Turbulence and CFD models: Theory and applications

Roadmap to Lecture 9

1. **Favre averaging**
2. **Wall functions for heat transfer**
3. **Wall functions – Additional observations**
4. **Surface roughness**

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Favre averaging

- When dealing with compressible flows (or variable density flows), besides the velocity and pressure fluctuations, we must also account for density and temperature fluctuations.
- After applying the Reynolds decomposition and time-averaging to the exact Navier-Stokes equations, additional fluctuating correlations arise.
- To illustrate this, let us consider the conservation of mass equation for a compressible flow,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

- Now, let us use the Reynolds decomposition for the primitive variables (density and velocity),

$$\phi(\mathbf{x}, t) = \bar{\phi}(\mathbf{x}) + \phi'(\mathbf{x}, t) \qquad \text{or} \qquad \phi(\mathbf{x}, t) = \bar{\phi}(\mathbf{x}, t) + \phi'(\mathbf{x}, t)$$

Favre averaging

- Substituting the Reynolds decomposition into the continuity equation yields,

$$\frac{\partial(\bar{\rho} + \rho')}{\partial t} + \nabla \cdot (\bar{\rho}\mathbf{u} + \rho'\mathbf{u} + \bar{\rho}\mathbf{u}' + \rho'\mathbf{u}') = 0$$

- By time averaging the previous equation and using the Reynolds averaging rules, we arrive to the Reynolds-averaged continuity equation for compressible flows,

$$\frac{\partial(\bar{\rho})}{\partial t} + \nabla \cdot \left(\bar{\rho}\bar{\mathbf{u}} + \overline{\rho'\mathbf{u}'}\right) = 0$$

- To achieve closure, we need to somehow approximate the correlation between the fluctuating quantities ($\overline{\rho'\mathbf{u}'}$).
- The situation is more complicated for the momentum and energy equations, where triple correlations involving the density fluctuations appear.

Favre averaging

- To simplify the problem, we introduce the density-weighted averaging procedure suggested by Favre [1].
- That is, we introduce the mass-weighted average of the quantity ϕ , as follows,

$$\tilde{\phi} = \frac{1}{\bar{\rho}} \lim_{T \to \infty} \frac{1}{T} \int_{t}^{t+T} \rho(\mathbf{x}, t)\phi(\mathbf{x}, t)dt = \frac{\overline{\rho\phi}}{\bar{\rho}}$$

- For example, the mass-weighted average of the velocity field is given as follows,

Favre averaging → $\tilde{\mathbf{u}}$; Reynolds averaging → $\overline{\rho \mathbf{u}}$, $\bar{\rho}$

$$\tilde{\mathbf{u}} = \frac{\overline{\rho \mathbf{u}}}{\bar{\rho}} \quad \text{or} \quad \bar{\rho}\tilde{\mathbf{u}} = \overline{\rho \mathbf{u}} \qquad (1)$$

- In our notation, the overbar denotes Reynolds averaging and the tilde denotes Favre averaging.
- If we expand the RHS of equation 1, that is, we substitute the Reynolds decomposition and use the Reynolds averaging rules, we obtain,

$$\bar{\rho}\tilde{\mathbf{u}} = \bar{\rho}\bar{\mathbf{u}} + \overline{\rho' \mathbf{u}'}$$

[1] A. Favre. Equations des Gaz Turbulents Compressibles. Journal de Mecanique. 1965.

Favre averaging

- If we substitute the following relation,

$$\bar{\rho}\tilde{\mathbf{u}} = \bar{\rho}\bar{\mathbf{u}} + \overline{\rho'\mathbf{u}'}$$

- Into the Reynolds averaged compressible continuity equation,

$$\frac{\partial(\bar{\rho})}{\partial t} + \nabla \cdot \left(\bar{\rho}\bar{\mathbf{u}} + \overline{\rho'\mathbf{u}'}\right) = 0$$

- We obtain the following equation,

$$\frac{\partial(\bar{\rho})}{\partial t} + \nabla \cdot (\bar{\rho}\tilde{\mathbf{u}}) = 0$$

- This is the Favre averaged compressible continuity equation.
- Which looks very similar to the incompressible Reynolds averaged equations.
- As for the Reynolds averaged equations, we simply eliminated the correlation of the fluctuating quantities.
- We can proceed in a similar fashion for the momentum and energy equations.

Favre averaging

- Like the Reynolds decomposition, we can introduce a Favre decomposition of the variable ϕ, as follows,

$$\phi = \tilde{\phi} + \phi''$$

Notice that this fluctuating quantity also includes the effects of density fluctuations

- And to form a Favre average, we simply multiply by the density and do time average (in the same way as in Reynolds average),

$$\overline{\rho\phi} = \bar{\rho}\tilde{\phi} + \overline{\rho\phi''}$$

- And recall that the mass-weighted average of the field ϕ is given as follows,

$$\tilde{\phi} = \frac{\overline{\rho\phi}}{\bar{\rho}}$$

- It is important to mention that density and pressure are not mass-weighted averaged.
- For density and pressure, we use Reynolds averaging (that is, Reynolds decomposition).
- The rest of the field variables can be mass-weighted averaged, namely, u, v, w, T, h, H, e.

Favre averaging

- At this point, the rules of Favre averaging are summarized as follows,

$$\bar{\tilde{\phi}} = \tilde{\phi}$$

$$\overline{\rho\tilde{\phi}} = \bar{\rho}\tilde{\phi}$$

$$\overline{\rho\phi''} = 0$$

$$\overline{\phi''} = -\frac{\overline{\rho'\phi'}}{\bar{\rho}} \neq 0$$

$$\overline{\rho\phi\psi} = \bar{\rho}\tilde{\phi}\tilde{\psi} + \overline{\rho\phi''\psi''}$$

$$\widetilde{\rho\phi} = \bar{\rho}\tilde{\phi}$$

- Plus, the Reynolds averaging rules.
- Finally, if the density is constant, $\tilde{\phi} = \bar{\phi}$ and $\phi'' = \phi'$, and we recast the incompressible RANS equations.

Favre averaging

- The Favre-averaged governing equations can be written as follows (using index notation),

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i}\left(\bar{\rho}\tilde{u}_i\right) = 0$$

$$\frac{\partial \bar{\rho}\tilde{u}_i}{\partial t} + \frac{\partial \bar{\rho}\tilde{u}_j\tilde{u}_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\bar{\tau}_{ji} - \overline{\rho u_j'' u_i''}\right]$$

$$\frac{\partial}{\partial t}\left[\bar{\rho}\left(\tilde{e} + \frac{\tilde{u}_i\tilde{u}_i}{2}\right) + \frac{\overline{\rho u_i'' u_i''}}{2}\right] + \frac{\partial}{\partial x_j}\left[\bar{\rho}\tilde{u}_j\left(\tilde{h} + \frac{\tilde{u}_i\tilde{u}_i}{2}\right) + \tilde{u}_j\frac{\overline{\rho u_i'' u_i''}}{2}\right] =$$

$$\frac{\partial}{\partial x_j}\left[-q_{Lj} - \overline{\rho u_j'' h''} + \overline{\tau_{ji} u_j''} - \overline{\rho u_j'' \frac{1}{2} u_i'' u_i''}\right] + \frac{\partial}{\partial x_j}\left[\tilde{u}_i\left(\bar{\tau}_{ij} - \overline{\rho u_i'' u_j''}\right)\right]$$

$$P = \bar{\rho} R \tilde{T}$$

- These are the exact FANS equations (Favre-averaged Navier-Stokes).
- As for the RANS equations, we need to add approximations to derive the solvable equations.

Favre averaging

- Let us review the most commonly used approximations for compressible flows.
  - The Reynolds-Stress tensor (or Favre-Stress tensor), can be modeled as follows,

Sometimes ignored for non-supersonic flows

$$\bar{\rho}\tau_{ij}^{F} = \overline{-\rho u_i'' u_i''} = \mu_T \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} - \frac{2}{3}\frac{\partial \tilde{u}_k}{\partial x_k}\delta_{ij} \right) - \frac{2}{3}\bar{\rho} k \delta_{ij}$$

In incompressible flows this term is equal to zero

- This is the Boussinesq assumption but formulated for compressible flows.

Favre averaging

- Let us review the most commonly used approximations for compressible flows.
    - Turbulent heat-flux vector,

$$q_{Tj} = \overline{\rho u_j'' h''} = -\frac{\mu_T c_p}{Pr_T} \frac{\partial \tilde{T}}{\partial x_j} = -\frac{\mu_T}{Pr_T} \frac{\partial \tilde{h}}{\partial x_j}$$

    - Molecular diffusion and turbulent transport,

$$\overline{\tau_{ji} u_i''} - \overline{\rho u_j'' \frac{1}{2} u_i'' u_i''} = \left(\mu + \frac{\mu_T}{\sigma_k}\right) \frac{\partial k}{\partial x_j}$$

- These two terms are modeled using the gradient diffusion hypothesis.

