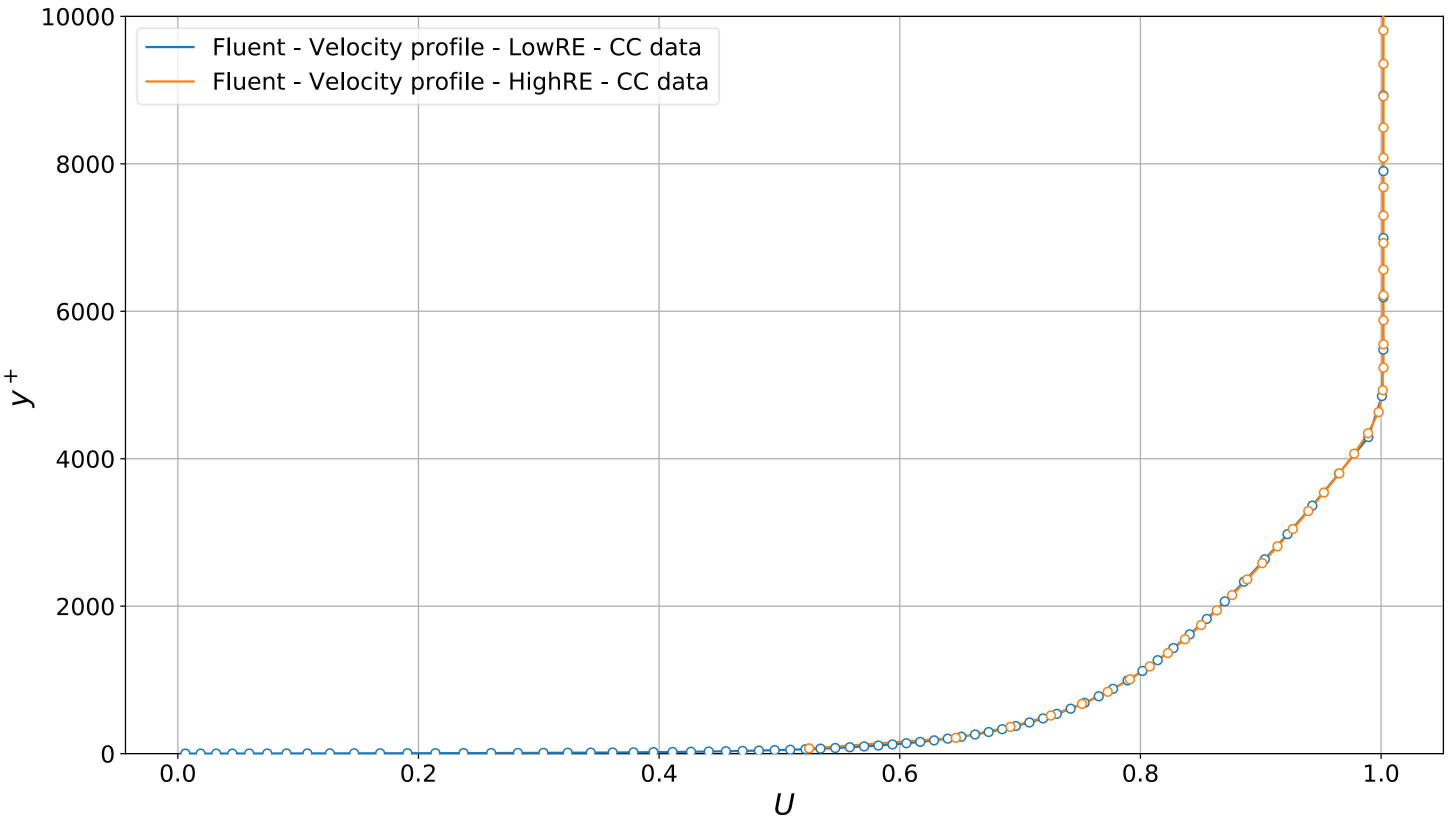
- Velocity profile close to the wall – y^+ vs. U
- Each circle represents a cell center.



Near wall treatment

2D Zero pressure gradient flat plate

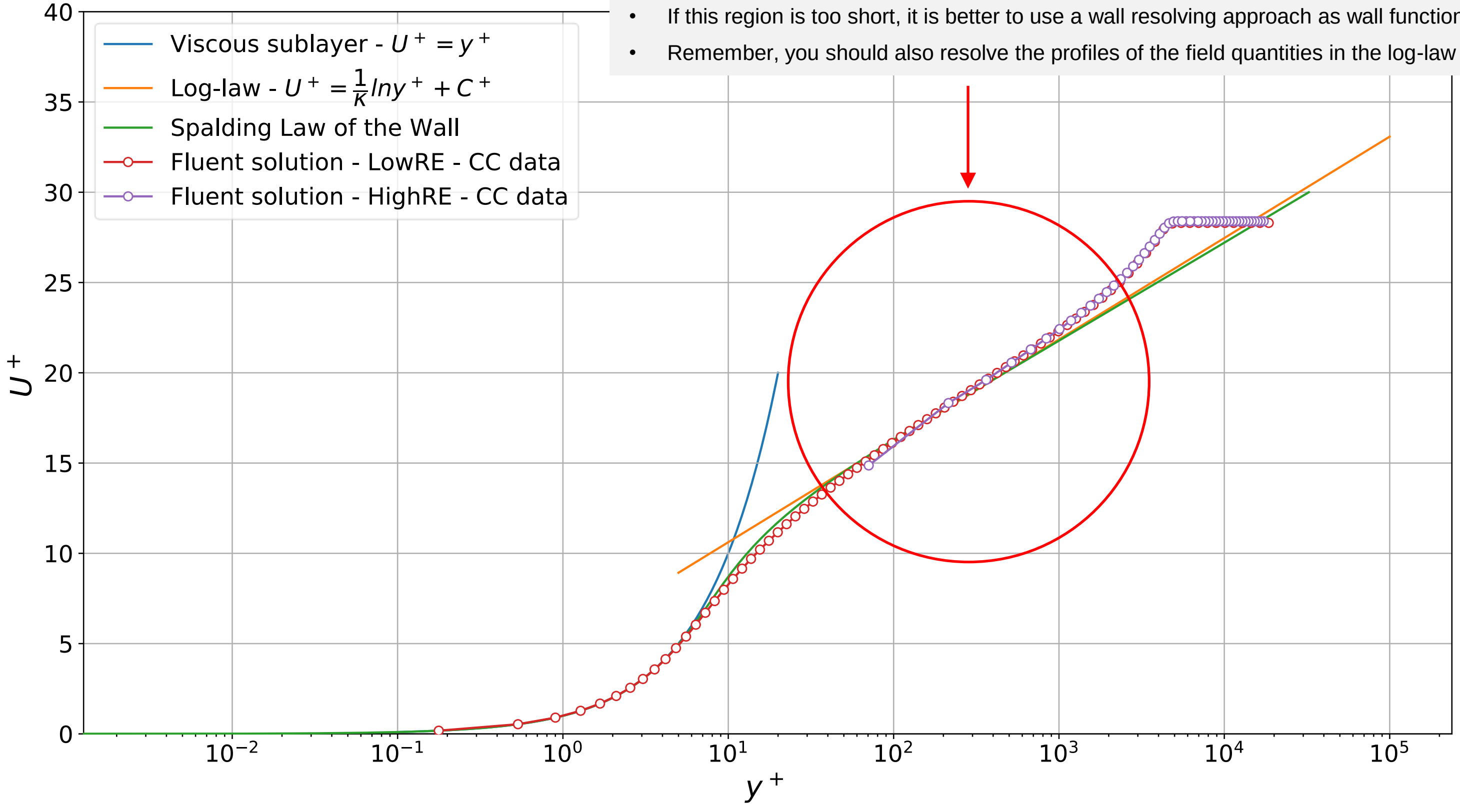
- Velocity profile close to the wall – y^+ vs. U
- Each circle represents a cell center.



