Improving wave model validation based on RMSE

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Motivation

- This work has been developed during a tuning of WWIII in the Mediterranean sea.

- RMSE, NRMSE and SI provided unsatisfactory indication of wave model performances during a validation of WWIII in the Mediterranean sea in storm conditions.

- **A best fit of the model based on RMSE led to parametrizations affected by strong negative bias.**

Conclusions

- RMSE, NRMSE and SI tend to be **systematically better for simulations affected by negative bias**. This is mostly evident when:
    - We are tuning parameters involving an amplification of the results.
    - Correlation coefficient appreciably smaller than 1 , $\rho < 0.9$.
    - Standard deviation of the observations of the same order of the average, $\sigma_O / \overline{O} \sim 1$.

- The indicator HH introduced by Hanna and Heinold (1985), defined by:

$$HH = \sqrt{\frac{\sum (S_i - O_i)^2}{\sum S_i O_i}} = \sqrt{\frac{\overline{(S - O)^2}}{\overline{SO}}}$$

provides more reliable information, being minimum for simulations unaffected by bias.

Validation of wave model in the Mediterranean sea

Model Wavewatch III

- Ardhuin et al. (2010) source terms (41 parameterizations).
- Tolman and Chalikov (1996) source terms, not tuned to the Mediterranean sea conditions.

Statistical indicators used for validation

Normalized Bias (NBI)

$$NBI = \frac{(\overline{S} - \overline{O})}{\overline{O}}$$

Root Mean Square Error

$$RMSE = \sqrt{\overline{(S - O)^2}}$$

Normalized Root Mean Square Error (NRMSE)

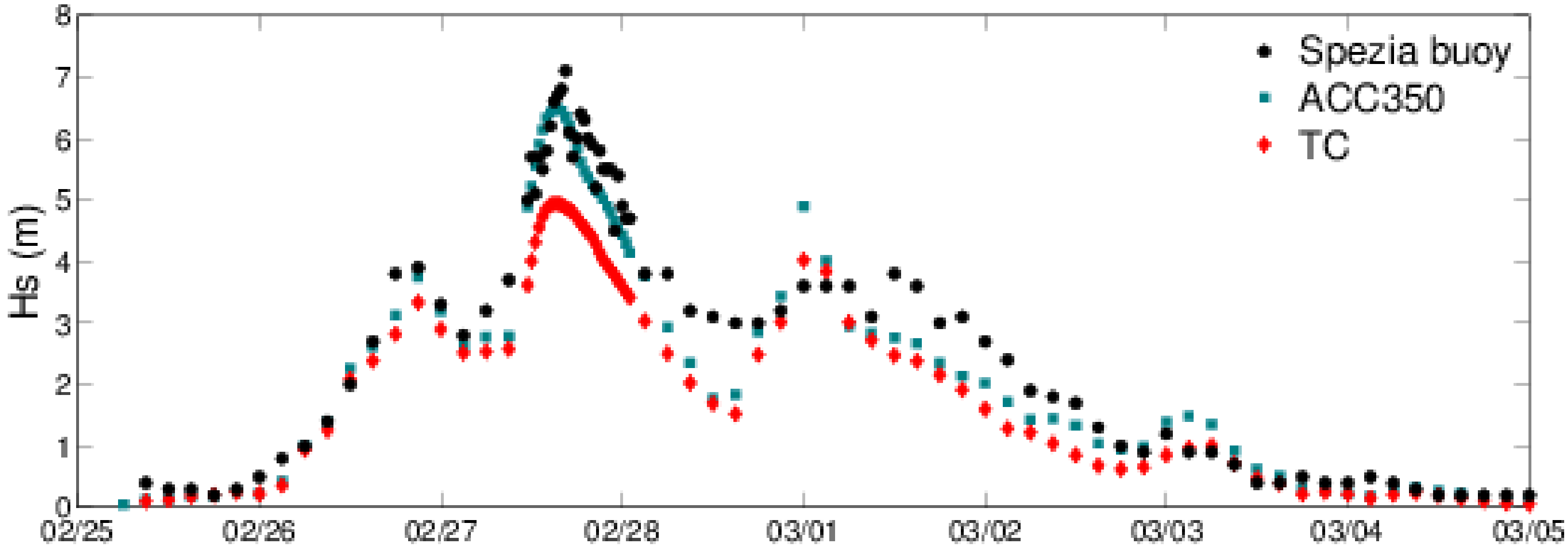
$$NRMSE = \sqrt{\frac{\overline{(S - O)^2}}{\overline{O^2}}}$$