Favre averaging

- It is important to mention that for compressible flows, the non-zero divergence of the Favre averaged velocity modifies the mean strain rate term in the RHS of the Reynolds-Stress tensor.
- Therefore, the Reynolds-Stress tensor is manipulated in such a way to guarantee that its trace is equal to $-2k$.
- This implies that the second eddy viscosity is equal to,

$$\frac{2}{3}\mu_T$$

- The turbulent kinetic energy can be computed a follows,

$$\bar{\rho}k = \frac{1}{2}\overline{\rho u_i'' u_i''}$$

- The viscous stress tensor τ_{ij} and the molecular heat flux q_L (or laminar) are computed as follows,

$$\tau_{ij} = \mu\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial \tilde{u}_k}{\partial x_k}\delta ij \qquad q_L = -\kappa\frac{\partial \tilde{T}}{\partial x_j}$$

Favre averaging

- As can be seen, the Favre averaging is very similar to the Reynolds averaging.
- To derive the solvable equations, we must approximate all the terms that involve correlations of fluctuating quantities.
- Similar to the incompressible equations, we can derive the exact transport equations for the turbulent kinetic energy and Reynolds stresses.
- Favre averaging is used for compressible flows, mixture of gases, species concentration, and combustion.
- Favre averaging eliminates density fluctuations from the averaged equations.
- However, it does not remove the effect the density fluctuations have on the turbulence.
- Favre averaging is a mathematical simplification, not a physical one.
- Remember, when using Favre averaging, the density and pressure are not mass-weighted averaged.
    - For density and pressure, we use Reynolds averaging (that is, Reynolds decomposition).
    - The rest of the field variables can be mass-weighted averaged, namely, u, v, w, T, h, H, e.

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Wall functions for heat transfer

- As for the momentum (or viscous) wall functions, there is also a treatment for the thermal wall functions.
- Depending on the value of y*, the value of the non-dimensional temperature T* (equivalent to the concept of u*), can be computed as follows,

Thermal viscous sublayer where conduction is important

$$T^* = \begin{cases} Pr\, y^* & y^* < 5 \\ Pr_t \left[\dfrac{1}{\kappa} \ln \left(E y^* \right) + P \right] & y^* > 30 \end{cases}$$

Logarithmic law for the turbulent region where effects of turbulence dominate conduction

Pr

Molecular (or laminar) Prandtl number

Pr_t

Turbulent Prandtl number

Wall functions for heat transfer

- The following nondimensional temperature relation (in the log-law region) is used to relate the temperature at the cell center T_P to the heat flux at the wall q_w.

$$T^* = Pr_t \left[\frac{1}{\kappa} \ln \left(E y^* \right) + P \right]$$

- This is similar to what we did with the viscous wall function, where we related U_P to τ_w.

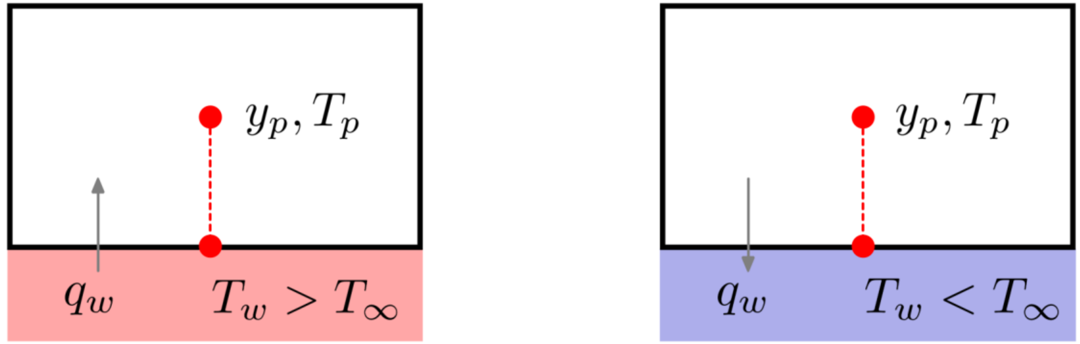
Wall functions for heat transfer

- When using thermal wall functions, we are interested in computing the thermal diffusivity used to approximate the wall heat transfer q_w,

Thermal conductivity (k) — Specific heat (c_p) — Thermal diffusivity (α)

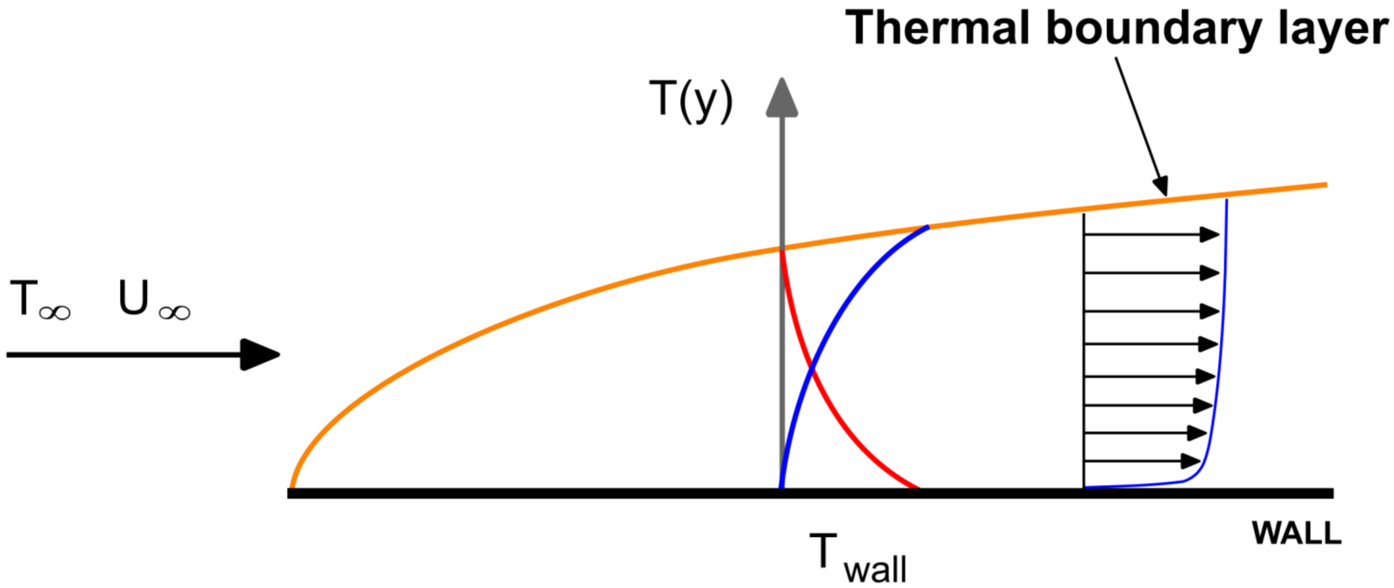
$$q_w = k\frac{\partial T}{\partial y} = \rho c_p \alpha \frac{\partial T}{\partial y} = \rho c_p \alpha \frac{T_w - T_p}{y_p - y_w}$$

- We need to relate the temperature at the cell center T_P to the heat flux at the wall q_w.



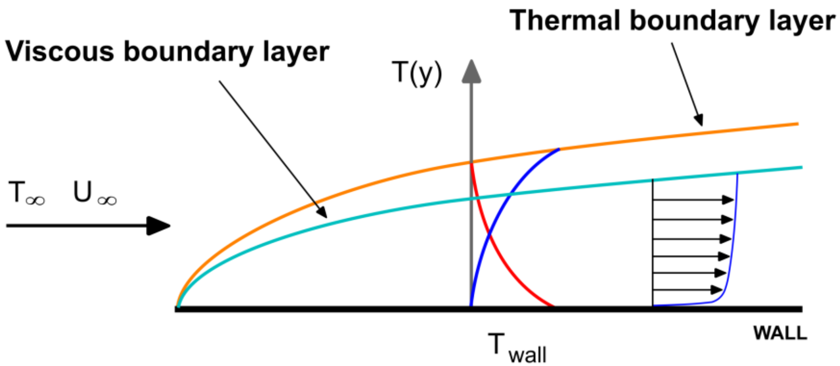
Wall functions for heat transfer

- Recall that in the momentum boundary layer, the velocity at the walls is zero.
- In the thermal boundary layer, the temperature at the wall can be higher or lower than the freestream temperature.
- Therefore, we can have very different temperature profiles growing from the walls.
- The temperature at the walls also has an influence of the wall shear stresses.