Near wall treatment

2D Zero pressure gradient flat plate

- Normalized velocity profile close to the wall – U^+ vs. y^+
- Each circle represents a cell center.

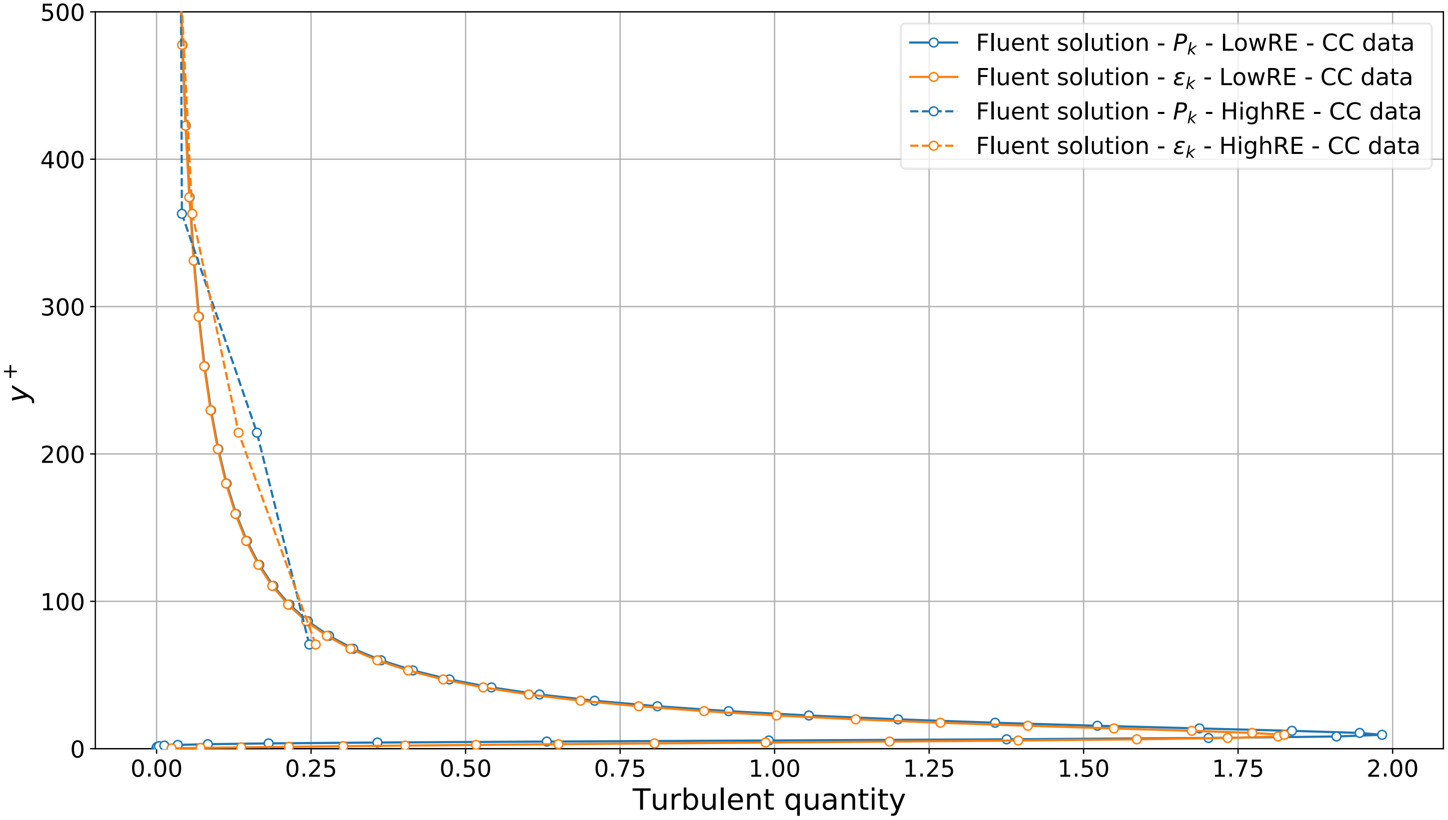


- The extension of the log-law region depends on the Reynolds number.
- If this region is too short, it is better to use a wall resolving approach as wall functions can be inaccurate.
- Remember, you should also resolve the profiles of the field quantities in the log-law region.

