Three-dimensional stability of the flow past a rotating cylinder

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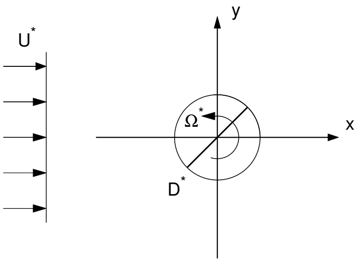
Problem formulation

Incompressible flow

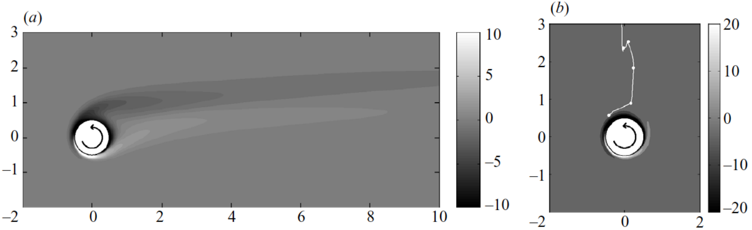


$$Re = \frac{U^* D^*}{\nu^*}$$

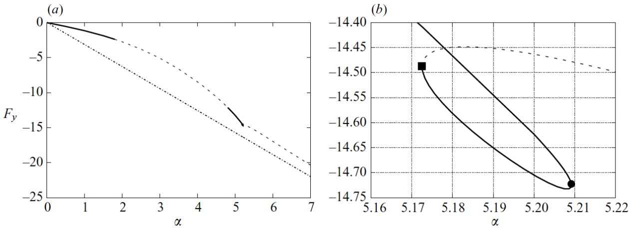
$$\alpha = \frac{\Omega^* D^*}{2U^*}$$

$$\mathbf{U}(x,y,z,t) = \mathbf{U}_b(x,y) + \epsilon \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathbf{u}(x,y,\kappa) \exp(i\kappa z + \sigma t) d\kappa$$

Steady flow



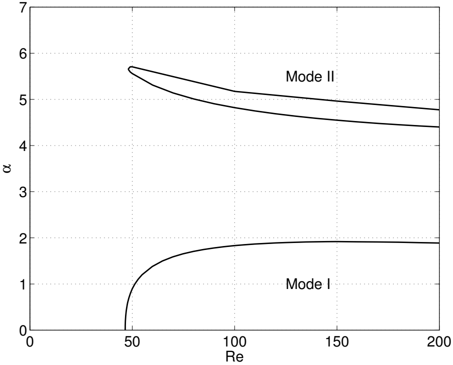
Vorticity field at $Re = 100$ and (a) $\alpha = 1.8$, (b) $\alpha = 4.85$.



Force in cross-stream direction

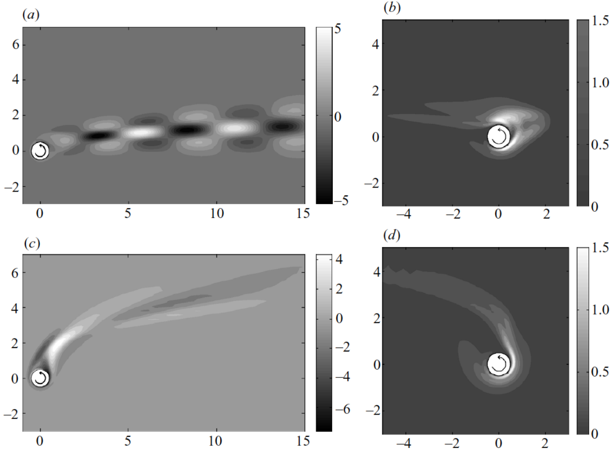
Neutral curve

Two-dimensional perturbations ($\kappa = 0$)



Previous studies Pralits et al. (2010), Stojkovic et al. (2002), Mittal (2003), Kang et al. (1999)

Shedding mode I & II



$Re = 100$, Vorticity field (a) $\alpha = 1.8$, (c) $\alpha = 4.85$,
Adjoint field (b) $\alpha = 1.8$, (d) $\alpha = 4.85$

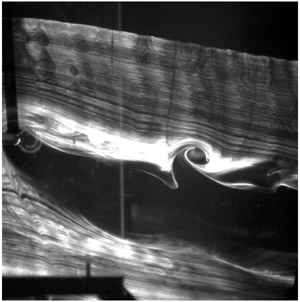
Motivation

Missing bits...