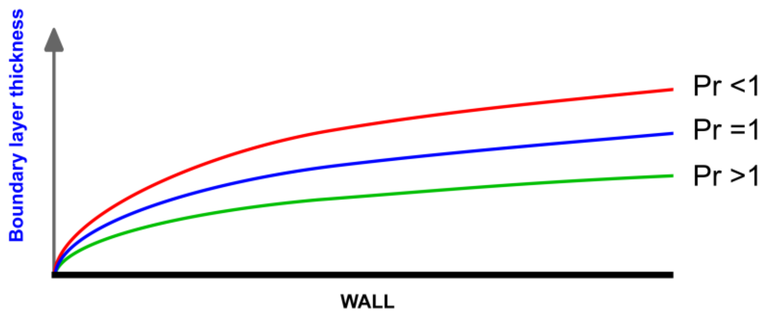


Wall functions for heat transfer

Momentum and thermal boundary layer



Thermal boundary layer vs. Viscous boundary layer
Forced convection

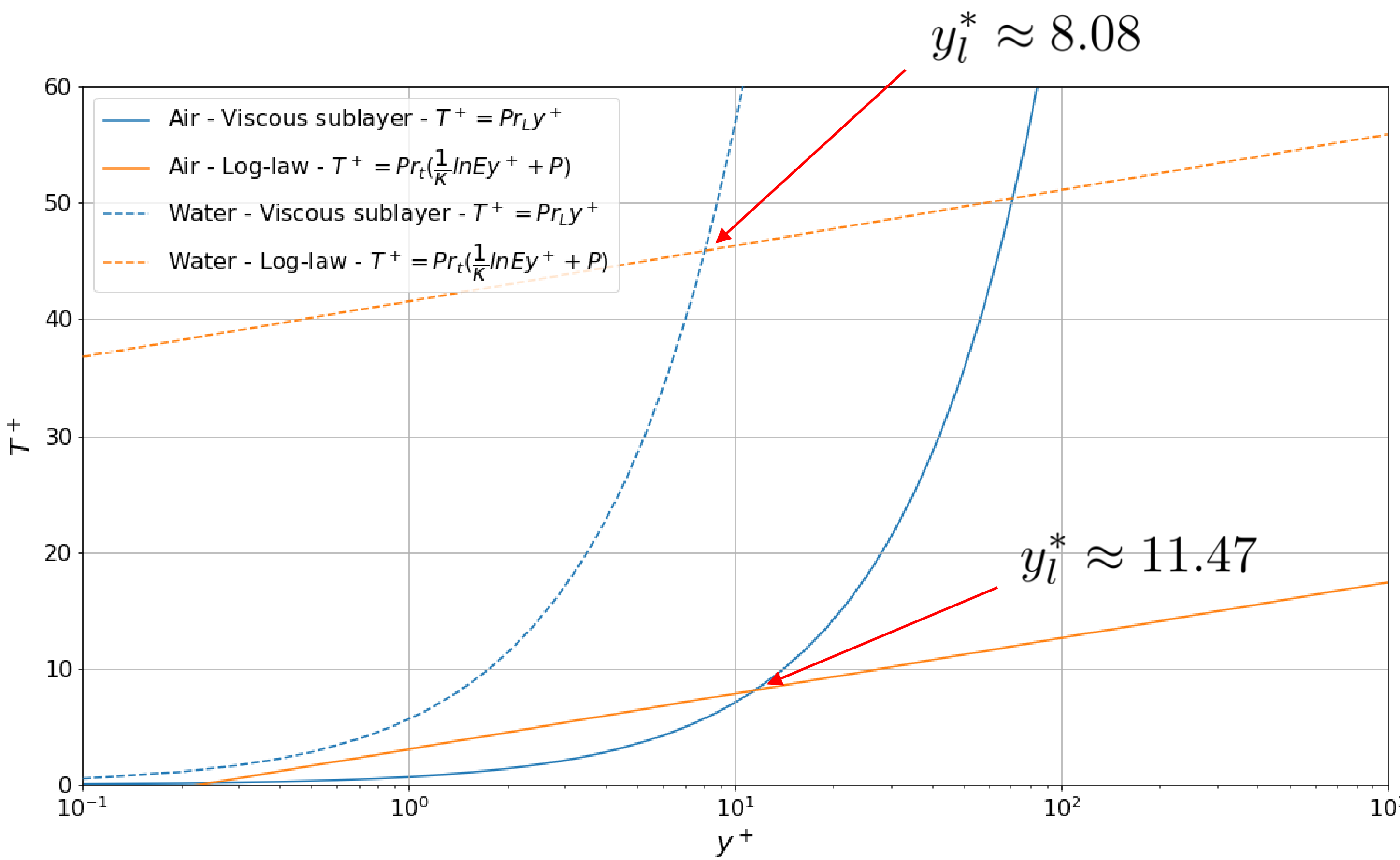


Thermal boundary layer in function of Prandtl number (Pr)

- Just as there is a viscous boundary layer in the velocity distribution (or momentum), there is also a thermal boundary layer.
- Thermal boundary layer thickness is different from the thickness of the viscous sublayer (momentum) and is fluid dependent.
- The thickness of the thermal sublayer for a high Prandtl number fluid (*e.g.*, water) is much less than the momentum sublayer thickness.
- For fluids of low Prandtl numbers (*e.g.*, air), it is much larger than the momentum sublayer thickness.
- For Prandtl number equal 1, the thermal boundary layer is equal to the momentum boundary layer.

Wall functions for heat transfer

- The normalized temperature plot (T^+ vs. y^+), depends on the Prandtl number.
- Different values of Prandtl number will result in different plots, with different intersection points between the viscous sublayer and the log-law region.
- The intersection point of the viscous sublayer and the log-law region is known as the non-dimensional thermal sublayer thickness or y_l^*.



Wall functions for heat transfer

- The thermal diffusivity coefficient α is computed as follows,

$$\alpha_w = \begin{cases} \alpha & y^* < y_l^* \\ \dfrac{u_\tau y_p}{Pr_t \left[\dfrac{1}{\kappa} ln\left(E y^*\right) + P \right]} & y^* > y_l^* \end{cases}$$

- Remember, the value of y_l^* depends on the Prandtl number.
- At this point, the heat flux q_w at the wall can be computed as follows,

$$q_w = \rho c_p \alpha_w \frac{T_w - T_p}{y_p - y_w}$$

- The turbulent Prandtl number Pr_t is usually equal to 0.85, but it highly depends on the flow properties and the flow physics.

Wall functions for heat transfer

- Let us revisit the nondimensional temperature function T^*,

$$T^* = Pr_t \left[\frac{1}{\kappa} \ln \left(E y^* \right) + P \right] \qquad y^* > 30$$

- The term *P* appearing in this relation, has a strong dependence on the Prandtl number (molecular and turbulent).
- One way to approximate this term is by using Jayatilleke function [1],

$$P = 9.24 \left[\left(\frac{Pr}{Pr_t} \right)^{3/4} - 1 \right] \left[1 + 0.28 e^{-0.007 Pr/Pr_t} \right]$$

- Kader [2] and Patankar and Spalding [3] also proposed alternative formulations for computing P.

[1] C. Jayatilleke. The influence of Prandtl number and surface roughness on the resistance of the laminar sublayer to momentum and heat transfer. Prog. Heat Mass Transfer, 1, 193-329. 1969.
[2] B. Kader. Temperature and concentration profiles in fully turbulent boundary layers. Int. J. Heat Mass Transfer. 24(9). 1981.
[3] S. Patankar and D. Spalding. A calculation procedure for heat, mass and momentum transfer in three dimensional parabolic flows. Int. J. Heat Mass Transfer, 15(10). 1972.

Wall functions for heat transfer

- Let us summarize the steps needed to compute the wall heat flux q_w using wall functions,
  - Compute y*.
  - Compute the Prandtl number,

$$Pr = \frac{\nu}{\alpha}$$

  - Compute the intersection point y_l^*.
  - Compute the thermal diffusivity at the wall.

$$\alpha_w = \begin{cases} \alpha & y^* < y_l^* \\ \dfrac{u_\tau y_p}{Pr_t \left[\dfrac{1}{\kappa} ln\left(E y^*\right) + P \right]} & y^* > y_l^* \end{cases}$$

  - Compute the heat flux at the wall.

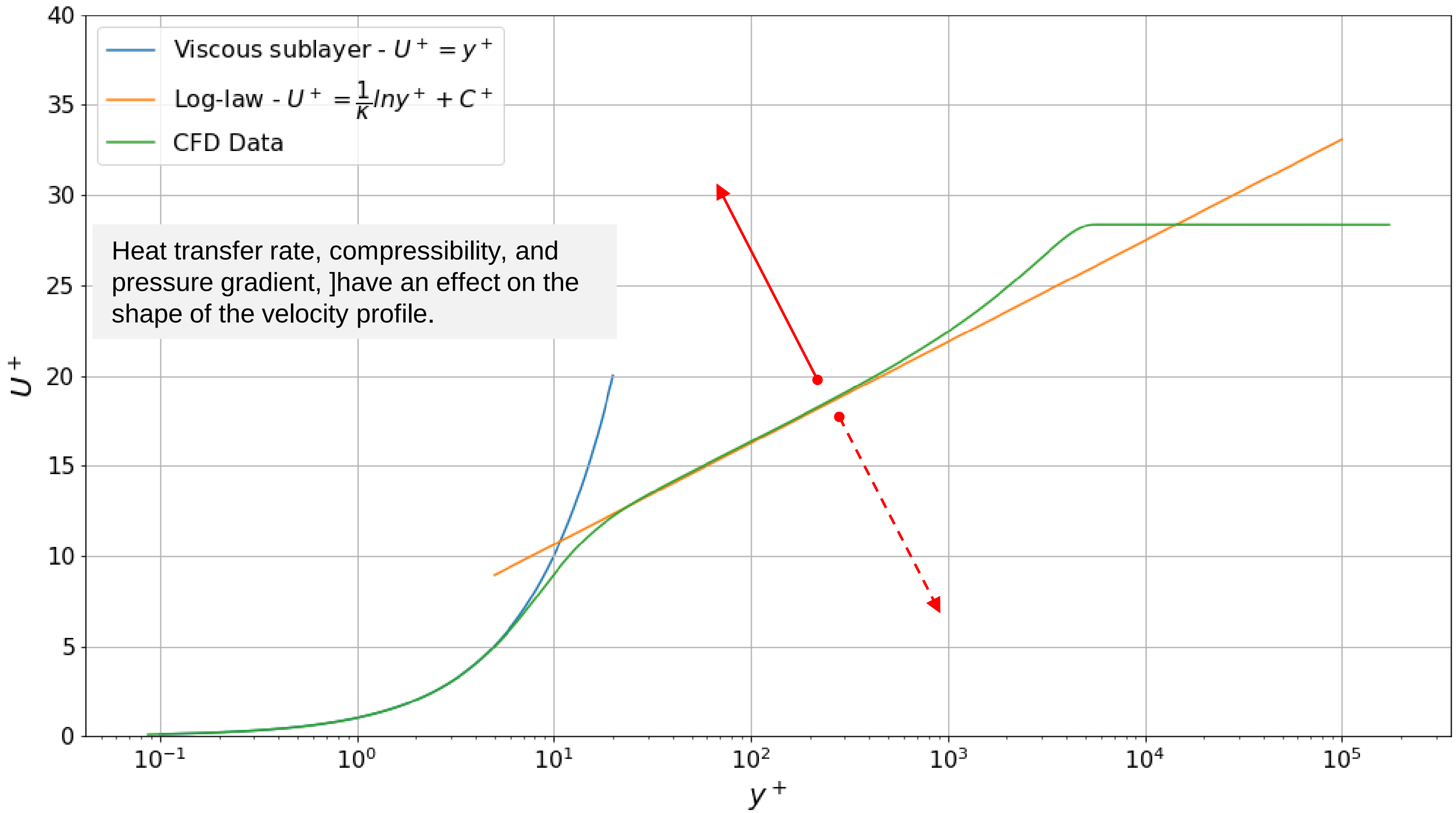
$$q_w = \rho c_p \alpha_w \frac{T_w - T_p}{y_p - y_w}$$

