Turbulence and CFD models: Theory and applications

Roadmap to Lecture 6

Part 5

1. RANS models deficiencies, palliatives, and corrections

RANS models deficiencies, palliatives, and corrections

- The Boussinesq approximation (or EVM) is a brutal simplification of reality, and this can be a major source of predictive defects.
- And this is regardless of the closure approximations used (turbulence models), which are, in themselves, the cause of additional errors.
- The Boussinesq approximation lies in the belief that the Reynolds Stress tensor behaves in a similar fashion as the Newtonian viscous stress tensor.
- The EVM models assume a linear behavior of the Reynolds stress tensor.
- The main deficiencies of the EVM models are:
    - The assumption of isotropy in shear flows (which is not strictly true),

$$\overline{u'^2} = \overline{v'^2} = \overline{w'^2} = \frac{2}{3}k$$

    - The possibility of predicting negative normal shear stresses.
    - The use of a scalar eddy viscosity for every component of the Reynolds stress tensor.

RANS models deficiencies, palliatives, and corrections

- Let us recall the Boussinesq approximation.
- Remember, this approximation is the core of all eddy viscosity models (EVM).

$$\tau^R = -\rho\left(\overline{\mathbf{u}'\mathbf{u}'}\right) = 2\mu_t \bar{\mathbf{S}}^R - \frac{2}{3}\rho k \mathbf{I} = \mu_t \left[\nabla \bar{\mathbf{u}} + \nabla \bar{\mathbf{u}}^T\right] - \frac{2}{3}\rho k \mathbf{I}$$

Reynolds averaged strain-rate tensor

Identity matrix (or Kronecker delta).

$$\bar{\mathbf{S}}^R = \frac{1}{2}\left[\nabla \bar{\mathbf{u}} + \nabla \bar{\mathbf{u}}^T\right]$$

$$k = \frac{1}{2}\overline{\mathbf{u}' \cdot \mathbf{u}'} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Which is equivalent to the Kronecker delta

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{if } i \neq j. \end{cases}$$

RANS models deficiencies, palliatives, and corrections

- Let us summarize a few applications where the Boussinesq approximation is not very accurate,
  - Poor performance in flows with large extra strains, *e.g.*, curved surfaces, strong vorticity, swirling flows.
  - Rotating flows, *e.g.*, turbomachinery, wind turbines.
  - Impinging flows.
  - Highly anisotropic flows and flows with secondary motions, *e.g.*, fully developed flows in non-circular ducts.
  - Non-local equilibrium and flow separation, *e.g.*, airfoil in stall, dynamic stall.
  - Complex three-dimensional flows.
  - Residual turbulent viscosity near the walls.

- Many EVM models has been developed and improved along the years, so they address the shortcomings of the Boussinesq approximation.

RANS models deficiencies, palliatives, and corrections

- In spite of the theoretical weakness of the Boussinesq approximation, it does produce reasonable results for a large number of flows.
- EVM models are the cornerstone of turbulence modeling in industrial applications.
- EVM is an area of active research and new ideas and palliatives to the know deficiencies continues to emerge.
- The deficiencies are found when comparing the numerical results with experimental results, by identifying abnormal behaviors when analyzing the budgets of the transport equations of the turbulence quantities and the shear stresses budget, or by noticing deviations of the non-dimensional velocities from the empirical correlations (law of the wall), among many.
- Many of the corrections take the form of:
    - Additional source terms.
    - Extra terms in the transport equations.
    - Corrective factors (damping, blending, limiting) in some of the terms of the transport equations.