- Shedding mode II (2D) has not been verified in experiments (ex. Yildirim et al.)
- 3D effects might be important

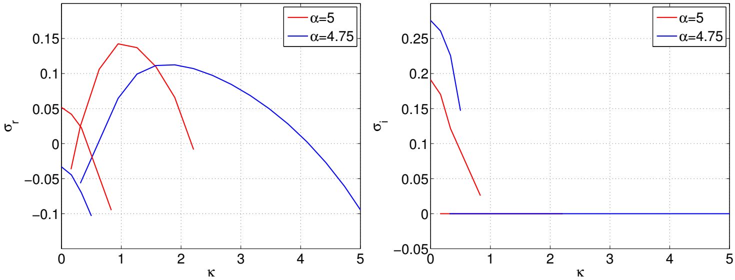
Main goal

- Determine if 3D effects are important
- Draw a neutral curve accounting for 3D instabilities
- Determine critical conditions
- Bifurcation diagram at branch II
- Sensitivity analysis
- Compare with experiments by Linh (2011)



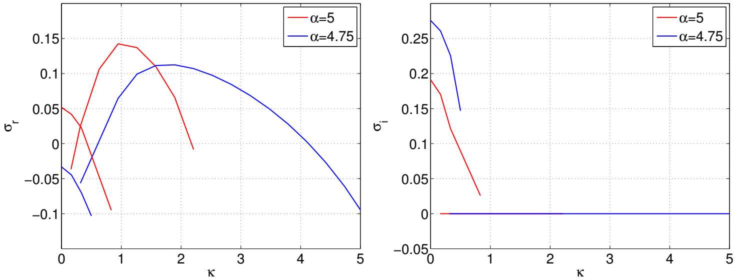
Experiments: Yildirim et al. EFMC 7

Shedding mode II



Growth rate σ_r (left) and Strouhal number $St = \sigma_i/(2\pi)$ (right) for $Re = 100$ as a function of the rotation rate and spanwise wave number.

Shedding mode II



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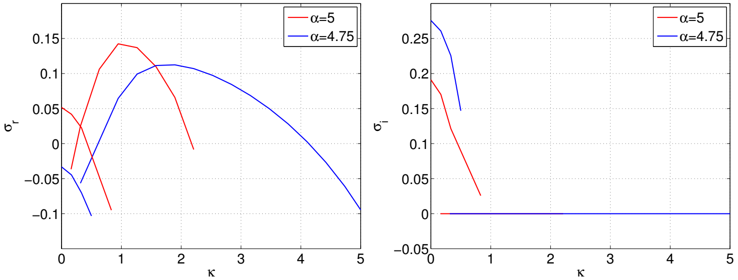
A new stationary three-dimensional mode II is found...

Shedding mode II



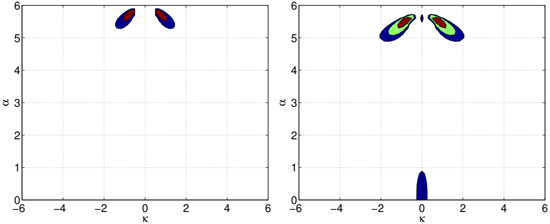
Growth rate σ_r (left) and Strouhal number $St = \sigma_i/(2\pi)$ (right) for $Re = 100$ as a function of the rotation rate and spanwise wave number.

A new stationary three-dimensional mode II is found...

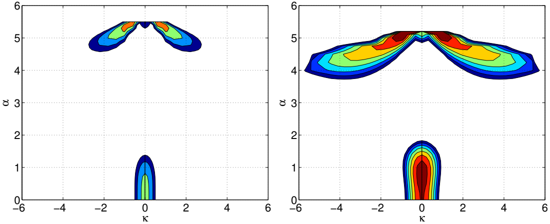
...with approximately 3 times the growth rate of the 2D unsteady one.