Roadmap to Lecture 9

1. ~~Favre averaging~~
2. ~~Wall functions for heat transfer~~
3. **Wall functions – Additional observations**
4. Surface roughness

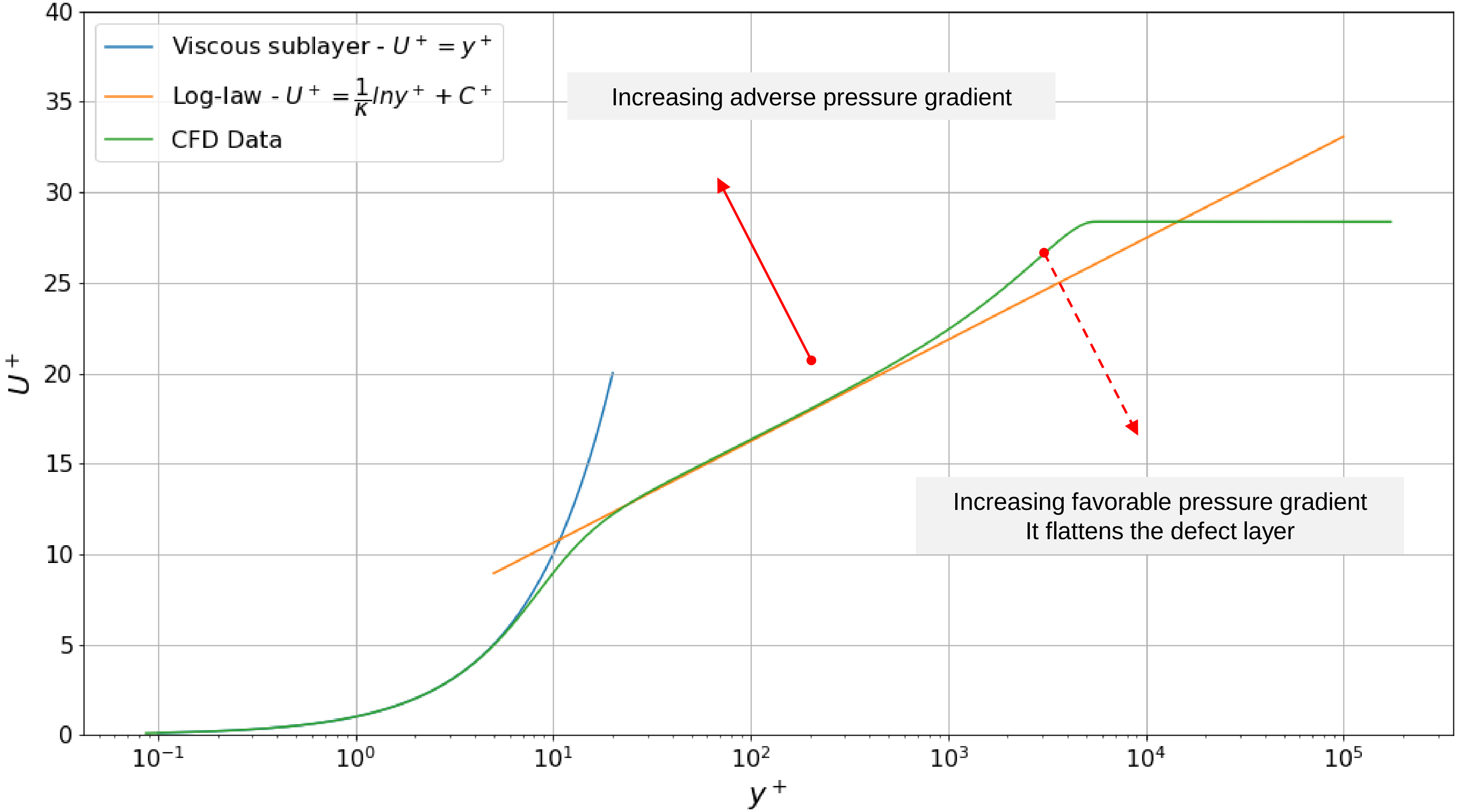
Wall functions – Additional observations

- When studying the boundary layer, heat transfer rate, compressibility, pressure gradient, mach number, and surface roughness (among many factors), they all have an effect on the velocity profile.
- It is not obvious what effect each of these parameters have on the velocity profile.
- Hereafter, we present a few observations or general guidelines (use with care).



Wall functions – Additional observations

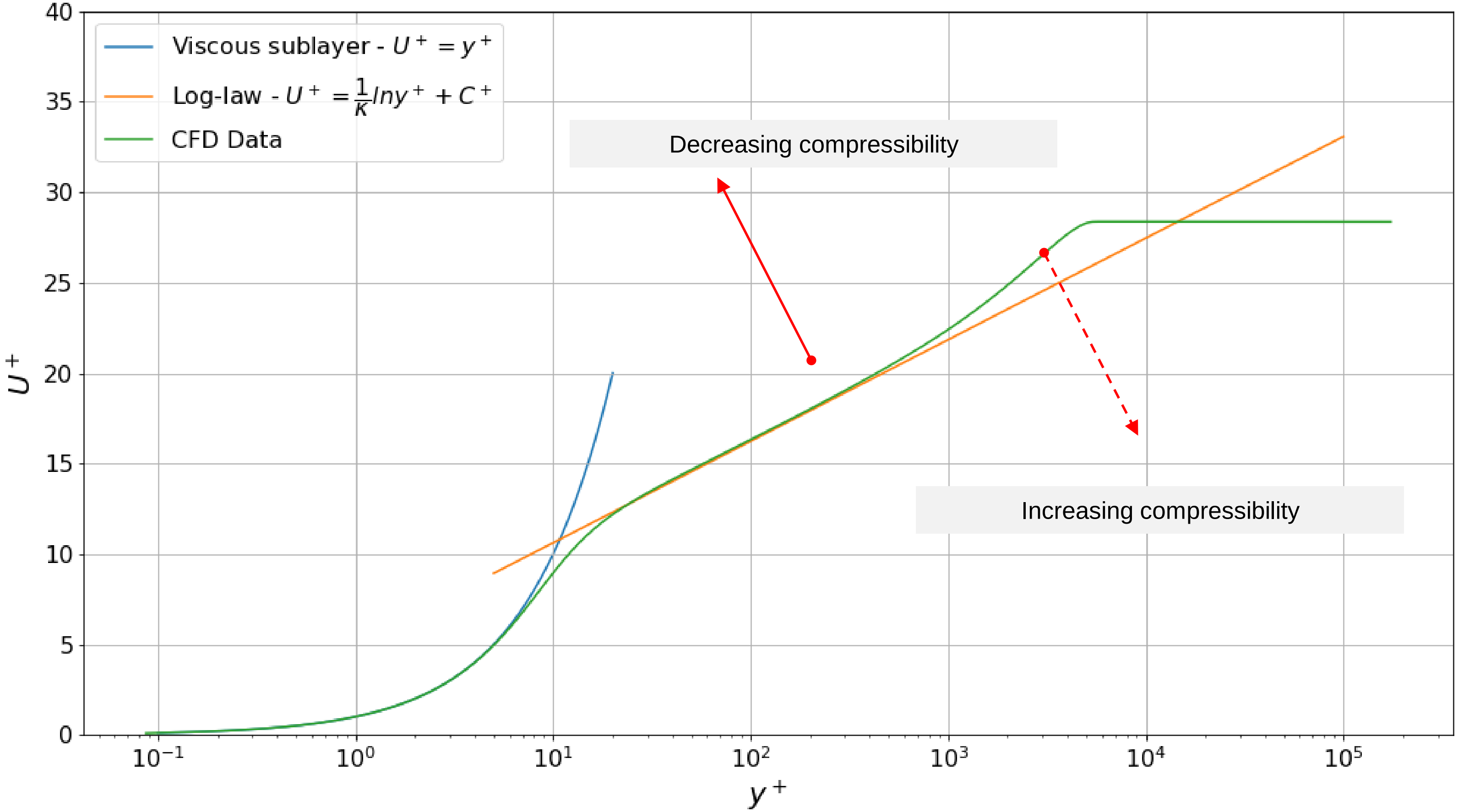
- Effect of adverse pressure gradient on the law of the wall.



- The boundary layer thickens, and the skin friction decreases as the adverse pressure gradient is increased.