RANS models deficiencies, palliatives, and corrections

- Let us briefly overview a few of the remedies to some of the problems found with EVM.
- Remember, these corrections work by adding source terms, extra terms, or corrective factors (damping, blending, limiting) in some of the terms of the transport equations.
- By looking at the general form of the transport equations of the turbulent quantities, any term can be corrected.

$$\underbrace{\nabla_t \phi}_{\text{Transient term}} + \underbrace{\nabla \cdot \bar{\mathbf{u}} \phi}_{\text{Convection}} = \underbrace{P^{\phi}}_{\text{Production}} + \underbrace{\epsilon^{\phi}}_{\text{Dissipation}} + \underbrace{D^{\phi}}_{\text{Diffusion}} + \underbrace{S^{\phi}}_{\text{Source terms}}$$

- Finally, we will briefly outline some corrective methods, we are not going to elaborate on the closure coefficients or the closure relationships.
- If you are interested in getting more details about one of the methods, we invite you to read the references cited.

RANS models deficiencies, palliatives, and corrections

Production limiters – Stagnation point anomaly

- A disadvantage of two-equation turbulence models is the excessive generation of the turbulent kinetic energy in the vicinity of stagnation points.
- This is known as the stagnation point anomaly [1], where spuriously high levels of TKE are seen as a flow approaches a stagnation point or is subjected to high rates of strain.
- Many corrections have been proposed to avoid this problem.
- Let us discuss a production limiter approach originally proposed by Menter [2].
- In order to avoid the buildup of turbulent kinetic energy in the stagnation regions, the production term in the turbulence equations can be limited as follows,

$$P_k = \min\left(P_k, C_{lim}\rho\epsilon\right)$$

- Where the coefficient C_{lim} has a default value of 10.
- This limiter does not affect the shear layer performance of the model, it only avoids the buildup of TKE in stagnation points in aerodynamic simulations.
- Another formulation is based on the work of Kato and Launder [3].

References:
[1] P. Durbin. On the k−3 stagnation point anomaly. International Journal of Heat and Fluid Flow 17, 1996.
[2] F. R. Menter. Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications. AIAA Journal. 32(8). 1598–1605. August 1994.
[3] M. Kato and B. E. Launder. The modelling of turbulent flow around stationary and vibrating square cylinders. Ninth Symposium on Turbulent Shear Flows. Kyoto, Japan. August 16-18, 1993.

RANS models deficiencies, palliatives, and corrections

Curvature correction

- One drawback of the EVM is that they are insensitive to streamline curvature and system rotation, which play a significant role in many turbulent flows of practical interest.
- Spalart and Shur [1], Shur et al. [2], and Smirnov and Menter [3], have proposed a modification to the production term in order to sensitize EVM models to the effects of streamline curvature and system rotation,

$$P_k \to P_k f_r$$

- The correction basically imposes a multiplier f_r to limit the production term based on the effects of streamline curvature and system rotation.
- The parameter f_r is computed using the following functions,

$$f_r = \max\left[0, 1 + C_{curv}\left(\tilde{f}_r - 1\right)\right]$$

$$\tilde{f}_r = \max\left[\min\left(f_{rotation}, 1.25\right), 0\right]$$

References:
[1] P. R. Spalart, M. L. Shur. On the Sensitization of Turbulence Models to Rotation and Curvature. Aerospace Sci. Tech. 1(5). 297–302. 1997.
[2] M. L. Shur, M. K. Strelets, A. K. Travin, P. R. Spalart. Turbulence Modeling in Rotating and Curved Channels: Assessing the Spalart-Shur Correction. AIAA Journal. 38(5). 2000.
[3] P. E. Smirnov, F. R. Menter. Sensitization of the SST Turbulence Model to Rotation and Curvature by Applying the Spalart-Shur Correction Term. ASME Paper GT 2008-50480. 2008.