Correlation coeff. (ρ)

$$\rho = \frac{\sum (S_i - \overline{S})(O_i - \overline{O})}{N \sigma_S \sigma_O}$$

Statistics on 17 storms and 23 buoys

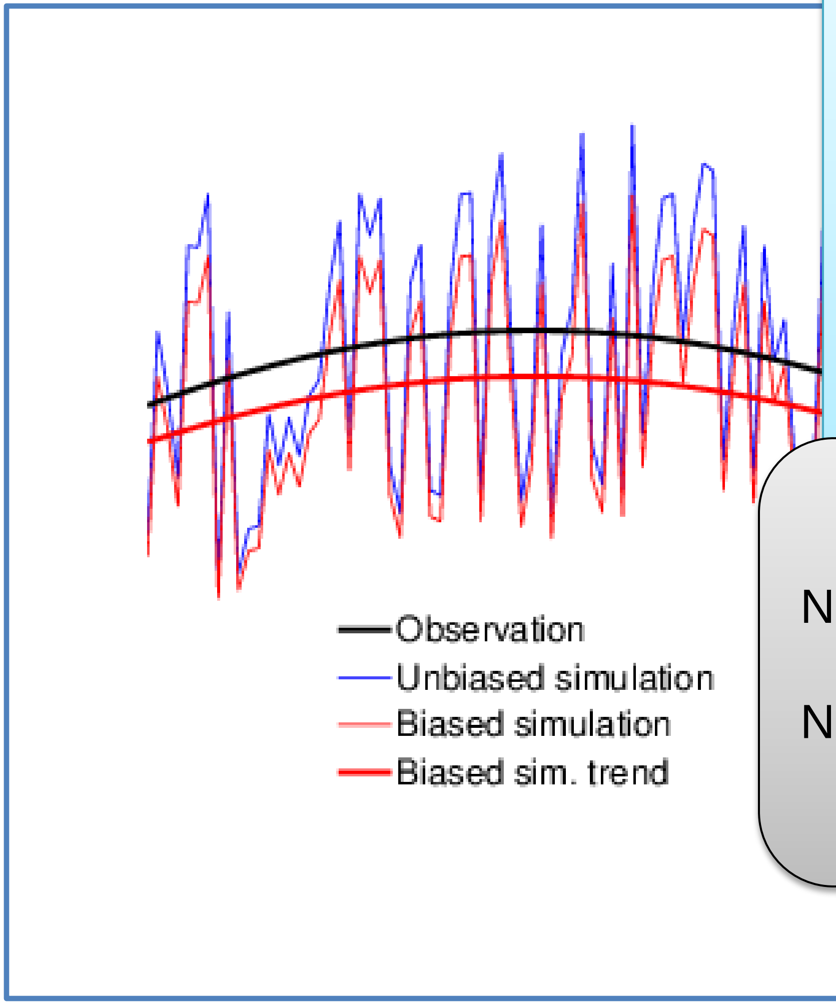
	ACC350	BJA	T & C
NBI	2.1%	-4.6%	-11.2%
ρ	0.889	0.885	0.883
NRMSE	0.2864	0.2800	0.2798 (-2.3%)



February 1990 storm

Drawback of using RMSE

A numerical example



ρ=0.614 for both of the simulations

NBI = -12% for the red simulation

One would say the best simulation is the blue one.