Shedding mode I & II $\sigma_r(\alpha, \kappa) \geq 0$

$Re = 40$ (left) and $Re = 50$ (right)

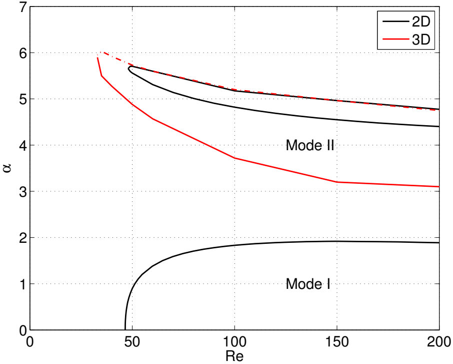


$Re = 60$ (left) and $Re = 100$ (right)



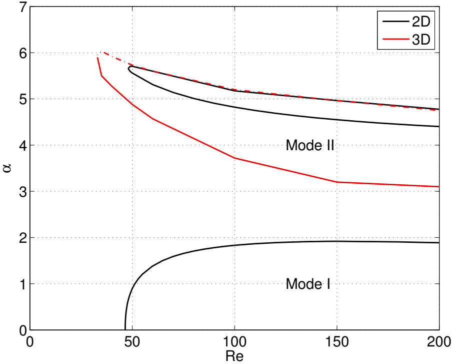
Contours: 0, 0.02, 0.04, 0.06, 0.08, 0.1, 0.12

Neutral curve



Neutral stability curve of two and three-dimensional perturbations.

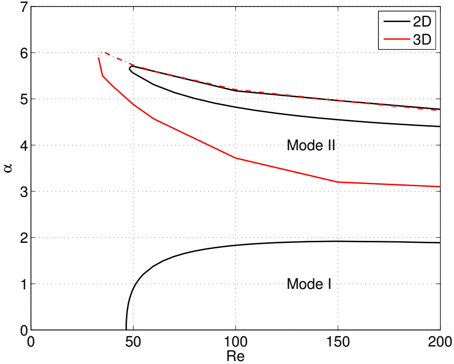
Neutral curve



Neutral stability curve of two and three-dimensional perturbations.

Critical condition: $Re_c \approx 33$, $\alpha_c \approx 5.8$

Neutral curve



Neutral stability curve of two and three-dimensional perturbations.

Critical condition: $Re_c \approx 33,\ \alpha_c \approx 5.8$

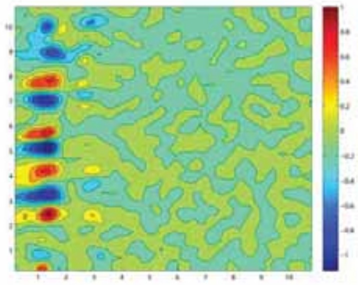
A pitchfork bifurcation occurs at critical conditions and the instability is 3D

Experiments by Linh, PhD thesis, NUS, 2011

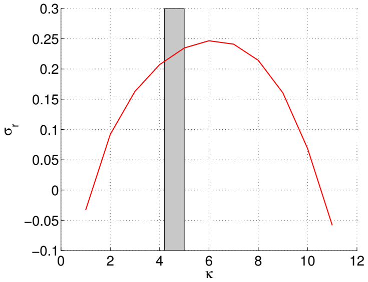
... spent more than 10 times the duration to capture the regular vortex shedding in order to wait for that vortex to appear again, but no regular one-sided vortex shedding could be detected. It might be due to the three dimensional disturbances of the current experimental flow because of the finite cylinder length and differences in cylinder end conditions. To the best of the authors knowledge, there is no published three dimensional numerical study to confirm the existence of such instability in 3-D flow. Thus it is reasonable to speculate that the second instability is a two-dimensional flow phenomenon with evidences gathered so far. **From Linh (2011)**