RANS models deficiencies, palliatives, and corrections

Curvature correction

- The function $\tilde{f}_r$ is limited in the range from 0 up to 1.25.
  - 0 corresponds, for example, to strong convex curvature (stabilized flow, no turbulence production).
  - 1.25 corresponds, for example, to strong concave curvature (enhanced turbulence production).
- The lower limit in $\tilde{f}_r$ is introduced for numerical stability reasons, whereas the upper limit is needed to avoid over-generation of the eddy viscosity in flows with a destabilizing curvature/rotation.
- The coefficient C_{curv} in the function f_r is used to modify the influence of the strength of the curvature correction (if needed for a specific flow).
- Typically, the value of the coefficient C_{curv} is constant and equal to 1.
- The function $f_{rotation}$ (the empirical function used to account for streamline curvature and system rotation) in $\tilde{f}_r$ is defined as follows,

$$f_{rotation} = (1 + C_{r1}) \frac{2r^*}{1 + r^*} \left[1 - C_{r3} \tan^{-1} \left(C_{r2} \tilde{r}\right)\right] - C_{r1}$$

RANS models deficiencies, palliatives, and corrections

Curvature correction

- The rest of the closure coefficient and relationships can be computed as follows [1],

$$r^* = \frac{S}{\Omega}$$

Terms related to reference frame rotation
(measure of the system rotation)

$$\tilde{r} = 2\Omega_{ik} S_{jk} \left[\frac{DS_{ij}}{Dt} + \left(\epsilon_{imn} S_{jn} + \epsilon_{jmn} S_{in} \right) \Omega_m^{rot} \right] \frac{1}{\tilde{D}}$$

- Components of the Lagrangian derivative of the strain rate tensor [2,3].

$$D_t \mathbf{S} = \partial_t \mathbf{S} + U \cdot \nabla \mathbf{S}$$

- Implementing this term can be quite complex and expensive.
- Sometimes this term is simplified by using the Richardson number or ignoring it completely.

Components of the system rotation rate vector

Levi-Civita or permutation operator

References:
[1] P. R. Spalart, M. L. Shur. On the Sensitization of Turbulence Models to Rotation and Curvature. Aerospace Sci. Tech. 1(5). 297–302. 1997.
[2] S. Arolla, P. Durbin. A rotation/curvature correction for turbulence models for applied CFD. Progress in Computational Fluid Dynamics, an International Journal, Vol. 14, No. 6, 2014.
[3] P. E. Smirnov, F. R. Menter. Sensitization of the SST Turbulence Model to Rotation and Curvature by Applying the Spalart-Shur Correction Term. ASME Paper GT 2008-50480. 2008.

Curvature correction

- The rest of the closure coefficient and relationships can be computed as follows [1],

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) + \underbrace{2\epsilon_{mji}\Omega_m^{rot}}$$

Terms related to reference frame rotation
(measure of the system rotation)

$$\tilde{D} = D^4$$

$$D^2 = 0.5\left(S^2 - \Omega^2\right)$$

$$C_{r1} = 1.0$$

$$C_{r2} = 12.0$$

$$C_{r3} = 1.0$$

For the Spalart-Allmaras model

$$\Omega^2 = 2\Omega_{ij}\Omega_{ij}$$

$$S^2 = 2S_{ij}S_{ij}$$

References:
[1] P. R. Spalart, M. L. Shur. On the Sensitization of Turbulence Models to Rotation and Curvature. Aerospace Sci. Tech. 1(5). 297–302. 1997.

RANS models deficiencies, palliatives, and corrections

Curvature correction

- The rest of the closure coefficient and relationships can be computed as follows [1],

$$\tilde{D} = \Omega D^3$$

$$D^2 = \max\left(S^2, 0.09\omega^2\right)$$

$$C_{r1} = 1.0$$

$$C_{r2} = 2.0$$

$$C_{r3} = 1.0$$

For all other turbulence models [2]

$$\Omega^2 = 2\Omega_{ij}\Omega_{ij}$$

$$S^2 = 2S_{ij}S_{ij}$$

- Note that both the RNG and Realizable $k-\epsilon$ turbulence models already have their own terms to include rotational or swirl effects.
- The curvature correction option should therefore be used with caution for these two models.

References:
[1] P. R. Spalart, M. L. Shur. On the Sensitization of Turbulence Models to Rotation and Curvature. Aerospace Sci. Tech. 1(5). 297–302. 1997.
[2] P. E. Smirnov, F. R. Menter. Sensitization of the SST Turbulence Model to Rotation and Curvature by Applying the Spalart-Shur Correction Term. ASME Paper GT 2008-50480. 2008.

