SOLUZIONE COMPITINO — 11 aprile 2011

ESERCIZIO 1

Per un fluido termodinamico:

$$\rho = \rho(p,T)$$

$$d\rho = \left.\frac{\partial \rho}{\partial p}\right|_T dp + \left.\frac{\partial \rho}{\partial T}\right|_p dT \Rightarrow \frac{d\rho}{\rho} = \frac{1}{\rho}\left.\frac{\partial \rho}{\partial p}\right|_T dp + \frac{1}{\rho}\left.\frac{\partial \rho}{\partial T}\right|_p dT$$

$$= \epsilon^{-1} dp - \alpha\, dT$$

Trasformazione isoterma: $dT = 0$

$$\frac{d\rho}{\rho} = \epsilon^{-1} dp = a p^2\, dp$$

$$\ln\frac{\rho_f}{\rho_0} = a\frac{(p_f^3 - p_0^3)}{3} \Rightarrow \rho_f = \rho_0 \exp\left[\frac{a}{3}(p_f^3 - p_0^3)\right]$$

FILA A

$$\rho_f = 5000 \exp\left[\frac{10^{-4}}{3}(10^3 - 1^3)\right] = 5169{,}3\ \mathrm{kg/m^3}$$

FILA B

$$\rho_f = 4000 \exp\left[\frac{10^{-4}}{3}(8^3 - 1^3)\right] = 4068{,}7\ \mathrm{kg/m^3}$$

ESERCIZIO 2

$$\vec{\nabla} p = \rho\vec{f} - \rho\vec{a} \qquad \vec{f} = (0, -g) \qquad \vec{a} = (a, 0)$$

$$\begin{cases} \dfrac{\partial p}{\partial x} = \rho f_x - \rho a_x = -\rho a \\ \dfrac{\partial p}{\partial z} = \rho f_z - \rho a_z = -\rho g \end{cases}$$

$$dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial z} dz = -\rho a\, dx - \rho g\, dz$$

Sulla superficie libera deve essere $dp = 0$, dunque:

$$dp = -\rho a\, dx - \rho g\, dz_i = 0 \rightarrow \frac{dz_i}{dx} = -\frac{a}{g} \rightarrow z_i = -\frac{a}{g}x + z_0$$

Dal bilancio di massa:

$$\frac{\left(z_i|_{x=0} + z_i|_{x=L}\right)L}{2} = L h_0 \Rightarrow \frac{\left(z_0 - \frac{a}{g}L + z_0\right)}{2} = h_0 \Rightarrow z_0 = h_0 + \frac{a}{g}L/2$$

Dunque: $z_i = \frac{a}{g}\left(\frac{L}{2} - x\right) + h_0$

FILA A

$$z_i\Big|_{x=0} = \frac{a}{g}\frac{L}{2} + h_o = h$$

$$\frac{a}{g} = \frac{2}{L}(h - h_o) = \frac{2}{5}\left(3 - \frac{2}{3}3\right) = 0.4$$

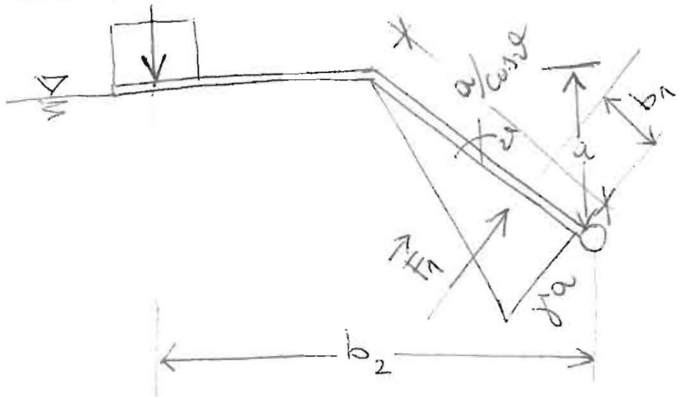
$$a = 0.4 \cdot 9.81 = 3.924\ m/s^2$$

FILA B

$$\frac{a}{g} = \frac{2}{6}\left(3{,}5 - \frac{2}{3}3{,}5\right) = 0{,}39$$

$$a = 3{,}815\ m/s^2$$

ESERCIZIO 3



$$|F_1| = \frac{\gamma a\, a}{2\cos\vartheta} \qquad b_1 = \frac{a}{\cos\vartheta}\frac{1}{3}$$