$$Re = 206,\ \alpha = 4$$

$$\lambda \approx 1.25D - 1.5D \ \rightarrow \kappa \approx \mathbf{5 - 4.2}$$



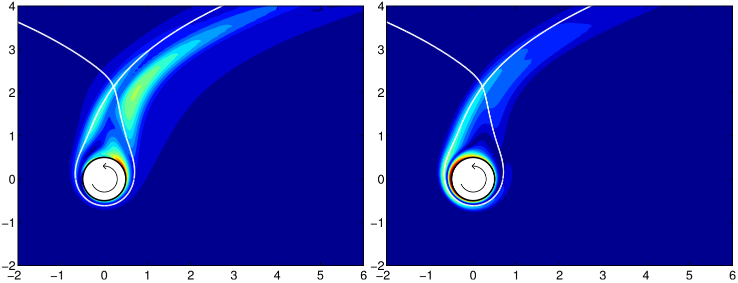
Vorticity (PIV)



Linear stability analysis , $\sigma_i = 0$

Shedding mode II comparison: # I

Absolute value of the perturbation vorticity field for $Re = 100$, $\alpha = 5$



$\kappa = 0$ $\kappa = 1$

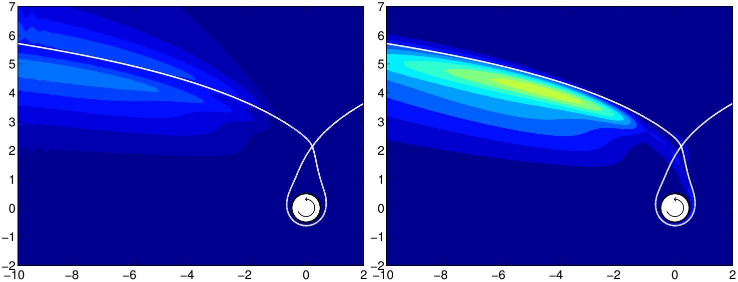
$$\max|\omega(x, y, \kappa = 1)| / \max|\omega(x, y, \kappa = 0)| \approx 2.$$

The white curve shows the stagnation line.

Note that for $\kappa = 1$ the vorticity is aligned with maximum strain.

Shedding mode II comparison: # II

Absolute value of the adjoint velocity field for $Re = 100$, $\alpha = 5$



$\kappa = 0$ $\kappa = 1$

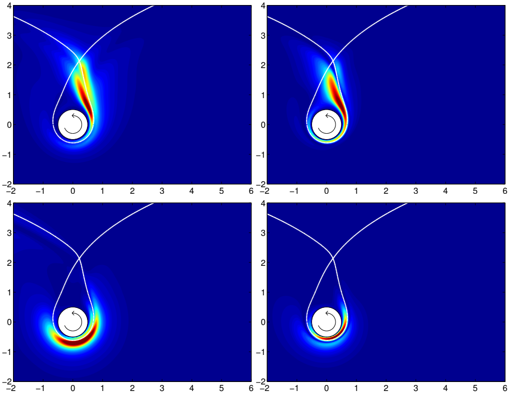
$$\max |\mathbf{u}^\star(x, y, \kappa = 1)| / \max |\mathbf{u}^\star(x, y, \kappa = 0)| \approx 2.$$

The white curve shows the stagnation line.

Note that for $\kappa = 1$ the vorticity is aligned with compression

Shedding mode II comparison: # III

Structural sensitivity for $Re = 100$, $\alpha = 5$



$\kappa = 0$ $\kappa = 1$

$$\mathbf{S}_p(x_0, y_0) = \frac{\mathbf{u}^\star(x_0, y_0)\,\mathbf{u}(x_0, y_0)}{\displaystyle\int_\Omega \mathbf{u}^\star \cdot \mathbf{u}\, dS},$$

$$\frac{\max \|\mathbf{S}_p(\kappa = 1)\|}{\max \|\mathbf{S}_p(\kappa = 0)\|} \approx 3$$

$$\mathbf{S}_b(x_0, y_0) = \frac{\mathbf{U}_b^\star(x_0, y_0)\,\mathbf{U}_b(x_0, y_0)}{\displaystyle\int_\Omega \mathbf{u}^\star \cdot \mathbf{u}\, dS},$$

$$\frac{\max \|\mathbf{S}_b(\kappa = 1)\|}{\max \|\mathbf{S}_b(\kappa = 0)\|} \approx 7$$