RANS models deficiencies, palliatives, and corrections

Prediction/correction of excessive heat transfer

- In separated or impinging flows, the near-wall length-scale can become too large, resulting in excessively high levels of near-wall turbulence.
- To remedy this behavior, Yap [1] introduced an extra source term into the dissipation rate equation.
- The modification, which is used on the ϵ transport equation is given as follows,

$$S_\epsilon = 0.83\frac{\epsilon^2}{k}\max\left[\left(\frac{l}{\epsilon l_e}-1\right)\left(\frac{l}{\epsilon l_e}\right)^2, 0\right] \quad \text{where} \quad l = \frac{k^{1.5}}{\epsilon} \qquad l_e = C_\mu^{-0.75}\kappa y$$

- This correction is useful to prevent excessive heat transfer at re-attachment and stagnation points.
- To eliminate the dependence of the above source term on the wall distance, a differential form of the length-scale correction was proposed in reference [2].

References:
[1] C. Yap. Turbulent heat and momentum transfer in recirculating and impinging flows . Ph.D. Thesis Manchester University. 1987.
[2] H. Iacovides, M. Raisee. Recent progress in the computation of flow and heat transfer in internal cooling passages of gas turbine blades. Int. J. Heat Fluid Flow 20:320–328. 1999.

RANS models deficiencies, palliatives, and corrections

Realizability

- The term realizability is related to physically tenable Reynolds stress predictions.
- In easier terms, a realizable stated is one in which none of the normal stresses becomes negative, *i.e.*,

$$\overline{u'^2}, \overline{v'^2}, \overline{w'^2} \geq 0$$

- In addition, realizability also satisfy the following condition,

$$\frac{\overline{u'v'}}{\sqrt{\overline{u'^2}}\sqrt{\overline{v'^2}}}, \frac{\overline{u'w'}}{\sqrt{\overline{u'^2}}\sqrt{\overline{w'^2}}}, \frac{\overline{v'w'}}{\sqrt{\overline{v'^2}}\sqrt{\overline{w'^2}}} < 1$$

- The previous relations are referred to as the Schwartz inequality.
- These inequalities can be enforced in EVM models.
- The most known model that satisfy this inequality is the Realizable $k - \epsilon$ [1,2,3].

References:
[1] T. Shih, W. Liou, A. Shabbir, Z. Yang, J. Zhu. A New k-epsilon Eddy-Viscosity Model for High Reynolds Number Turbulent Flows. Computers Fluids. 24(3). 227–238. 1995.
[2] T. Shih, W. Liou, A. Shabbir, Z. Yang, J. Zhu. A New k-epsilon Eddy-Viscosity Model for High Reynolds Number Turbulent Flows – Model Development and Validation. NASA TM 106721. 1994.
[3] T. Shih, J. Zhu. A New Reynolds Stress Algebraic Equation Model. NASA TM 106644. 1994.

RANS models deficiencies, palliatives, and corrections

Damping functions near the walls

- Near the walls, in the viscous sublayer, the turbulent viscosity should exponentially damp to zero.
- Some EVM models might have prediction capability issues since they predict non-zero values (or large values) of turbulent viscosity near the walls.
- To correct this behavior, many EVM use damping functions near the walls.
- For the example, the Van Driest damping function commonly used in mixing length models and some LES models, reads a follows,

$$l = \kappa y \left[1 - e^{-y^+/A^+} \right]$$

l is use as the length scale to compute the turbulent viscosity

$$A^+ = 26$$

This coefficient depends on the pressure gradient

- Another damping function used in the low-RE $k - \epsilon$ turbulence model by Jones and Launder [1], is written as follows,

$$\nu_t = C_\mu f_\mu \frac{k^2}{\epsilon} \qquad f_\mu = e^{-2.5/(1+Re_T/50)} \qquad Re_T = \frac{k^2}{\nu \epsilon}$$

References:

[1] W. Jones, B. Launder. The prediction of laminarization with a two-equation model of turbulence. Intl. J. Heat and Mass Transfer, 15, 301-314. 1972.