Wall functions – Additional observations

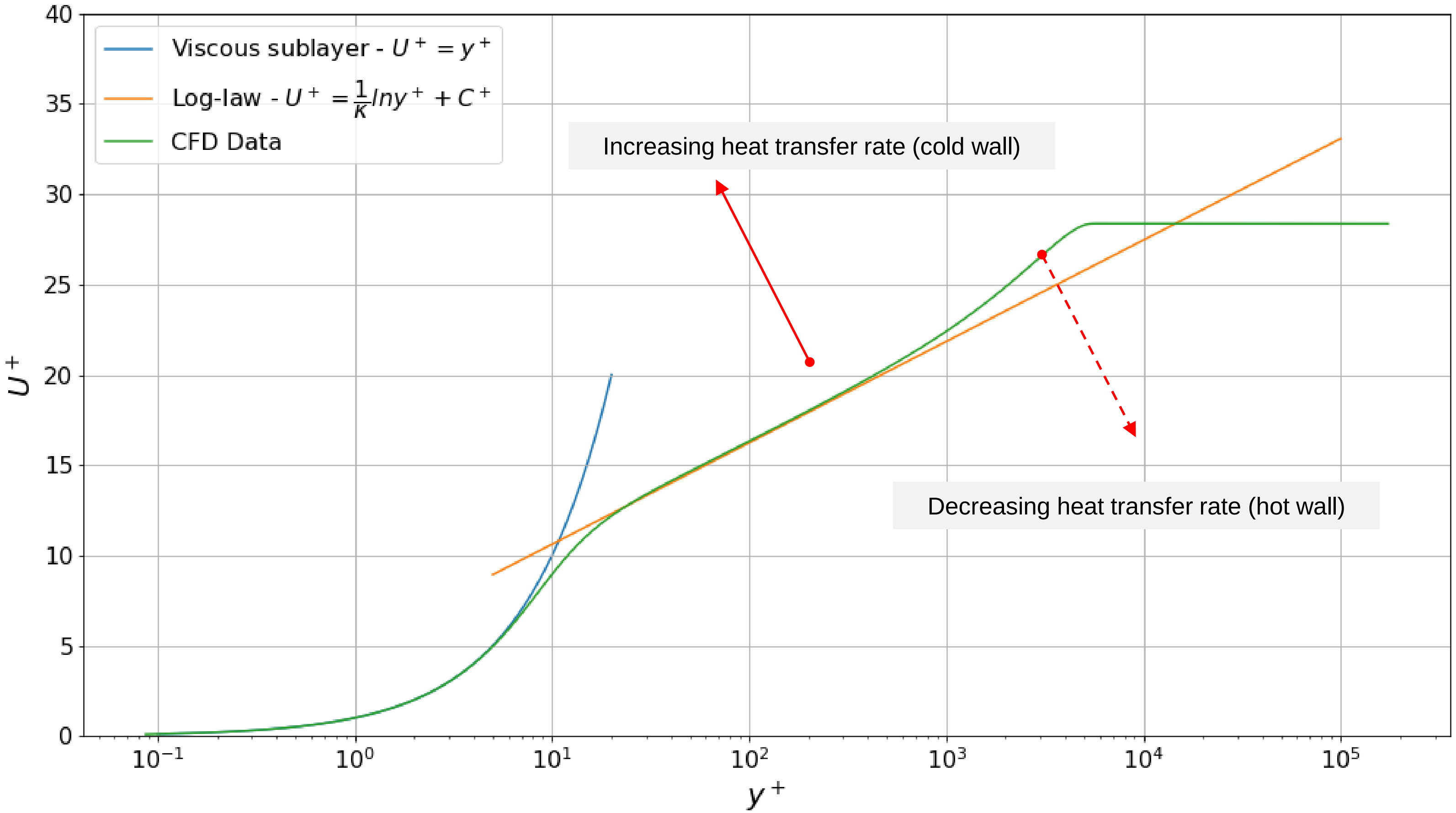
- Effect of compressibility on the law of the wall.



- Increasing compressibility causes the skin friction to decrease.

Wall functions – Additional observations

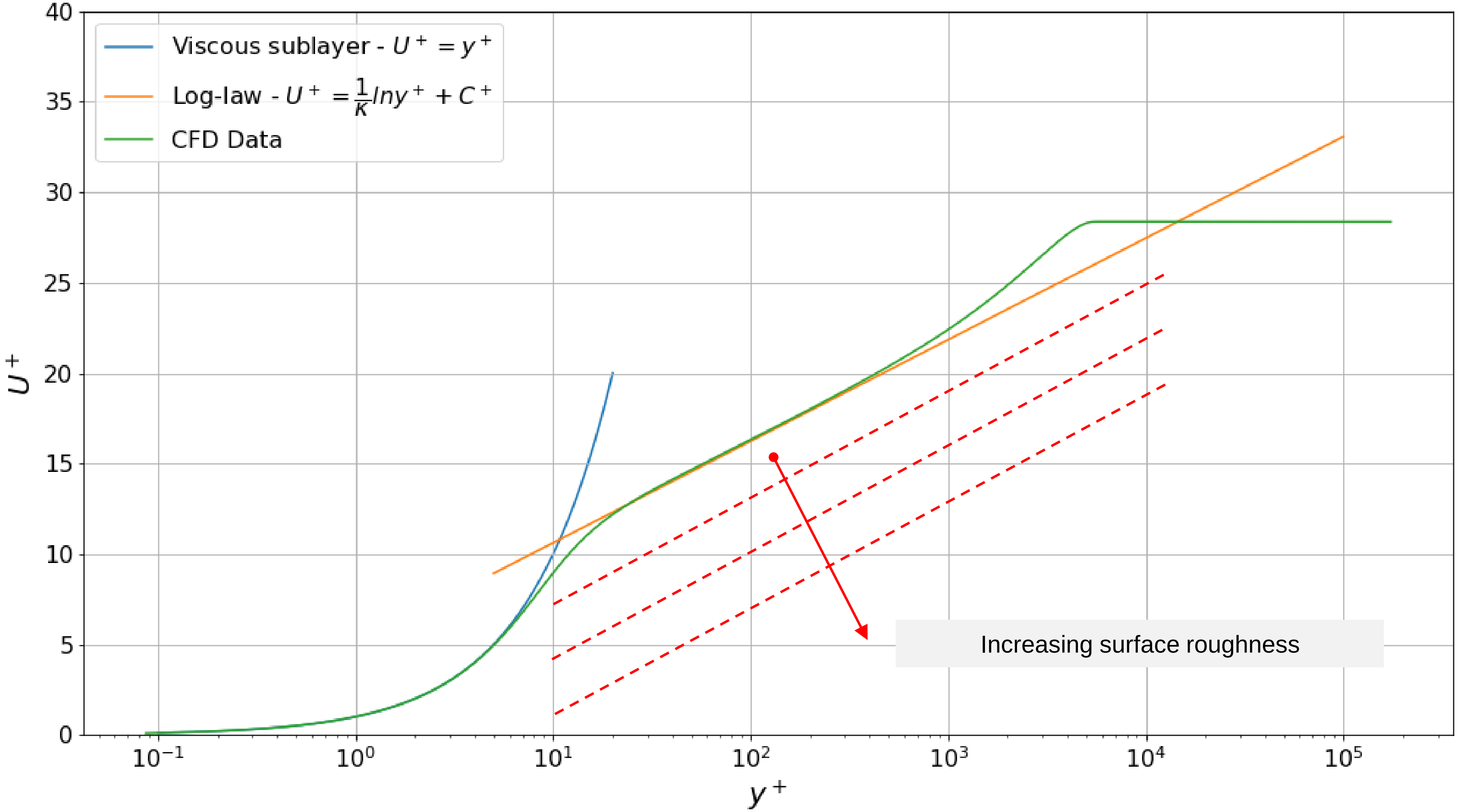
- Effect of heat transfer on the law of the wall.



- Notice that depending on the Prandtl number, the boundary layer can be thicker or thinner.
- A cold wall creates a thinner boundary layer and increases the skin friction
- A hot wall thickens the boundary layer and decreases the skin friction.

Wall functions – Additional observations

- Effect of surface roughness on the law of the wall.



- Surface roughness increases the skin friction.
- It shifts the velocity profile downwards.

Wall functions – Additional observations

- When dealing with compressible high-speed flows and heat transfer, the coefficients appearing in the law of the wall relations and closure relations, have a strong dependence on the Mach number and the heat transfer rate.
- The coefficients change in function of the Mach number in the compressible law of the wall.
- No need to mention that the physical properties also depend on the Mach number and heat transfer rate.