NRMSE(blue) = 0.384

NRMSE(red) = 0.356 (~ -7.2%)

Geometrical decomposition of RMSE

Scatter component

$$SC = \sqrt{\sum [(S_i - \overline{S}) - (O_i - \overline{O})]^2}$$

Bias component $BI = \overline{S} - \overline{O}$

$$\boxed{RMSE^2 = SC^2 + BI^2}$$

Geometrical decomposition of NRMSE

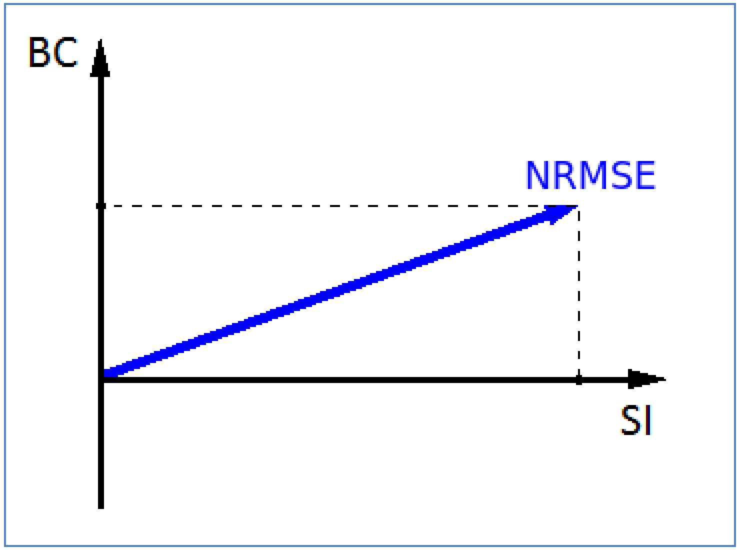
Scatter component (also called Scatter Index)

$$SI = \sqrt{\frac{\sum [(S_i - \overline{S}) - (O_i - \overline{O})]^2}{\sum O_i^2}}$$

Bias component

$$BC_{NRMSE} = \sqrt{\frac{\overline{O}^2}{\overline{O}^2 + \sigma_O^2}} NBI$$

$$\boxed{NRMSE^2 = SI^2 + BC_{NRMSE}^2}$$



Are SI and BC independent?

In general σ_S and $\bar{S}$ are not independent.

Let's assume this relation:

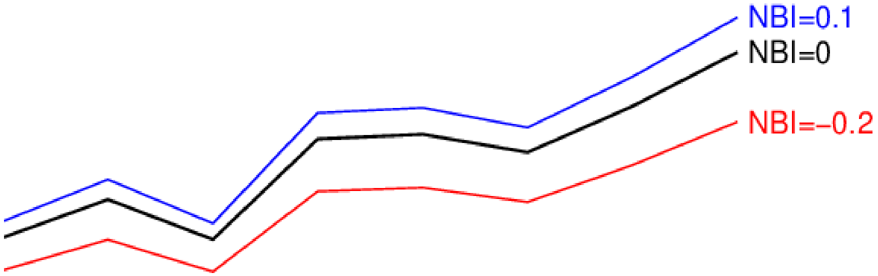
$$\boxed{\frac{\sigma_S}{\bar{S}} \sim const.}$$

Holds for amplifications

Example of set of simulations with constant ratio $\sigma_S / \bar{S}$:

$$S_{NBI\ i} = (1 + NBI)\, S_{0\ i}$$

where $S_{0\ i}$ is the unbiased simulation.