Near wall treatment

2D Zero pressure gradient flat plate

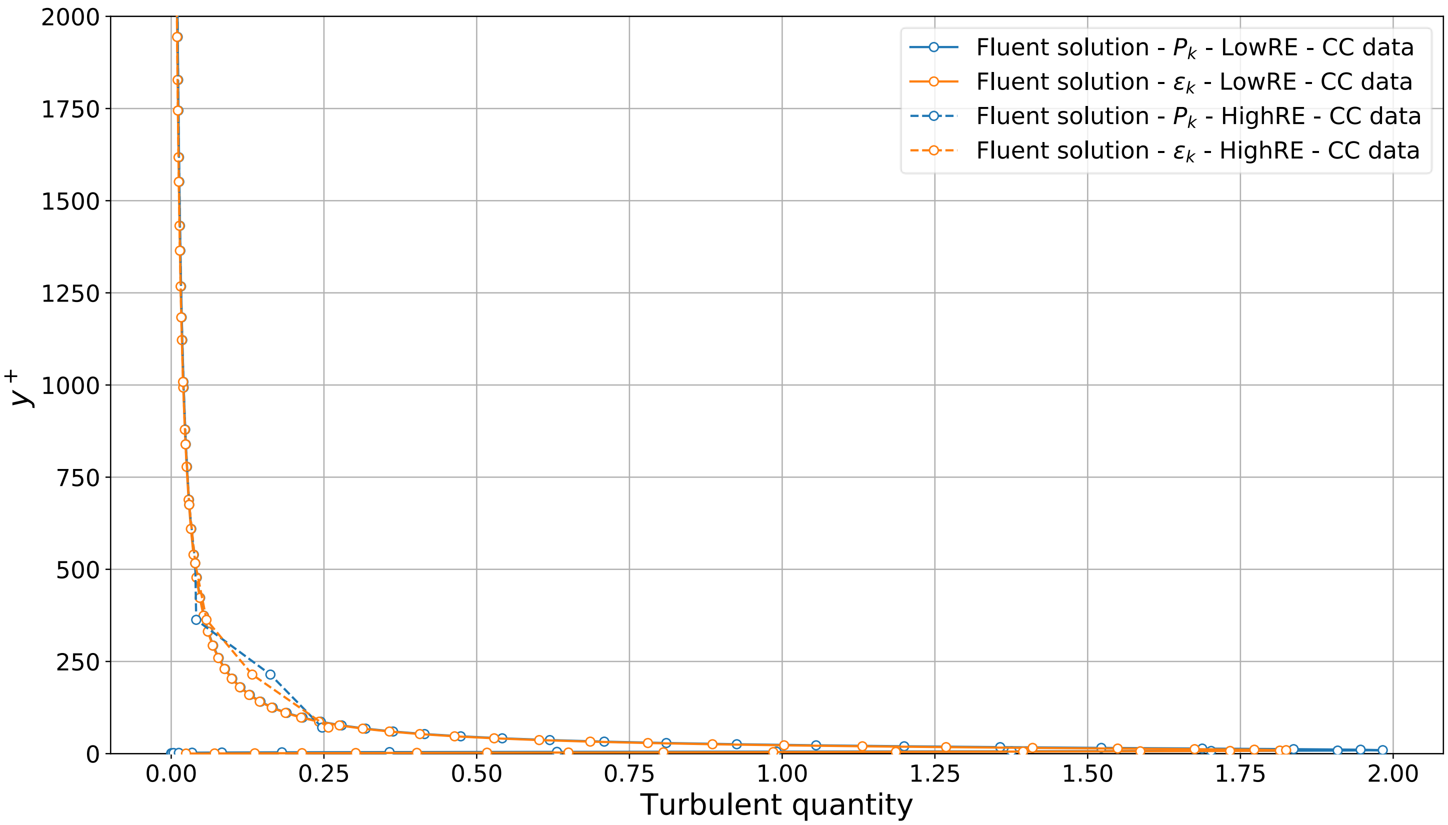
- Budget of turbulent kinetic energy production and dissipation close to the walls.
- Each circle represents a cell center.



Near wall treatment

2D Zero pressure gradient flat plate

- Budget of turbulent kinetic energy production and dissipation close to the walls.
- Each circle represents a cell center.