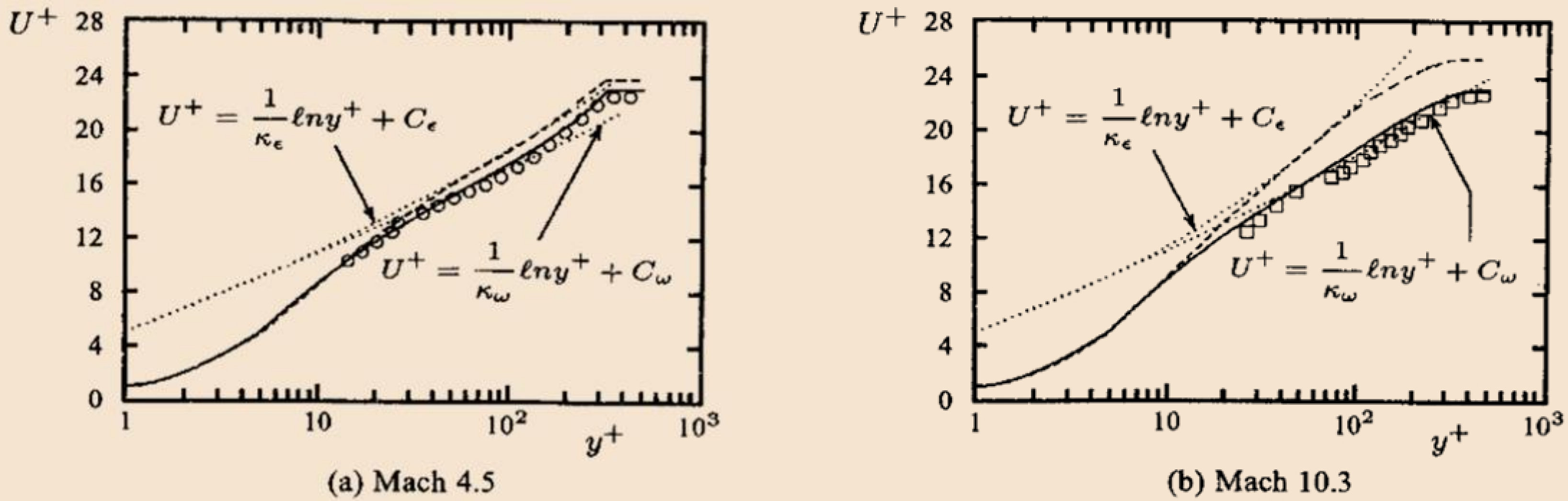


Figure 5.3: *Computed and measured velocity profiles for compressible flat-plate boundary layers:* —— *Wilcox (2006) k-ω; - - - Chien k-ϵ; ○ Coles; □ Watson.*

References:

[1] D. C. Wilcox. Turbulence Modeling for CFD. Third edition, DCW Industries, 2010.

Wall functions – Additional observations

- In the case of pressure gradients and flow separation (non-equilibrium conditions), the law of the wall can be sensitized to pressure gradients effects [1].

$$\frac{\tilde{U} C_\mu^{1/4} k^{1/2}}{\tau_w / \rho} = \frac{1}{\kappa} ln \left(E \frac{\rho C_\mu^{1/4} k^{1/2} y}{\mu} \right)$$

- Where,

$$\tilde{U} = U - \frac{1}{2} \frac{dp}{dx} \left[\frac{y_v}{\rho \kappa k^{1/2}} ln \left(\frac{y}{y_v} \right) + \frac{y - y_v}{\rho \kappa k^{1/2}} + \frac{y_v^2}{\mu} \right]$$

$$y_v = \frac{\mu y_v^*}{\rho C_\mu^{1/4} k_p^{1/2}} \qquad y_v^* = 11.225$$

- This correction is recommended for use in complex flows involving separation, reattachment, and impingement where the mean flow and turbulence are subjected to pressure gradients and rapid changes.
- In such flows, improvements can be obtained, particularly in the prediction of wall shear (skin-friction coefficient) and heat transfer (Nusselt or Stanton number).

[1] S. Kim, D. Choudhury. A Near-Wall Treatment Using Wall Functions Sensitized to Pressure Gradient. In ASME FED Vol. 217,Separated and Complex Flows. ASME. 1995.

Wall functions – Additional observations

- The non-dimensional temperature T*, can be further improved by adding the viscous heating contribution [1],

$$T^* = T_c^* + \frac{D}{q}$$

- Where,

$$T_c^* = \begin{cases} Pr\, y^* & y^* < y_l^* \\ Pr_T \left[\frac{1}{\kappa} \ln\left(E y^*\right) + P \right] & y^* > y_l^* \end{cases}$$

$$D = \begin{cases} \rho u^* \frac{1}{2} Pr U_p^2 & y^* < y_l^* \\ \rho u^* \frac{1}{2} \left[Pr_t U_p^2 + \left(Pr - Pr_t\right) U_c^2 \right] & y^* > y_l^* \end{cases}$$

- Where U_c is the mean velocity at the intersection point y_l^*.
- This correction is recommended for highly compressible flows, where heating by viscous dissipation can have a strong influence in the temperature distribution in the near-wall region.

[1] J. R. Viegas, M. W. Rubesin, C. C. Horstman. On the Use of Wall Functions as Boundary Conditions for Two-Dimensional Separated Compressible Flows. Technical Report AIAA-85-0180. 1985.

Roadmap to Lecture 9

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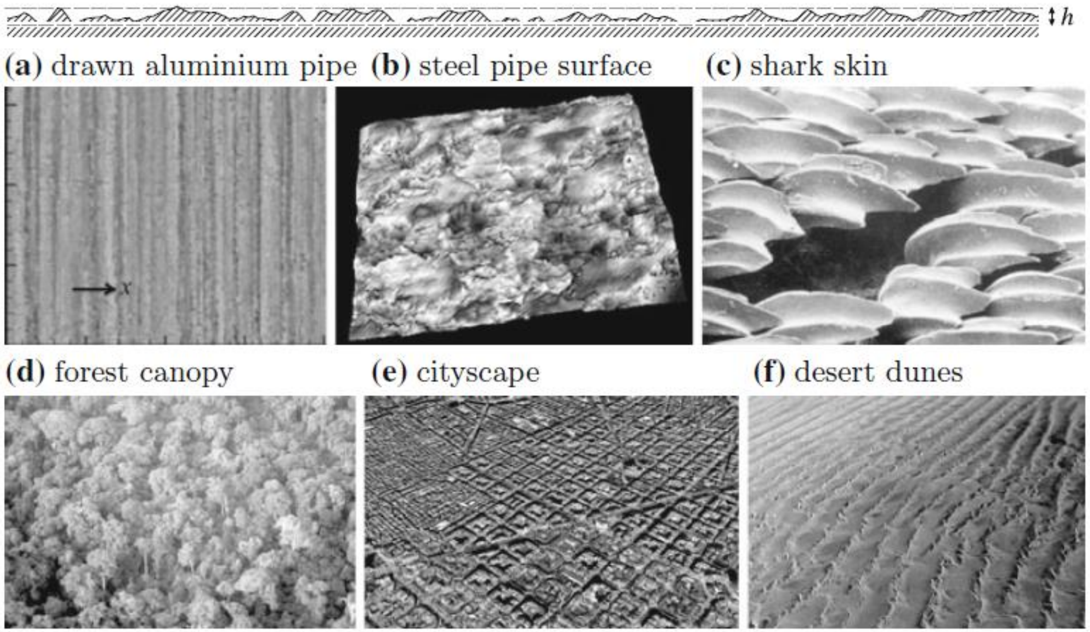
4. **Surface roughness**

Surface roughness

- So far, we have only considered smooth walls.
- In reality, every wall and material is characterized by small irregularities, which we refer to as roughness.
- The roughness has a characteristic height *h*, as shown in the figure below.
- Wall roughness increases the wall shear stress and heat transfer rate.

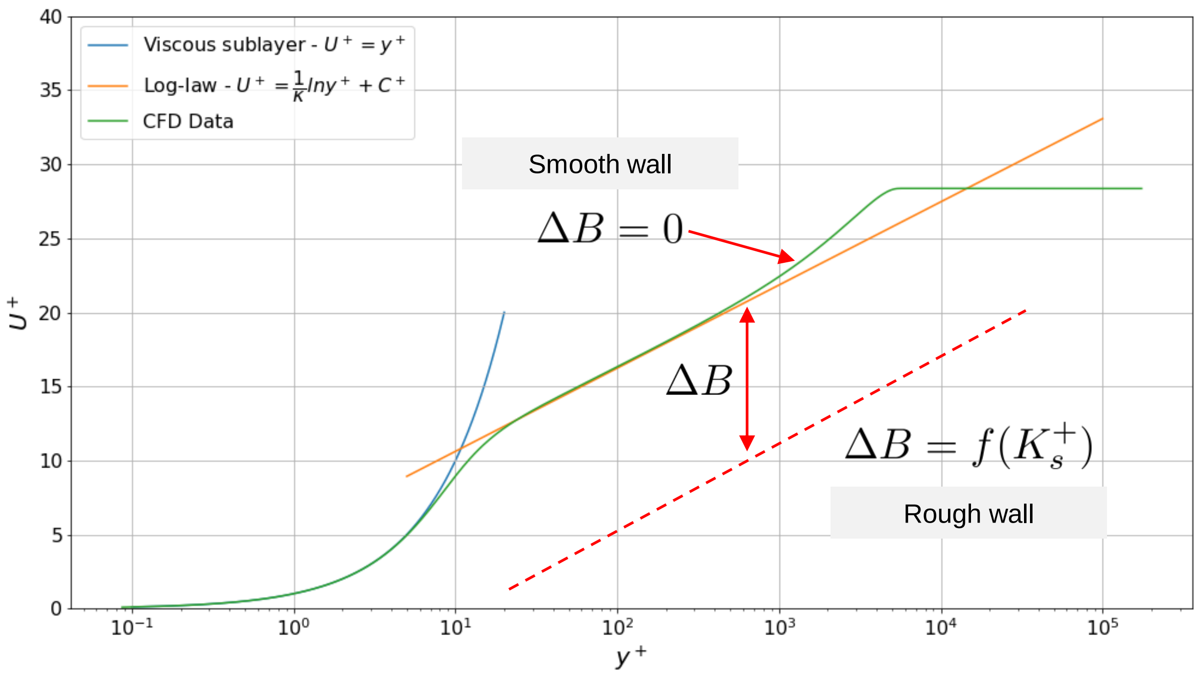


Examples of wall roughness. **(a)** Surface of an aluminum pipe: $h_{rms} \approx 0.16$ μm. **(b)** Scanned surface (1.4 x 1.1 mm^2) of a non-rusted commercial steel pipe: $h_{rms} \approx 5$ μm. **(c)** Scales of the great white shark: $h_{rms} \approx 0.1$ mm. **(d)** Aerial views of tropical forest in Gabon (h ≈ 0.1-10 m). **(d)** Barcelona landscape (h ≈ 10-100 m). **(f)** The Namib desert (h ≈ 10-500 m). Adapted from reference [1].

