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Analytical modelling of Conjugate Heat Transfer for the nasal flow

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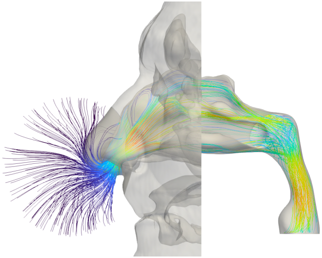
Master Thesis in Aeronautical Engineering

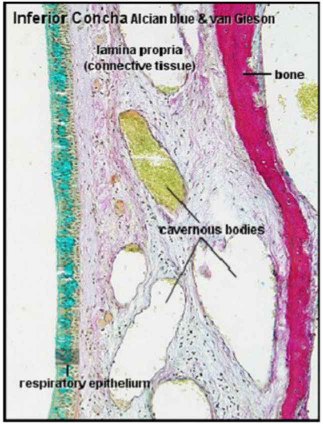


CFD in a clinical setting

- CFD allows detailed analysis of nasal flow characteristics, enhancing surgeries success rates for nasal airway issues
- OpenNOSE project aims to develop a reliable diagnostic procedure for nasal airway issues using CFD
- Heat transfer is a crucial aspect of the nasal flow, but often overlooked

Mucosal temperature boundary condition

- Airways walls at a constant temperature (T_{const}), 37 °C or piecewise constant based on empirically determined values, widely used in literature
- Conjugate heat transfer (CHT), between the thin mucous layer lining the airways walls and the airflow, allows overcoming the difficulty of prescribing the temperature



CHT: advantages and drawbacks

✓ More realistic results[1] : allows to identify the coldest areas of nose surface

✗ Higher computational cost and time: additional equations and mesh cells

✗ Higher complexity of the geometrical model due to mucous layer addition

GOAL: Develop an analytical model to obtain results analogously to CHT, without its drawbacks

[1] Mangani F. “Effetto della temperatura nella fluidodinamica nasale”. Master’s thesis Politecnico di Milano. (2020)

Proposed solution

Homogenization theory is used to derive an analytical equivalent boundary condition, imposed at the interface to mimic the mucous layer presence.

Equivalent boundary condition:

- derived for forced airflow in straight channel delimited by a smooth solid boundary (methodology inspired by Ahmed *et al.*[2] for rib-roughened surfaces)
- valid for well separated length scales

[2] Ahmed E.N., Bottaro A., Tanda G. “Conjugate natural convection along regularly ribbed vertical surfaces: A homogenization-based study”. Numerical Heat Transfer, Part A: Applications. (2023)

Governing equations

Fluid region (β) :

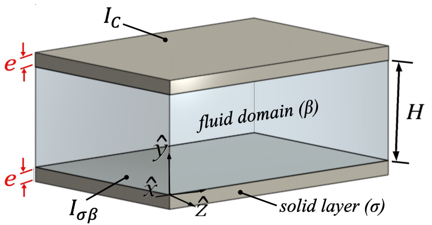
$$\begin{cases} \dfrac{\partial \hat{u}_i}{\partial \hat{x}_i} = 0 \\ \rho\, \hat{u}_j \dfrac{\partial \hat{u}_i}{\partial \hat{x}_j} = -\dfrac{\partial \hat{P}}{\partial \hat{x}_i} + \mu \dfrac{\partial^2 \hat{u}_i}{\partial \hat{x}_j^2} \\ \hat{u}_j \dfrac{\partial \hat{T}}{\partial \hat{x}_j} = \left(\dfrac{k_f}{\rho c_p}\right) \dfrac{\partial^2 \hat{T}}{\partial \hat{x}_j^2} \end{cases} \qquad (1)$$

Solid region (σ) :

$$\frac{\partial^2 \hat{\mathcal{T}}}{\partial \hat{x}_j^2} = 0 \qquad (2)$$

Temperature boundary conditions:

$$\begin{cases} \hat{\mathcal{T}} = \hat{\mathcal{T}}_C \quad \text{at } I_C \\ \hat{T} = \hat{\mathcal{T}}, \ \dfrac{\partial \hat{T}}{\partial \hat{n}} = \dfrac{k_s}{k_f} \dfrac{\partial \hat{\mathcal{T}}}{\partial \hat{n}} \quad \text{at } I_{\sigma\beta} \end{cases} \qquad (3)$$