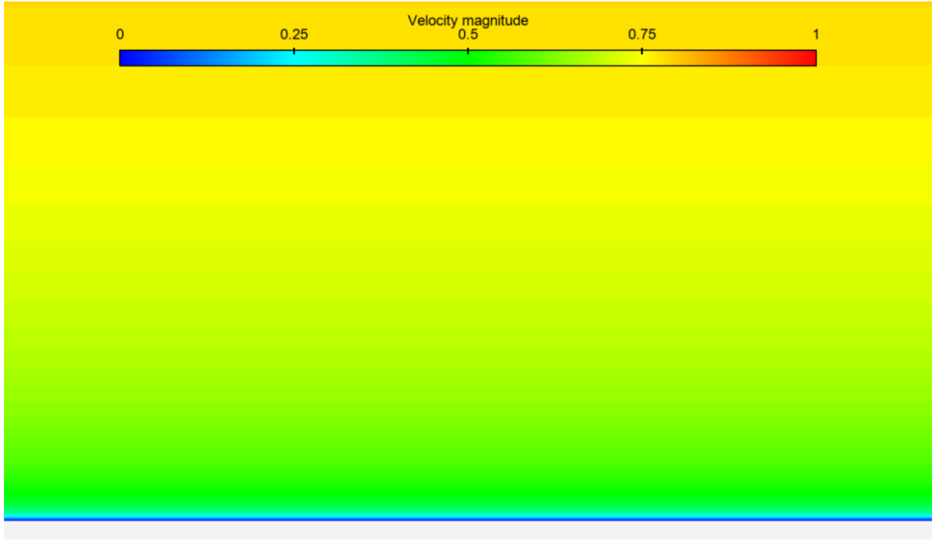
Near wall treatment

2D Zero pressure gradient flat plate

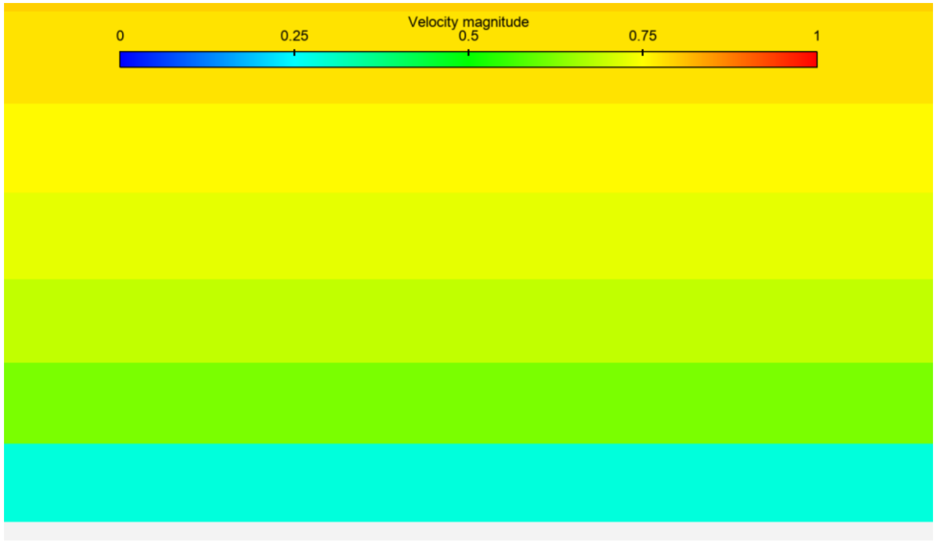
- Mesh comparison – Wall resolving mesh (left) and wall modeling mesh (right).
- It is important to mention that resolving the streamwise direction is also important.

Near wall treatment

2D Zero pressure gradient flat plate



Wall resolving mesh.

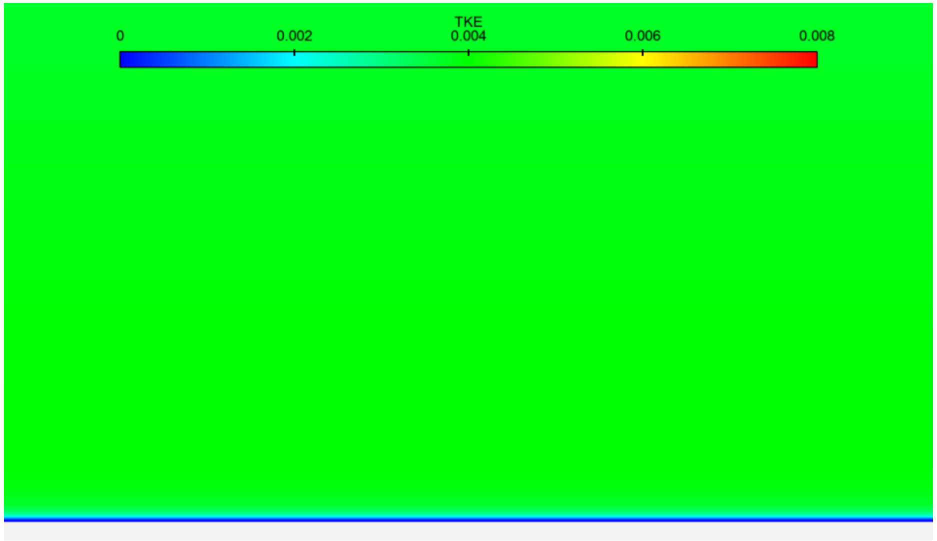


Wall modeling mesh.

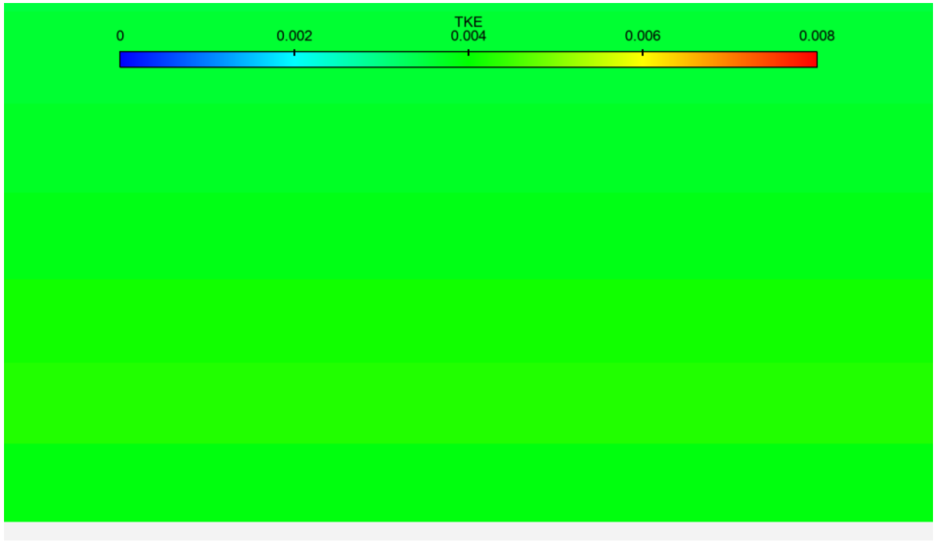
- Plot of velocity magnitude contours.

