On the Use of the Multiple Scale Method in Solving 'Difficult' Bifurcation Problems

Angelo Luongo
International Research Center M&MoCS, University of L'Aquila, L'Aquila, Italy

Several algorithms consisting in 'non-standard' versions of the Multiple Scale Method are illustrated for 'difficult' bifurcation problems.

Preliminary, the 'easy' case of bifurcation from a cluster of distinct eigenvalues is addressed, which requires using integer power expansions, and it leads to bifurcation equations all of the same order. Then, more complex problems are studied. The first class concerns bifurcation from a defective eigenvalue, which calls for using fractional power expansions and fractional time-scales, as well Jordan or Keldysh chains.

The second class regards the interaction between defective and non-defective eigenvalues. This problem also requires fractional powers, but it leads to differential equations which are of different order for the involved amplitudes. Both autonomous and parametrically excited non-autonomous systems are studied. Moreover, the transition from a codimension-3 to a codimension-2 bifurcation is explained.

As a third class of problems, singular systems possessing an evanescent mass, as Nonlinear Energy Sinks, are considered, and both autonomous systems undergoing Hopf bifurcation and non-autonomous systems under external resonant excitation, are studied.

The algorithm calls for a suitable combination of the Multiple Scale Method and the Harmonic Balance Method, this latter to be applied exclusively to the singular equations.

Several applications are shown, to test the effectiveness of the proposed methods.They include discrete and continuous systems, autonomous, parametrically excited and externally excited systems.