Homogenization-based upscaling

- Equivalent boundary condition sought at fluid-solid interface, where $\hat{\mathbf{u}} = 0$ $\Rightarrow$ boundary condition only for temperature
- Well-separated length scales ($\epsilon = e/H \ll 1$), problem decomposed in microscopic and macroscopic subdomains
- Microscopic problem (θ and ϕ non-dimensional fluid and solid temperatures):

$$\begin{cases} \dfrac{\partial^2 \theta}{\partial {x_j}^2} = 0, \quad \text{in } \beta \\ \dfrac{\partial^2 \phi}{\partial {x_j}^2} = 0 \quad \text{in } \sigma \\ +B.C. \end{cases} \tag{4}$$

Homogenized boundary condition

- The variables θ and ϕ are asymptotically expanded in terms of ϵ
  $\Rightarrow \theta = \theta^{(0)} + \epsilon\theta^{(1)} + \mathcal{O}(\epsilon^2)$ and $\phi = \phi^{(0)} + \epsilon\phi^{(1)} + \mathcal{O}(\epsilon^2)$
- Substituting the expansions in system (4), the solution in non-dimensional form is found reconstructing the problem at different orders

$\Longrightarrow$ In dimensional form, the homogenized boundary condition at the fluid-solid interface is:

$$\hat{T} = \hat{\mathcal{T}}_C + e\,\frac{k_f}{k_S}\,\frac{d\hat{T}}{d\hat{x}}\bigg|_{\hat{x}_0} \qquad (5)$$

Implementation in *OpenFOAM*

To implement the derived boundary condition in *OpenFOAM*, two files are created:

- *`homTempFvPatchScalarField.H`*: for defining the variables
- *`homTempFvPatchScalarField.C`*: for compiling the boundary condition

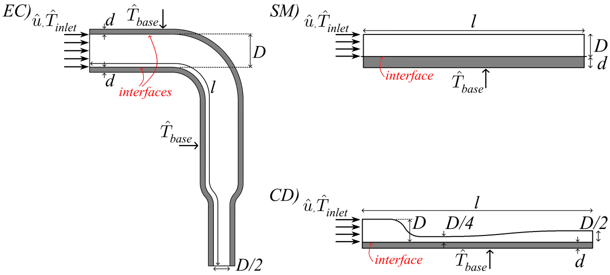
The boundary condition is then applied at the interface in homogenization-based simulations (*HOM*):

```
fluid_to_solid
    {
      type                            homTemp;
      kF                                0.026;
      kS                                0.598;
      solidThickness                   0.0005;
      baseTemperature             uniform 310;
      value                       uniform 310;
    }
```

Validation: Simple channels

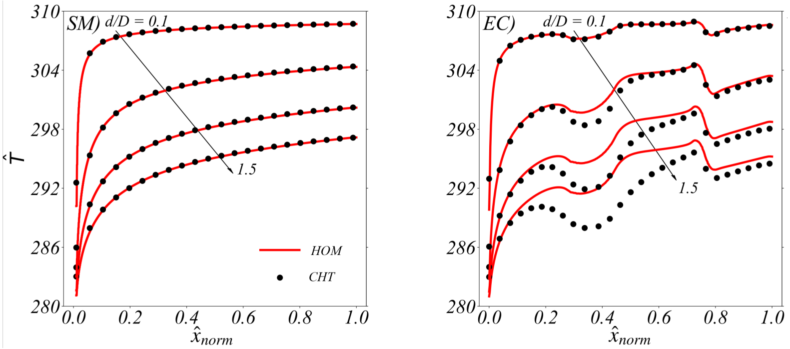
- Simple channel geometries, resembling shapes found in nasal anatomy: Elbow Convergent (*EC*), SMooth straight channel (*SM*) and Convergent Divergent (*CD*)