Amplification factor: $\boxed{\alpha = 1 + NBI}$

$$\frac{\sigma_S}{\bar{S}} \sim const. \quad \Longleftrightarrow \quad \rho \sim const.$$

Relationship SI – NBI

We can express both $\overline{S}$ and σ_S as functions of NBI

- $\overline{S} = \overline{O}(1 + NBI)$
- $\sigma_S / \overline{S} = const. \quad \Rightarrow \quad \sigma_S = \sigma_{S0}(1 + NBI)$

Also SI can be expressed as a function of NBI

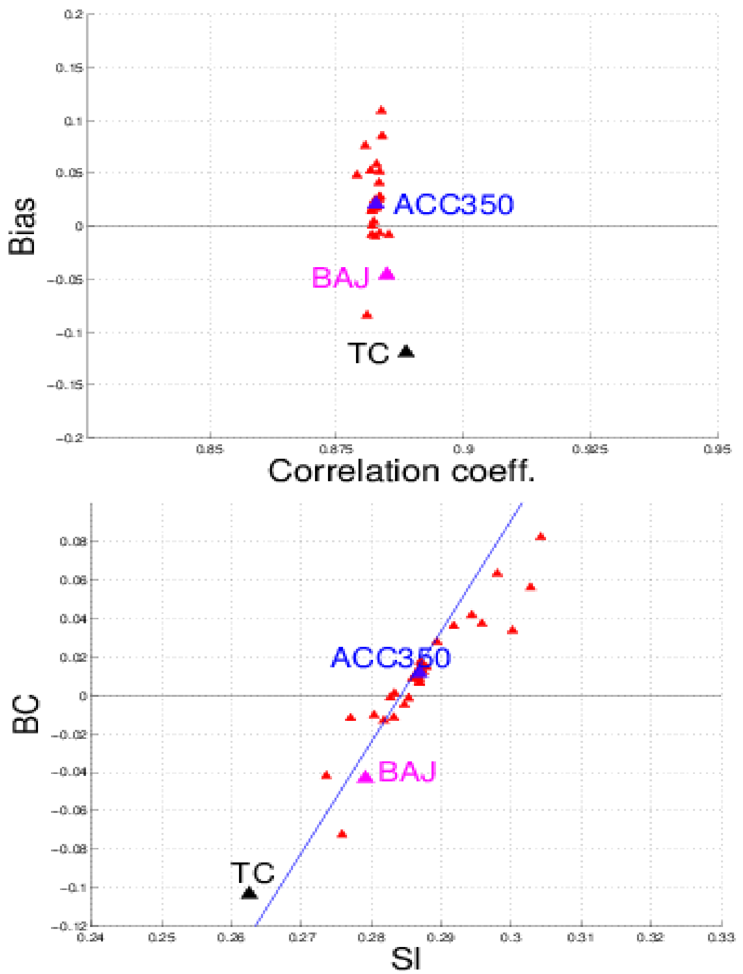
$$\boxed{\sigma_{S0} \sim \sigma_O} \quad \Rightarrow \quad \boxed{SI \sim SI_0 \left(1 + \frac{1}{2} NBI\right)}$$

$$\boxed{\left.\frac{\partial SI}{\partial NBI}\right|_{NBI = 0} \sim \frac{1}{2} SI_0 > 0}$$