Near wall treatment

2D Zero pressure gradient flat plate



Wall resolving mesh – $k = 0$ at the wall

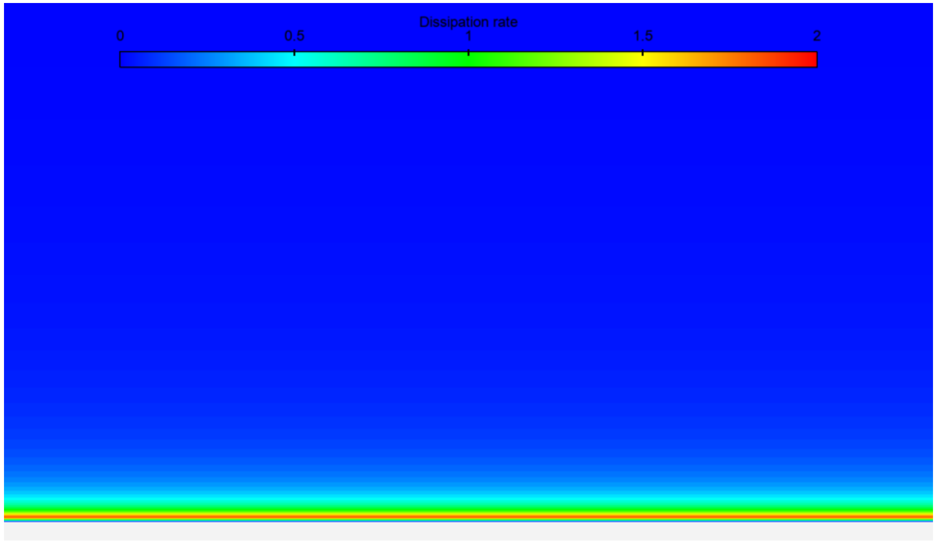


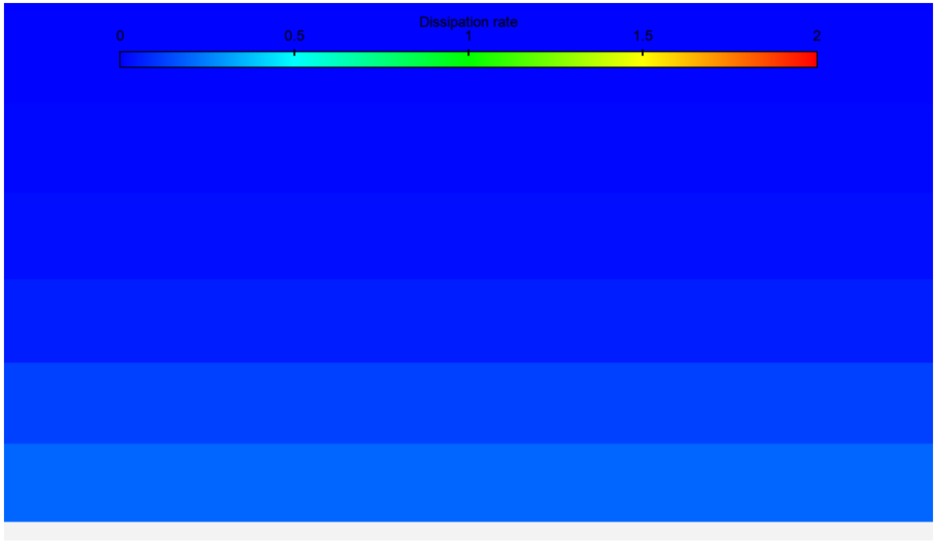
Wall modeling mesh – $\dfrac{\partial k}{\partial n} = 0$ at the wall

- Plot of turbulent kinetic energy contours.

Near wall treatment

2D Zero pressure gradient flat plate





Wall resolving mesh – $\epsilon = 2\nu \left(\frac{\partial \sqrt{k}}{\partial y} \right)^2$ at the wall*

Wall modeling mesh – $\epsilon_P = \frac{C_\mu^{3/4} k_P^{3/2}}{\kappa y_P}$

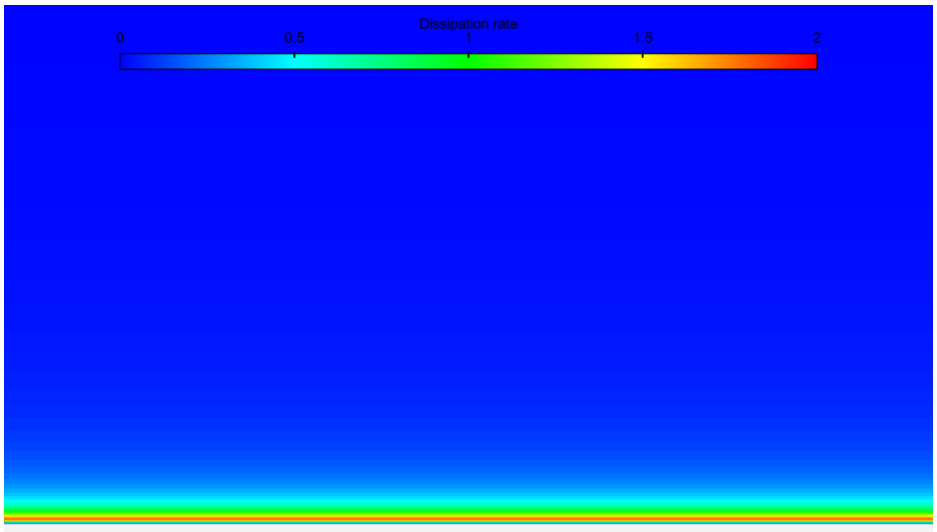
- Plot of turbulent dissipation rate contours.
- Defining the wall boundary conditions for the turbulent dissipation rate is not straightforward, in particular for wall resolving models (lowRE).
- There are different opinions and guidelines on this subject.

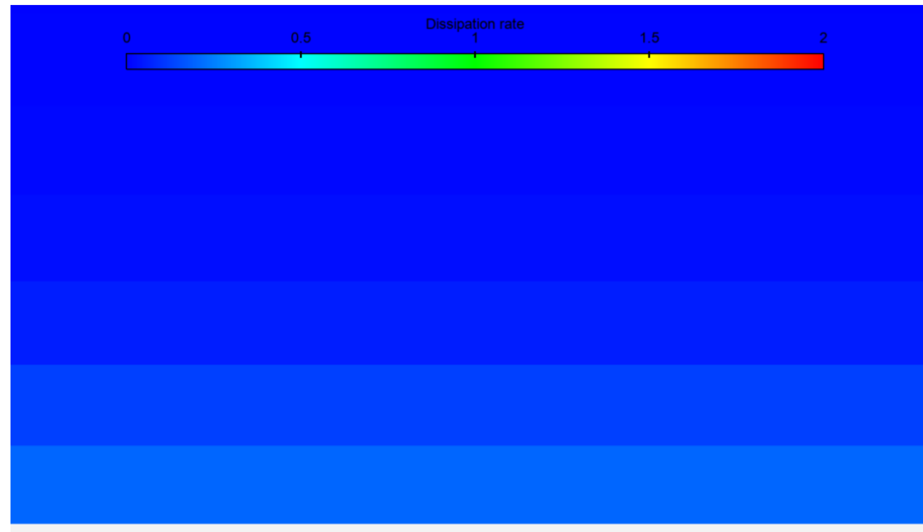
* This value is then used in the wall dissipation (sometimes called modified or isotropic dissipation) to get the value at the wall (which is zero) [1,2]. This approach is used in order to avoid numerical instabilities. It is not clear from the documentation of Ansys Fluent what approach is used but from the results, it appears that it is the method described in reference [2] or a variant.

[1] C. Speziale, R. Abid, E. Anderson. A Critical Evaluation of Two-Equation Models for Near Wall Turbulence. ICASE Report 90-46, 1990.
[2] W. Jones, B. Launder. The prediction of laminarization with a two-equation model of turbulence. Int. J. Heat Mass Transfer, vol. 15, pp. 301–314, 1972.