RANS models deficiencies, palliatives, and corrections

Multiphase flows corrections

- A multiphase flow is a fluid flow consisting of more than one phase component and has some level of phase separation above the molecular level.
- Multiphase flows exist in many different forms. For example, two phase flows can be classified according to the state of the different phases:
    - Gas-Liquid mixture.
    - Gas-Solid mixture.
    - Liquid-Solid mixture.
    - Immiscible liquid-liquid.
- To add turbulence, we only need to do the proper averaging (Reynolds or Favre) or use a filtering technique for LES simulations.
- However, there are may considerations that should be taken into account when using turbulence models with multiphase flows.
- These considerations can be related to reformulation of the turbulence models, recalibration of the closure requirements, meshing requirements, and so on.

RANS models deficiencies, palliatives, and corrections

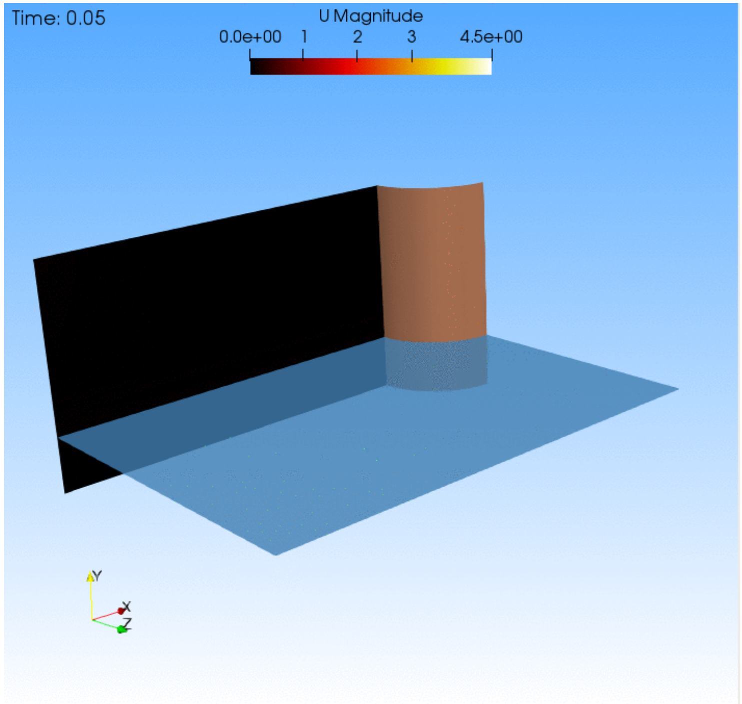
Multiphase flows corrections

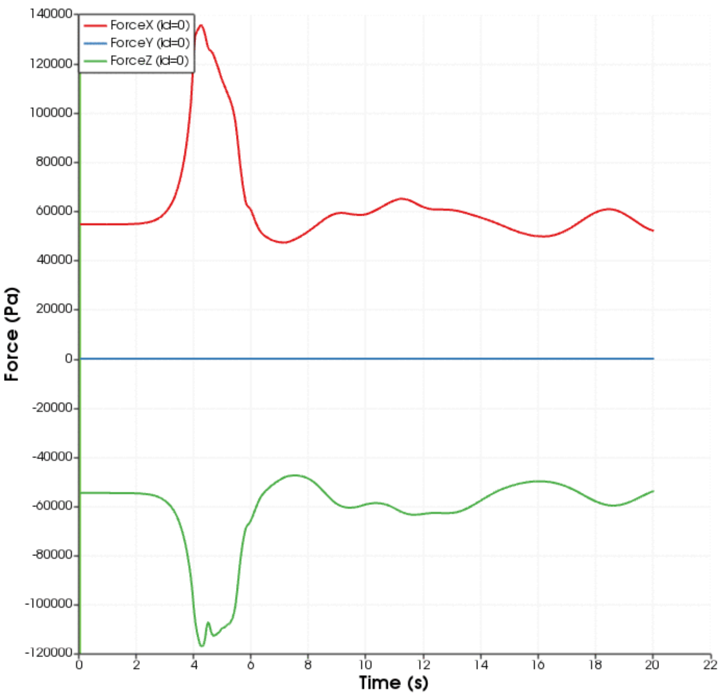
- In multiphase flows, spurious velocities or high velocity gradients at the interface between two fluids can result in high turbulence generation, in both phases.
- This well know deficiency of the turbulence models when dealing with multiphase flows can be corrected using multiphase stabilization techniques.
- In references [1,2,3,4] a few stabilization/damping techniques are presented.
- These techniques consist in applying corrections to the turbulent quantities, so spurious turbulent viscosity or parasite velocities are not produced at the interface.
- This is usually achieved by adding extra source term to the VOF equations in order to reduce the turbulence generated near a free-surface.
- Similar corrections can be applied to dispersed flows, granular flows, and Langrangian particles.