[1] F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer. 2016.

Surface roughness

- In CFD, the roughness must be modeled.
- In theory, it can be resolved with very fine meshes but the computational requirements are too high.
- Wall roughness increases the wall shear stress and heat transfer rate.
- It also breaks up the viscous sublayer.
- By looking at the nondimensional velocity plot, the wall roughness shifts the nondimensional velocity downwards by a factor of ΔB.



Surface roughness

- There are many ways to add roughness to the solution.
- Let us study maybe the most common wall function for roughness.
- This implementation is based on the standard wall functions.

STEP 1.

$$U^+ = \frac{1}{\kappa} \ln\left(Ey^+\right) - \Delta B$$

Roughness correction coefficient

STEP 2.

$$U^+ = \frac{1}{\kappa} \ln\left(Ey^+\right) - \ln\left(e^{\Delta B}\right)$$

STEP 3.

$$U^+ = \frac{1}{\kappa} \ln\left(\frac{Ey^+}{e^{\Delta B}}\right)$$

- Where we used the following logarithm rules to derive the previous relations,

$$\ln\left(e^x\right) = x \qquad \ln\left(x\right) - \ln\left(y\right) = \ln\left(\frac{x}{y}\right)$$

Surface roughness

- If we introduce the following relation,

$$E' = \frac{E}{e^{\Delta B}}$$

- Into the relation corrected for wall roughness,

$$U^+ = \frac{1}{\kappa} \ln \left(\frac{E y^+}{e^{\Delta B}} \right)$$

- We obtain the following equation,

$$U^+ = \frac{1}{\kappa} ln \left(E' y^+ \right)$$

- Which is identical to the standard wall function formulation (except for the variable E').
- We now have a way to work with smooth and rough walls using the same log-law relation.

Surface roughness

- Let us now address the roughness correction factor ΔB.
- This factor can be computed as follows [1, 2],

$$\Delta B = 0 \qquad K_s^+ < 2.25$$

Hydraulically smooth

$$\Delta B = \frac{1}{\kappa}\left(\frac{K_s^+ - 2.25}{87.75} + C_s K_s^+\right) \times \dots$$

$$\dots \times \sin\left[0.4258\left(\ln K_s^+ - 0.811\right)\right] \qquad 2.25 < K_s^+ < 90$$

Transitional

$$\Delta B = \frac{1}{\kappa}\ln\left(1 + C_s K_s^+\right) \qquad K_s^+ > 90$$

Fully rough

- Notice that ΔB is a function of the nondimensional roughness height K_s^+ and the roughness constant C_s.

[1] T. Cebeci, P. Bradshaw. Momentum Transfer in Boundary Layers. Hemisphere Publishing Corporation. 1977.
[2] P. Ligrani, R. Moffat. Structure of transitionally rough and fully rough turbulent boundary layers. J. of Fluid Mechanics, 162, 69-98. 1986.

Surface roughness

- The hydraulically smooth condition exists when roughness heights are so small that the roughness is buried in the viscous sublayer.
- The fully rough flow condition exists when the roughness elements are so large that the sublayer is completely eliminated, and the flow can be considered as independent of molecular viscosity; that is, the velocity shift is proportional to $\ln(K_s^+)$.
- The transitional region is characterized by reduced sublayer thickness, which is caused by diminishing effectiveness of wall damping.
- Because molecular viscosity still has some role in the transitional region, the geometry of roughness elements has a relatively large effect on the velocity shift.
- Surface roughness can be expressed in function of the nondimensional roughness height K_s^+ (dimension of the grains) and the roughness constant C_s (shape of the grains).

Surface roughness

- The nondimensional roughness height can be computed as follows,

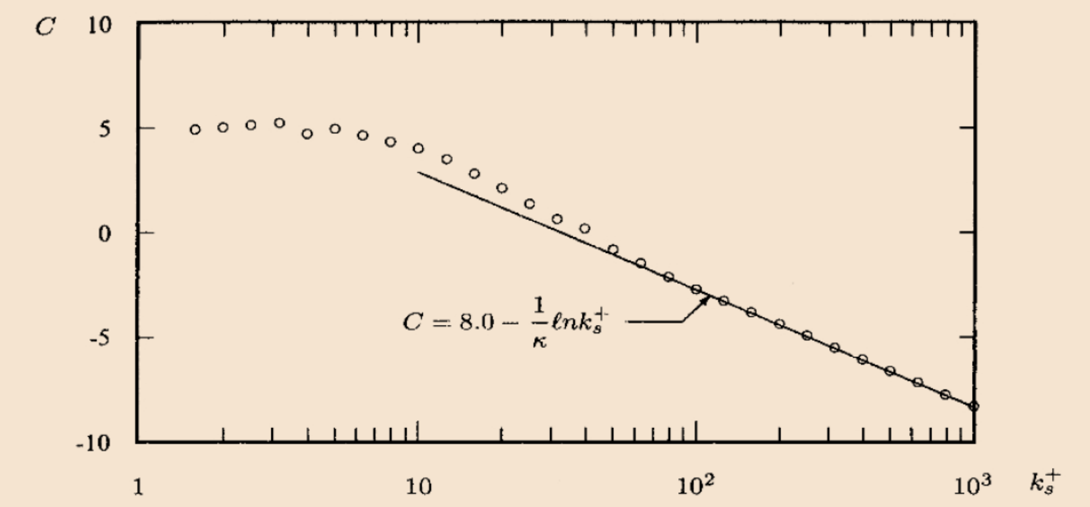
$$K_s^+ = \frac{\rho K_s u^*}{\mu}$$

← Similar to the approach taken for y^+

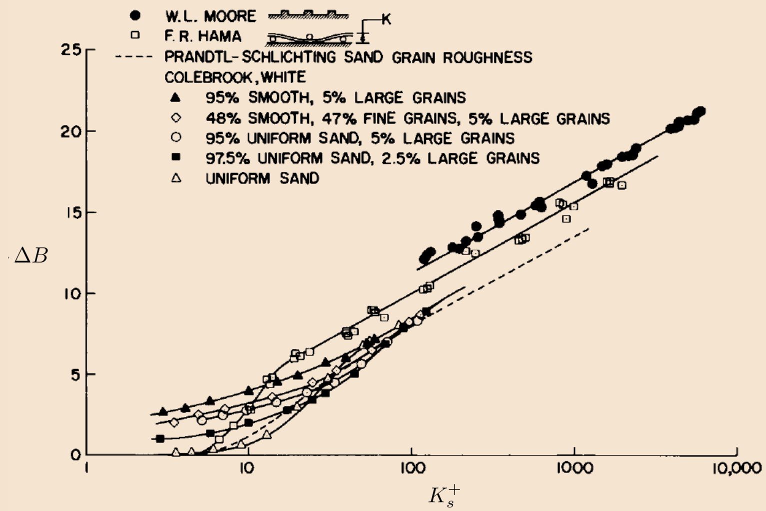
- Where K_s is the typical roughness height (sand grain diameter).
- The roughness constant C_s is often equal to 0.5. This constant represents the shape and distribution of the roughness elements (sand grains).
- It is recommended to fix C_s and adjust K_s^+.
- Remember, in our discussion, the roughness regime is subdivided into the three regimes.
    - Hydraulically smooth.
    - Transitional.
    - Fully rough.

Surface roughness

- The subdivision of the roughness regime and the dependence of the constant *C* (law of the wall) and the roughness correction factor ΔB are supported on experimental data (Nikuradse's data [1]).
- The constant *C* and the roughness correction factor depend on the roughness parameters K_s^+ and C_s.



Constant in the law of the wall as a function of surface roughness. Based on measurements of Nikuradse [1]. Adapted from [2].

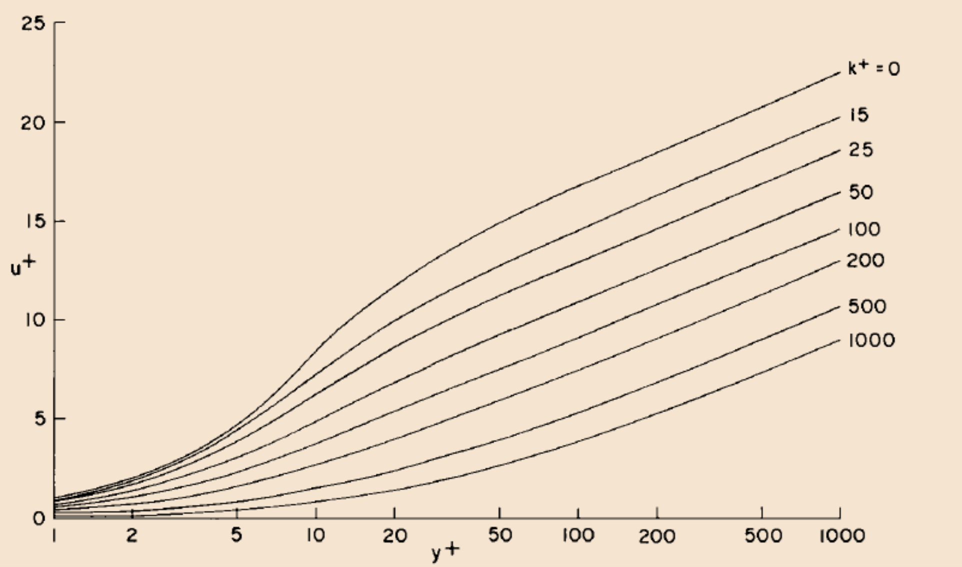


Effect of wall roughness on the roughness correction factor and universal velocity profiles [3].