Near wall treatment

2D Zero pressure gradient flat plate





Wall resolving mesh – $\epsilon = 2\nu \left(\frac{\partial \sqrt{k}}{\partial y} \right)^2$ at the wall*

Wall modeling mesh – $\epsilon_P = \frac{C_\mu^{3/4} k_P^{3/2}}{\kappa y_P}$

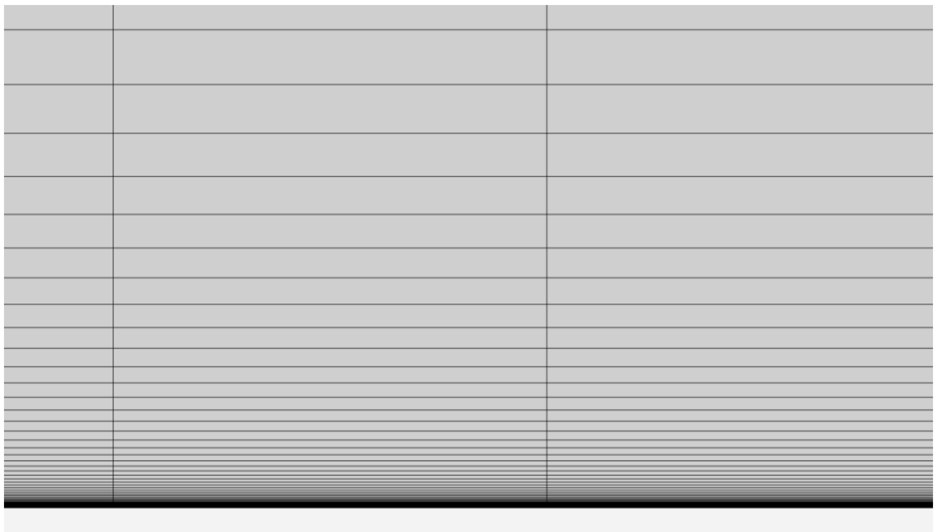
- In the wall modeling approach, the transport equation of the dissipation rate is not solved on the cell adjacent to the wall.
- To avoid numerical problems, the turbulent dissipation rate should not be zero at the wall in the wall resolving approach.
- In addition, TKE and turbulent dissipation rate can not be prescribed arbitrarily because their development is governed by the turbulence transport equations in boundary layer.

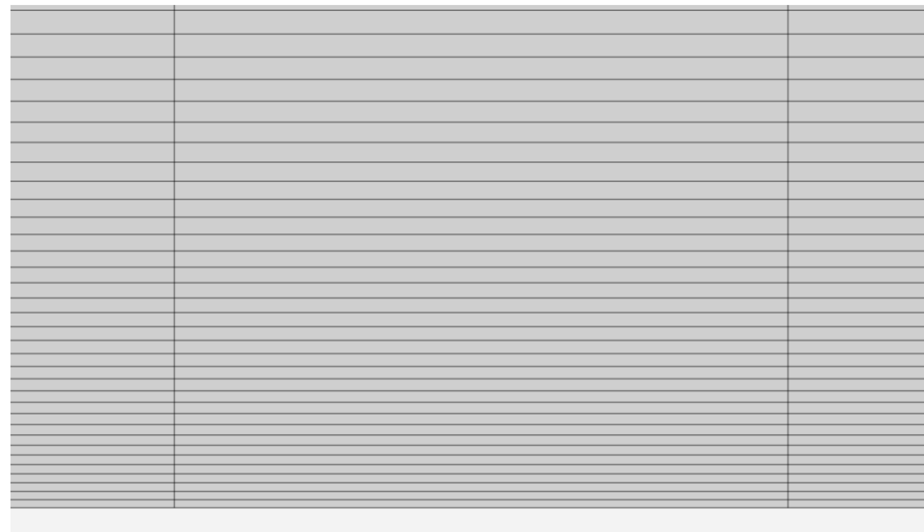
* This value is then used in the wall dissipation (sometimes called modified or isotropic dissipation) to get the value at the wall (which is zero) [1,2]. This approach is used in order to avoid numerical instabilities. It is not clear from the documentation of Ansys Fluent what approach is used but from the results, it appears that it is the method described in reference [2] or a variant.

[1] C. Speziale, R. Abid, E. Anderson. A Critical Evaluation of Two-Equation Models for Near Wall Turbulence. ICASE Report 90-46, 1990.
[2] W. Jones, B. Launder. The prediction of laminarization with a two-equation model of turbulence. Int. J. Heat Mass Transfer, vol. 15, pp. 301–314, 1972.