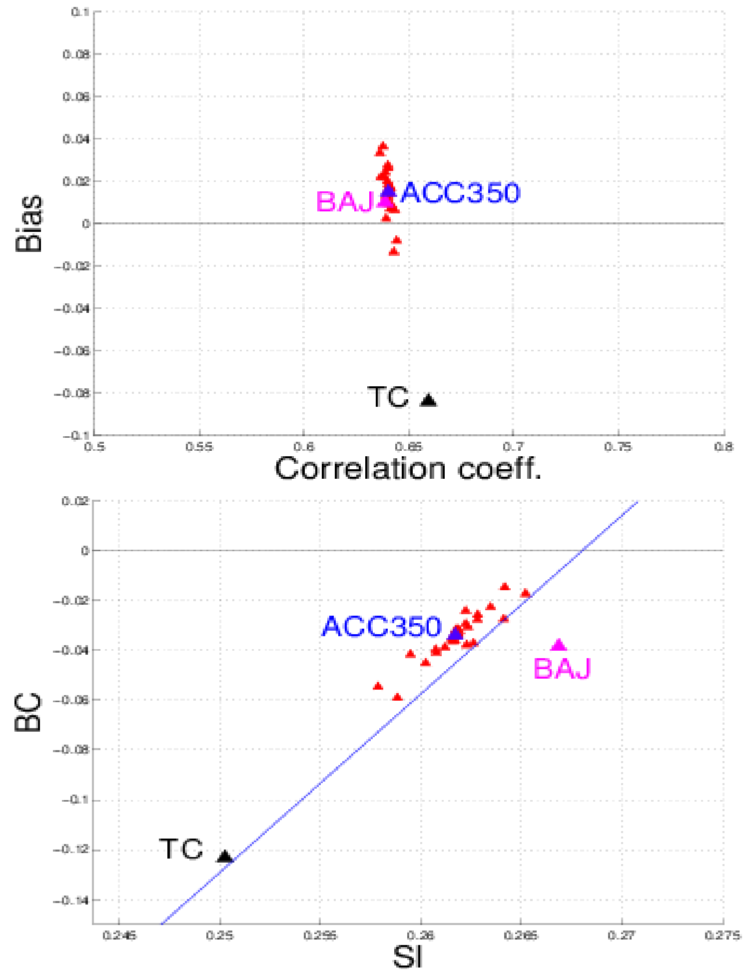
SI grows linearly in NBI around NBI=0

$$SI \sim SI_0 \left(1 + \frac{1}{2} NBI \right)$$

Fits well real world data



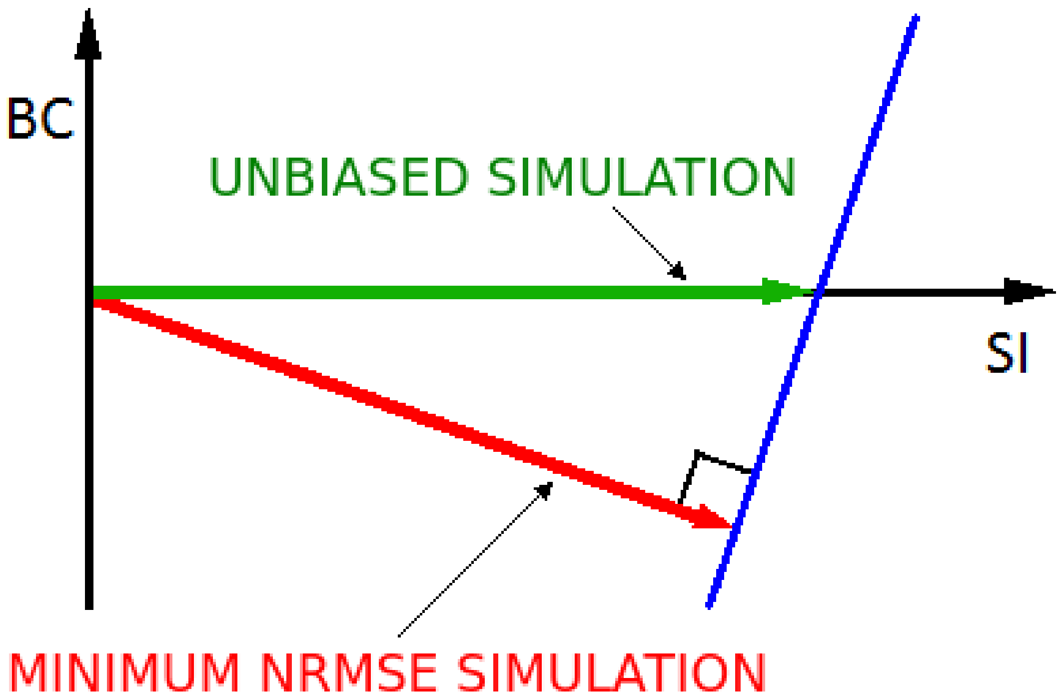
Significant wave height



Mean period

$$SI \sim SI_0\left(1+\frac{1}{2}NBI\right) \Rightarrow$$

The simulation with the minimum value of RMSE/NRMSE underestimates the average value



Conditions when this effect is most evident

- Standard deviation of the same order of the mean

$$\frac{\sigma_o}{\overline{O}} \approx 1$$

- Correlation appreciably smaller than 1 (0.7 - 0.9), since SI is minimum for

$$NBI = 1 - \rho$$

How to overcome this problem?