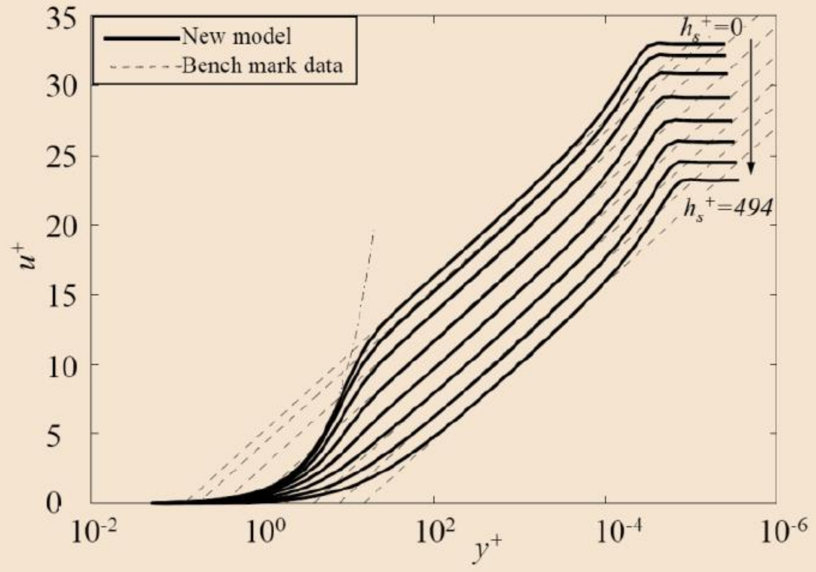
[1] J. Nikuradse. Law of Flow in Rough Pipes. Technical Memorandum 1292, National Advisory Committee for Aeronautics. 1950.
[2] D. Wilcox. Turbulence modeling for CFD. DCW Industries, Inc. 2006.
[3] F. Clauser. The turbulent boundary layer. Advan. Appl. Mech. 4, 1. 1956.

Surface roughness

- Plots of mean velocity distribution for uniform roughness at several K_s^+ values.



Plots of mean velocity distribution for uniform roughness at several values of nondimensional roughness height [1].



Plot of roughness mean velocity profiles [2].

[1] T. Cebeci, A. M. O. Smith. Analysis of turbulent boundary layers. Academic Press. 1974.
[2] U. Oriji, X. Yang, P. Tucker. Hybrid RANS/ILES for Aero Engine Intake. Proceedings of the ASME Turbo Expo 2014. Paper No: *GT2014-26472*. 2014.

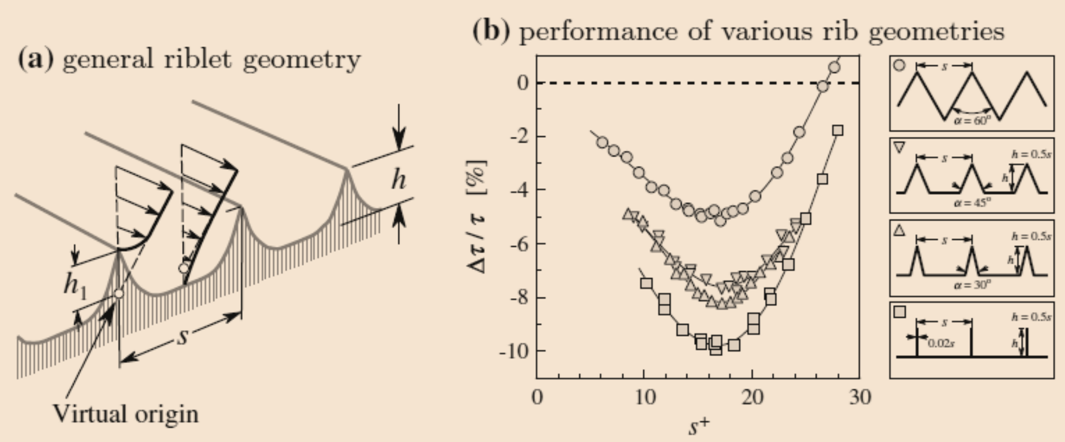
Surface roughness

- There is no universal roughness function valid for all types of roughness.
- Many methods to take into account the surface roughness are available in the literature, just to name a few,

  - [1] T. Cebeci, P. Bradshaw. Momentum transfer in boundary layers. Hemisphere Publishing Corporation. 1977.
  - [2] P. Ligrani, R. Moffat. Structure of transitionally rough and fully rough turbulent boundary layers. J. of Fluid Mechanics, 162, 69-98. 1986.
  - [3] B. Aupoix, R. Spalart. Extensions of the Spalart-Allmaras turbulence model to account for wall roughness. Int. J. of Heat and Fluid Flow. 24. 2003.
  - [4] D. Wilcox. Turbulence modeling for CFD. DCW Industries, Inc. 2006.
  - [5] U. Oriji, X. Yang, P. Tucker. Hybrid RANS/ILES for aero engine intake. Proceedings of the ASME Turbo Expo 2014. Paper No: *GT2014-26472*. 2014.
  - [6] B. Krishnappan. Laboratory verification of a turbulent flow model. J. of Hydraulic Eng. 110. 1984.
  - [7] J. Yoon, V. Patel. A numerical model of flow in channels with sand-dune beds and ice covers. IIHR Report no. 362. 1993
  - [8] J. Rotta. Turbulent boundary layers in incompressible flows. Progress in Aerospace Science. 1982.

Surface roughness

- Finally, as we have seen, a rough wall increases friction.
- But it happens that special rough surfaces can increase drag reduction.
- Well designed riblets can shift the nondimensional velocity profile upwards, and closer to the viscous law non-dimensional profile.
- It appears that small riblets that are aligned with the flow direction can achieve a significant drag reduction up to 10%.
- The maximum drag reduction occurs for a rib spacing s between 14 and 20 viscous wall units, i.e., $14 < s^+ < 20$, as illustrated in the figure.
- This is an example of biomimetics, since the development and application of ribbed surfaces has been inspired by the presence of ribbed scales on sharks

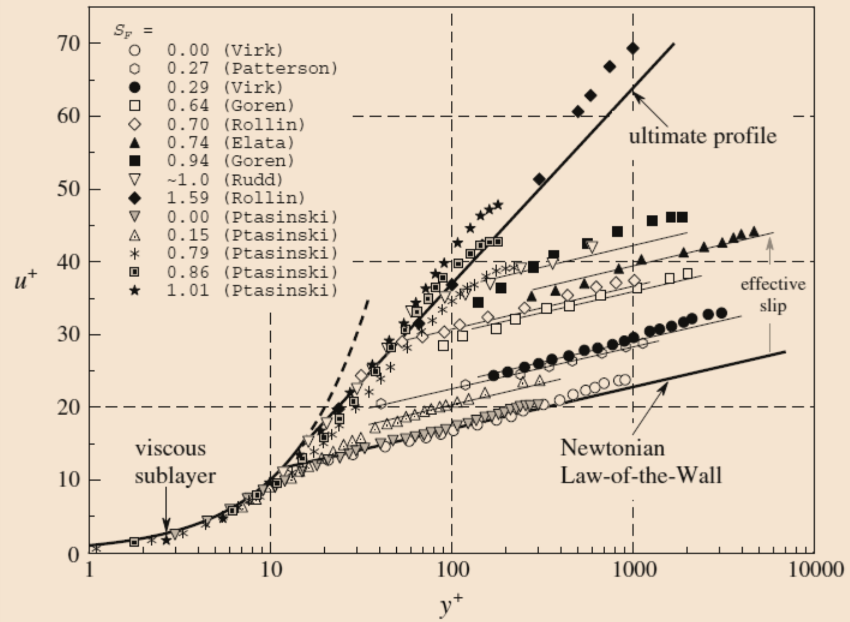


a. General geometry of a ribbed surface with a rib spacing s and rib height h. b. Drag reduction performance of various rib geometries. [1]

[1] F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer. 2016.

Surface roughness

- Besides using riblets for drag reduction, a similar effect can be achieved by polymer additives.
- Small amounts of certain polymer additives to fluids can achieve a significant reduction of friction drag, which is know as the Toms effect [1, 2, 3].
- The figure below shows experimental data of the measured velocity profiles for various flows with polymer additives.
  - The profiles for drag-reducing flows have logarithmic profiles with the same slope as for a Newtonian fluid, but with an offset, or effective slip.



The mean velocity profiles for various cases of polymer drag reduction.

[1] B. Toms. Some observations on the flow of linear polymer solutions through straight tubes at large Reynolds numbers. In Proc. 1st. Int. Congr. Rheol., vol. 2, pp. 135–141. 1948.
[2] P. Virk. Drag reduction fundamentals. AIChE J. 21, 625–656. 1975.
[3] F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer. 2016.