Near wall treatment

2D Zero pressure gradient flat plate

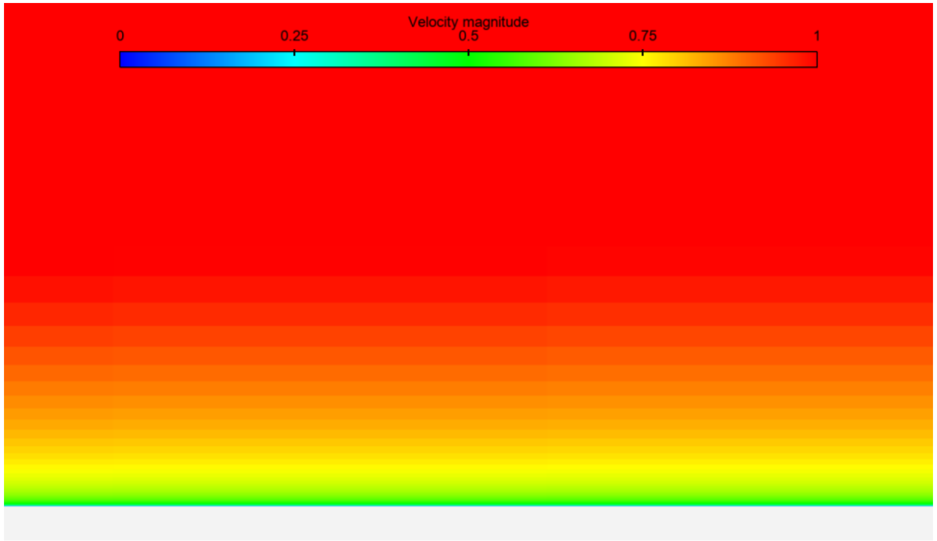




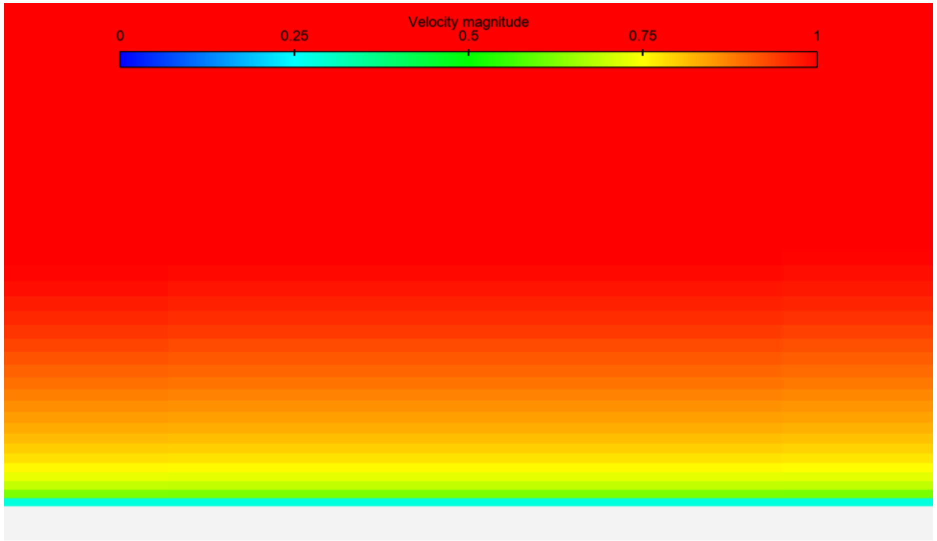
- Mesh comparison – Wall resolving mesh (left) and wall modeling mesh (right).
- We can look at the solution farther away so we can evidence the edge of the boundary layer
- It is important to mention that resolving the streamwise direction is also important.

Near wall treatment

2D Zero pressure gradient flat plate



Wall resolving mesh.

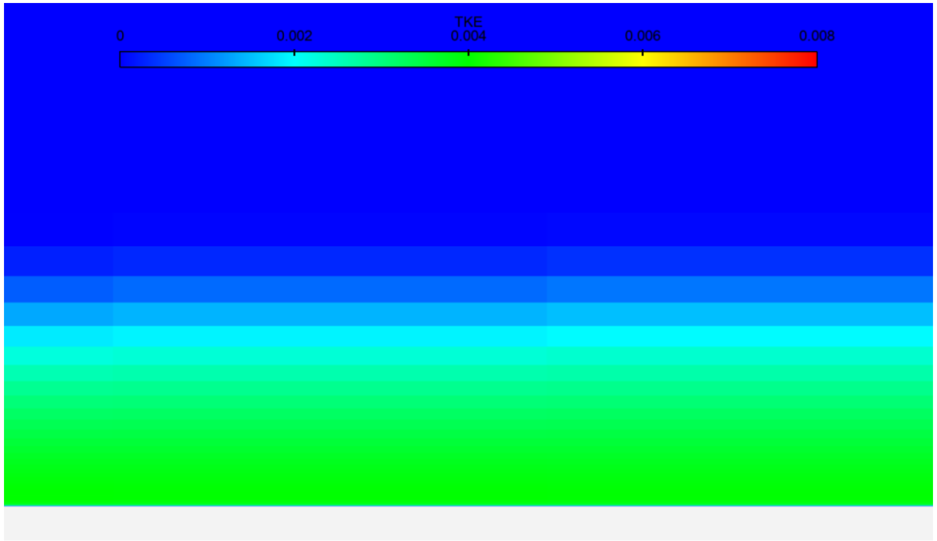


Wall modeling mesh.

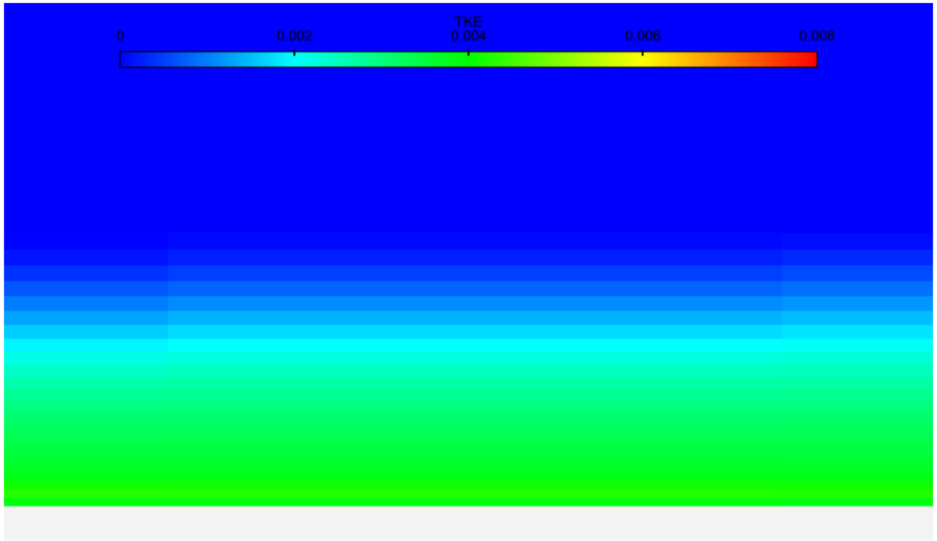
- Plot of velocity magnitude contours.

Near wall treatment

2D Zero pressure gradient flat plate



Wall resolving mesh.

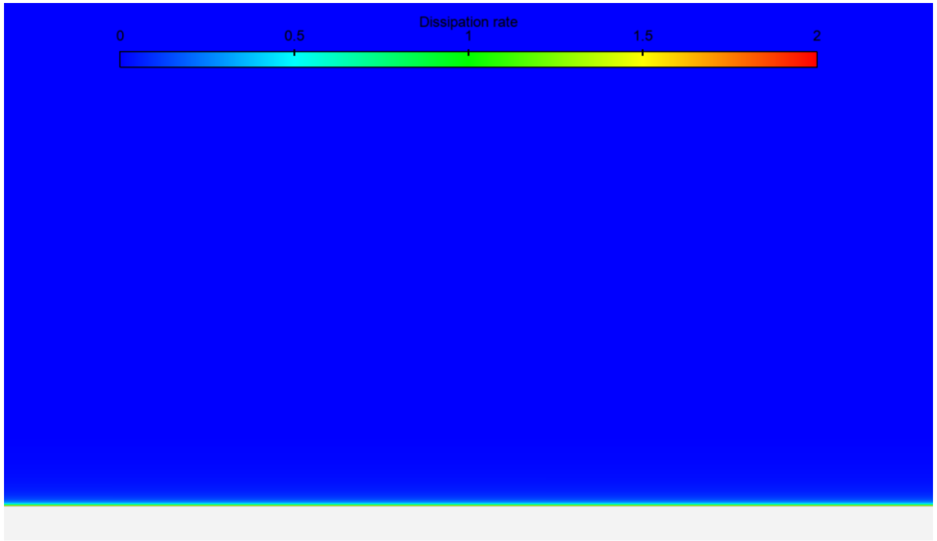


Wall modeling mesh.