Conclusions

- A new **stationary three-dimensional** mode II is found...
- ...with approximately 3 times the growth rate of the 2D unsteady one.
- A neutral curve has been shown in the plane (α, Re) which accounts for 3D perturbations
- Critical condition: $Re_c \approx 33$, $\alpha_c \approx 5.8$ (below classical Von Karman)
- A pitchfork bifurcation occurs at critical conditions and the instability is 3D
- The bifurcation at branch II of mode II is approximately 2D (as in Pralits et al., 2010)
- Flow is **linearly stable** for 3D disturbances when $\alpha > 2^{nd}$ turning point (as in 2D)
- **LST predicts the steady 3D mode observed in experiments by Linh (2011) at** $Re = 206$
- Need "**real**" 2D experiment, eg. SOAP FILM, to reproduce the non-stationary mode II
- Is the stationary 3D mode II an hyperbolic instability ? (stationary, aligned with direction of maximum strain, 3D, situated on a hyperbolic stagnation point.)

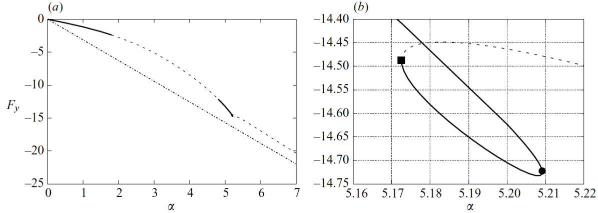
Extra slides

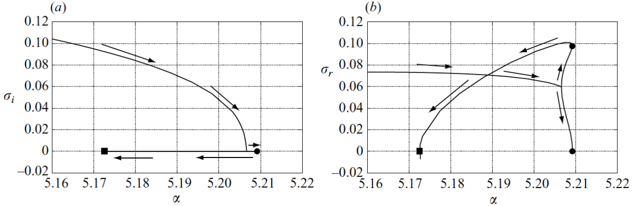
Numerical treatment I

- All equations are discretized using second-order finite-differences over a staggered, stretched, Cartesian mesh.
- An immersed-boundary technique is used to enforce the boundary conditions on the cylinder.
- The system of algebraic equations deriving from the disretization of the nonlinear mean-flow equations, along with their boundary conditions, is solved by a Newton-Raphson procedure.
- The eigenvalue problem is solved by Arnoldi routine, both right and left eigenvectors are solved simultaneously.

Upper branch of shedding mode II

Force of steady flow in the cross-stream direction





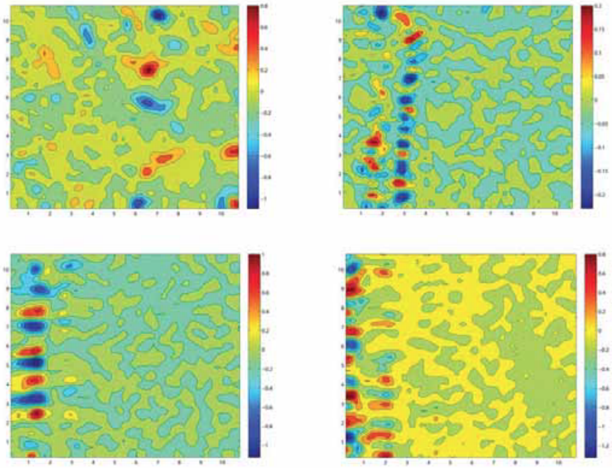
Frequency

Growth rate

Experiments by Linh, PhD thesis, NUS, 2011

PIV measurements: Spanwise vorticity, $Re = 206$, $\sigma_i = 0$

$\alpha = 1$ $\alpha = 3$



$\alpha = 4$ $\alpha = 5$