Hanna and Heinold (1985) indicator:

$$HH = \sqrt{\frac{\sum (S_i - O_i)^2}{\sum S_i O_i}} = \sqrt{\frac{\overline{(S - O)^2}}{\overline{SO}}}$$

Property of HH:

ρ constant $\Rightarrow$ HH minimum for null bias

$$HH^2 = \frac{\sum (S_i - O_i)^2}{\sum S_i O_i} = \frac{\overline{S}^2 + \sigma_S^2 + \overline{O}^2 + \sigma_O^2}{\overline{S}\,\overline{O} + \rho\sigma_S\sigma_O} - 2$$

- $\overline{S} = \overline{O}(1 + NBI)$
- $\sigma_S / \overline{S} = const. \quad \Rightarrow \quad \sigma_S = \sigma_{S0}(1 + NBI)$
- $\sigma_{S0} \sim \sigma_O$

$$\left.\frac{\partial HH^2}{\partial NBI}\right|_{NBI=0} \sim 0$$

$$\left.\frac{\partial^2 HH^2}{\partial NBI^2}\right|_{NBI=0} \sim 2\frac{\overline{O}^2 + \sigma_O^2}{\overline{O}^2 + \rho\sigma_O^2} > 0$$

$\Rightarrow$ HH has a minimum for null bias

Wavewatch III validation on the Mediterranean sea.

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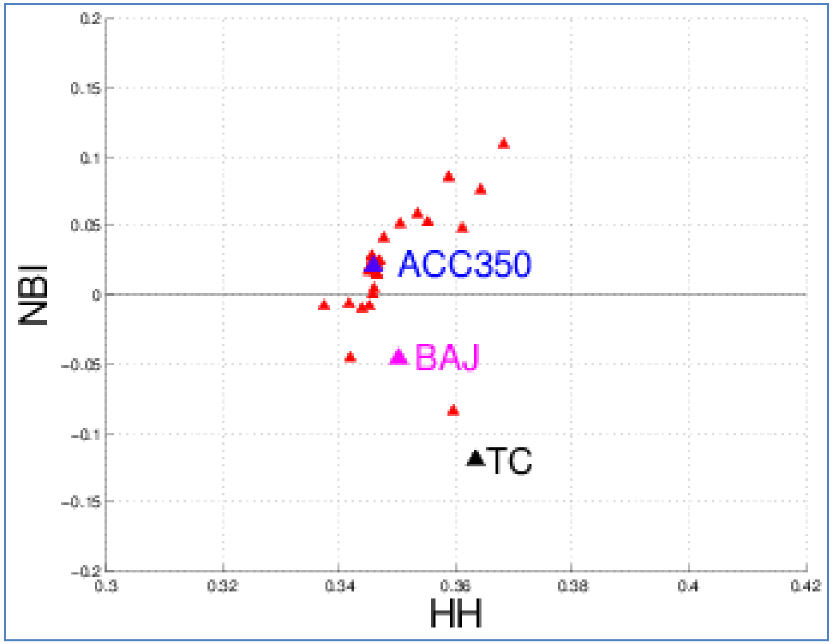
NBI	2.1%	-4.6%	-11.2%
ρ	0.889	0.885	0.883
NRMSE	0.2864	0.2800	0.2798 (-2.3%)
HH	0.3459	0.3502	0.3634 (+4.8%)

HH has a minimum for null bias

Mentaschi et al. 2013



Conclusions

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- HH indicator overcomes this problem introducing a different normalization of the root mean square error.

Thank you!