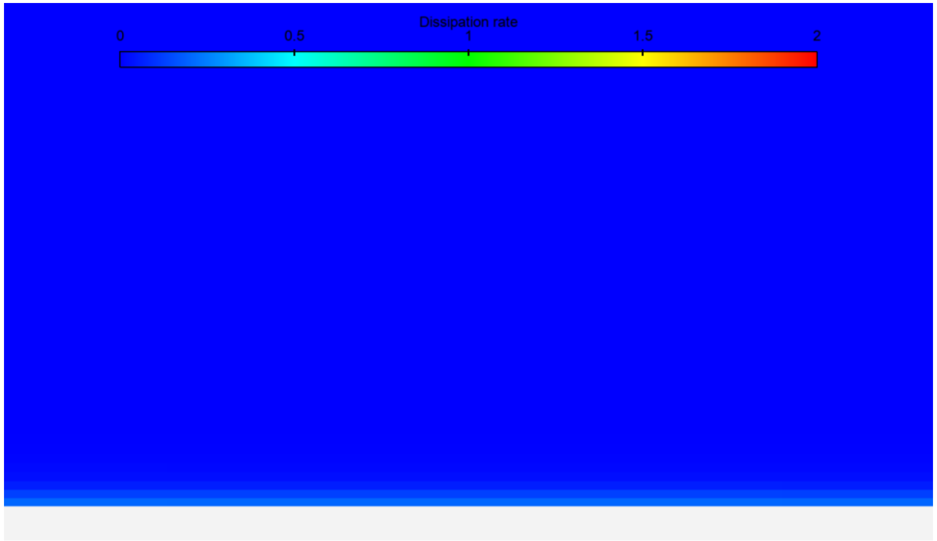
- Plot of turbulent kinetic energy contours.

Near wall treatment

2D Zero pressure gradient flat plate



Wall resolving mesh.

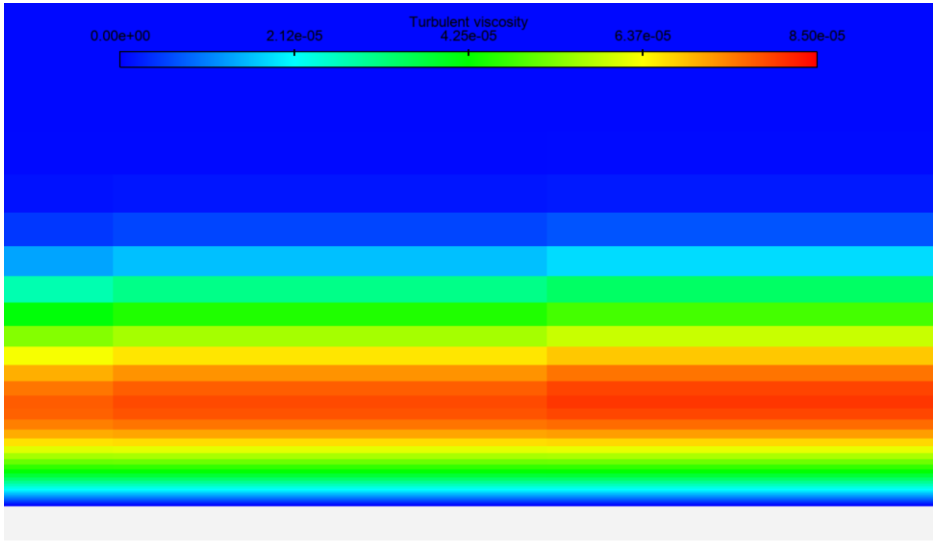


Wall modeling mesh.

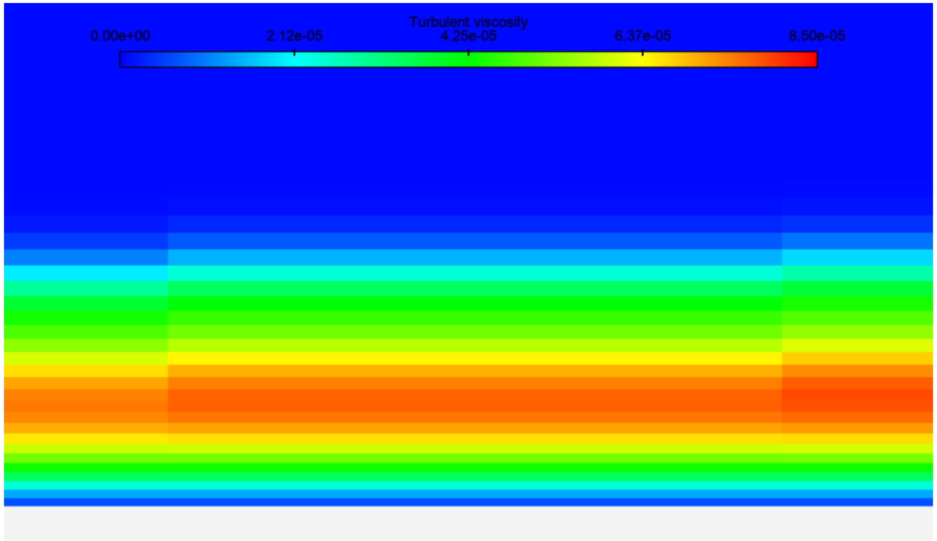
- Plot of turbulent dissipation rate contours.

Near wall treatment

2D Zero pressure gradient flat plate



Wall resolving mesh.



Wall modeling mesh.

- Plot of turbulent viscosity contours.
- The eddy viscosity ratio EVR (for general flows) or the turbulent viscosity (when dealing with incompressible flows) can be used to identify the boundary layer thickness as theirs values usually peaks in the middle of the boundary layer.