References:
[1] Devolder, B., Rauwoens, P., and Troch, P. (2017). Application of a buoyancy-modified k-w SST turbulence model to simulate wave run-up around a monopile subjected to regular waves using OpenFOAM. Coastal Engineering, 125, 81-94.
[2] Larsen, B.E. and Fuhrman, D.R. (2018). On the over-production of turbulence beneath surface waves in Reynolds-averaged Navier-Stokes models J. Fluid Mech, 853, 419-460
[3] Frederix, E. M. A., Mathur, A., Dovizio, D., Geurts, B. J., Komen, E. M. J. (2018). Reynolds-averaged modeling of turbulence damping near a large-scale interface in two-phase flow. Nuclear engineering and design, 333, 122-130.
[4] Egorov, Y (2204). Contact Condensation in Stratified Steam-Water Flow. EVOL-ECORDA-D 07.

Multiphase flows corrections

- In the figure, we show a simulation of the wave impact in a column using the VOF approach for separated flows.
- Note the spurious velocities and high velocity gradients at the interface between both fluids.
- Turbulence damping is required in the interfacial area to model the turbulent flow and the multiphase flow correctly.





http://www.wolfdynamics.com/training/mphase/gt1/hexa/ani3.gif

RANS models deficiencies, palliatives, and corrections

Effects of buoyancy on turbulence models

- The effects of buoyancy can be included in the turbulent transport equations.
- For example, in the $k-\omega$ model, the generation G (or production P) of turbulence due to buoyancy can be computed using the following relationship,

$$G_{\omega b} = \frac{\omega}{k} C_{1\epsilon}(C_{3\epsilon} G_b) - \frac{\omega}{k} G_b$$

- Where, as usual, we find many coefficients that requires calibration. Of all the coefficients, the most interesting one is,

$$C_{3\epsilon} = \tanh\left(\frac{v}{u}\right)$$

- This coefficient defines the degree to which the production term is affected by the buoyancy [1].
- In $C_{3\epsilon}$, *v* is the component of the flow velocity parallel to the gravitational vector and *u* is the component of the flow velocity perpendicular to the gravitational vector.
- That is, $C_{3\epsilon}$ will become 1 for buoyant shear layers for which the main flow direction is aligned with the direction of gravity.
- For buoyant shear layers that are perpendicular to the gravitational vector, $C_{3\epsilon}$ will become zero.

References:
[1] R. Henkes, F. van der Flugt, C. Hoogendoorn. Natural Convection Flow in a Square Cavity Calculated with Low-Reynolds-Number Turbulence Models. Int. J. Heat Mass Transfer. 34. 1991.

RANS models deficiencies, palliatives, and corrections

- We just briefly addressed a few of the deficiencies, palliatives, and corrections of turbulence models.
- These corrections are derived using a very rigorous mathematical framework.
- Have in mind that there are many more.
- Turbulence models are very flawed.
- Nevertheless, they do work.