- Tested four solid layer thicknesses for each geometry ($d/D = 0.1, 0.5, 1$ and 1.5)



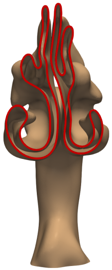
Validation: Results

Temperature distribution along solid-fluid interface ($\hat{x}_{norm} = x_{interface}/l$) for the extreme cases *SM* and *EC*:



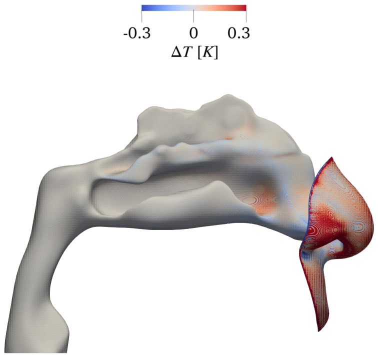
Nasal flow: Model

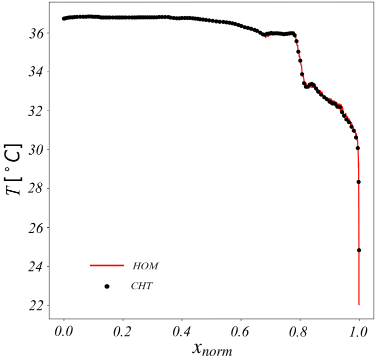
- Reconstructed from CT scan and simplified to add the mucous layer
- Mucous layer: constant thickness of 0.5 mm, water properties and base temperature of 37 °C
- Inspiration with flow rate of 16 l/min (resting condition) and different external temperatures
- Steady-state RANS with $RNG\ k - \epsilon$ model



Mucosal temperature difference

Temperature difference ($T_{HOM} - T_{CHT}$) and temperature distribution along the interface for $T_{inlet} = 7\,°C$:

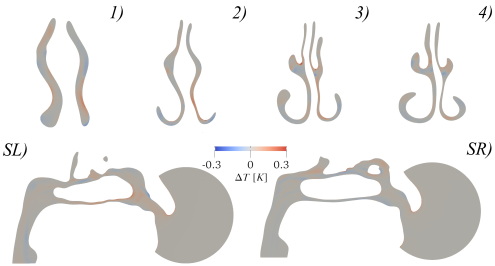




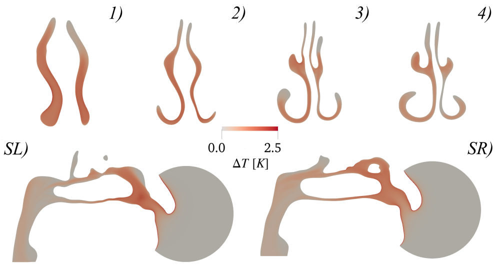
Internal temperature difference $(7\,°C)$

Temperature difference for $T_{inlet} = 7\,°C$:

$T_{HOM} - T_{CHT}$

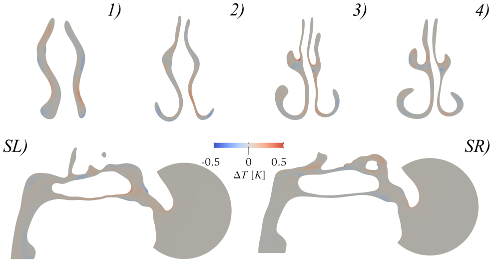


$T_{T_{const}} - T_{CHT}$

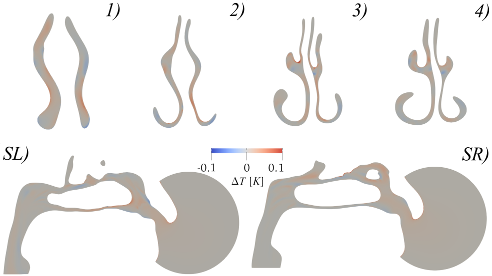


Temperature difference ($T_{HOM} - T_{CHT}$) for :

$T_{inlet} = -13\,°C$



$T_{inlet} = 27\,°C$



Final Remarks

HOM allows:

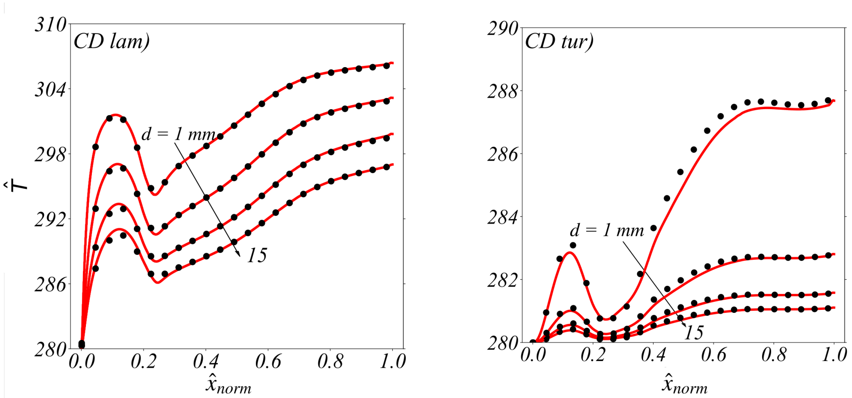
- more realistic results with respect to a constant interface temperature
- decrease in RAM use of 40% and in computational time of 30% with respect to CHT
- simpler model creation and mesh generation with respect to CHT

HOM limitations:

- in presence of highly curved surfaces and in narrower sections

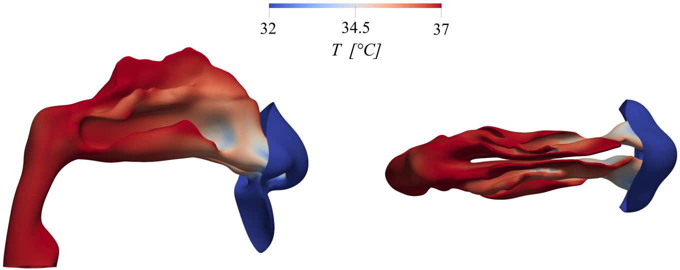
Thank for your attention

Validation: *CD* results



Mucosal temperature

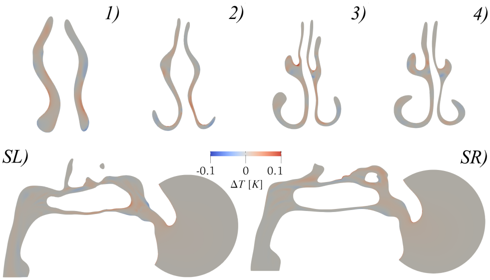
Temperature distribution along the interface obtained with the homogenized boundary condition (*HOM*) for $T_{inlet} = 7\,°C$:



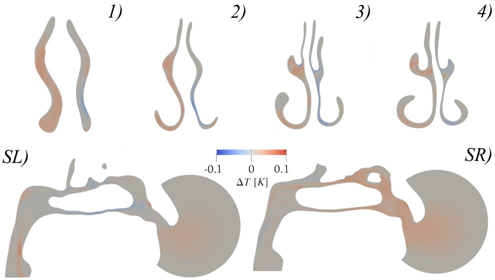
Internal temperature difference ($27\,°C$, $47\,°C$)

Temperature difference ($T_{HOM} - T_{CHT}$) for :

$T_{inlet} = 27\,°C$

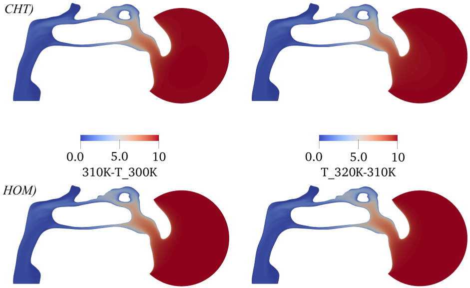


$T_{inlet} = 47\,°C$



Nose heating/cooling

$310K - T_{300K}$ $\quad$ $T_{320K} - 310K$



$(T_{320K} - 310K) - (310K - T_{300K})$