$$|F_1| b_1 = m g\, b_2$$

$$b_2 = a\, tg\vartheta + L - \frac{b}{2}$$

$$m = \frac{|F_1| b_1}{g\, b_2}$$

FILA A

$|F_1| = 47{,}777$ kN

$b_1 = 1{,}126$ m

$b_2 = 4{,}414$ m

$m = 1242{,}3$ kg

FILA B

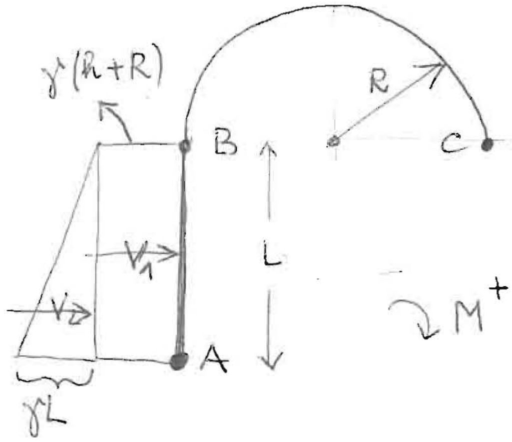
$|F_1| = 24{,}499$ kN

$b_1 = 0{,}793$ m

$b_2 = 4{,}767$ m

$m = 415{,}4$ kg

ESERCIZIO 4



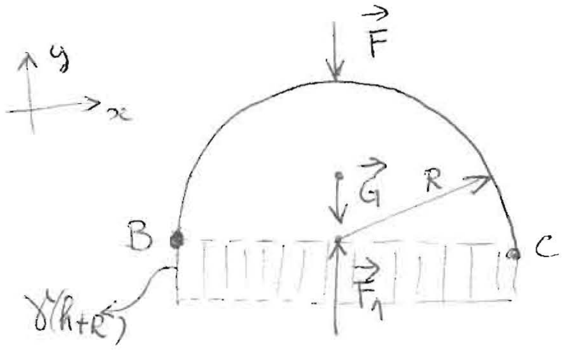
$F_{AB} = V_1 + V_2$

$V_1 = \gamma(h+R)L \qquad b_1 = L/2$ (braccio) rispetto ad A

$V_2 = \gamma L^2/2 \qquad b_2 = L/3$

Momento rispetto al polo A :

$M_{AB} = V_1 b_1 + V_2 b_2$



Per l'equilibrio deve risultare:

$\vec{F} + \vec{G} + \vec{F}_1 = 0$

$\vec{F} = (F_x, F_y) \qquad \vec{G} = (0, -G)$

$\vec{F}_1 = (0, F_1)$

$F_1 = \gamma(h+R)2R$

$G = \frac{\pi R^2}{2}\gamma$

$$\begin{array}{l} \boxed{x} \\ \boxed{y} \end{array} \begin{cases} F_x = 0 \\ F_y + F_1 - G = 0 \end{cases} \qquad \begin{cases} F_x = 0 \\ F_y = G - F_1 \end{cases}$$

Momento di $\vec{F}$ rispetto al polo A:

$M_{BC} = \sqrt{F_x^2 + F_y^2} \cdot R$ (braccio di F è pari a R)

Le forze idrostatiche complessive agenti su ABC saranno:

(orizzontali) $F_{Tx} = F_{AB} + 0 = V_1 + V_2$

(verticali) $F_{Ty} = 0 + F_{BC} = G - F_1$

Il momento rispetto al polo A:

$M_A = M_{AB} + M_{BC} = V_1 b_1 + V_2 b_2 + |G - F_1| R =$

FILA A

$F_{Tx} = 1746,2$ kN

$F_{Ty} = -5387,1$ kN

$M_A = 43843,2$ kN·m

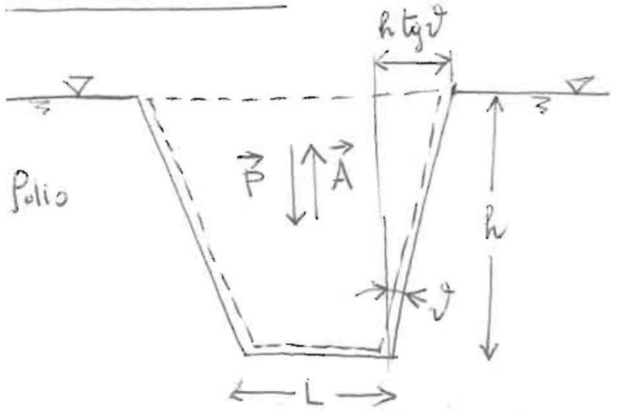
FILA B

$F_{Tx} = 1151,9$ kN

$F_{Ty} = -4123,9$ kN

$M_A = 28510,5$ kN·m

ESERCIZIO 5



$$|\vec{P}| = \rho_{H_2O} V_{H_2O}\, g + Mg \qquad \text{(peso } H_2O + \text{ '' recipiente)}$$

$$|\vec{A}| = \rho_{olio}\, g \left[Lh + h^2 \operatorname{tg}\vartheta\right] b \qquad \text{(spinta di Archimede)}$$

Per l'equilibrio deve risultare :

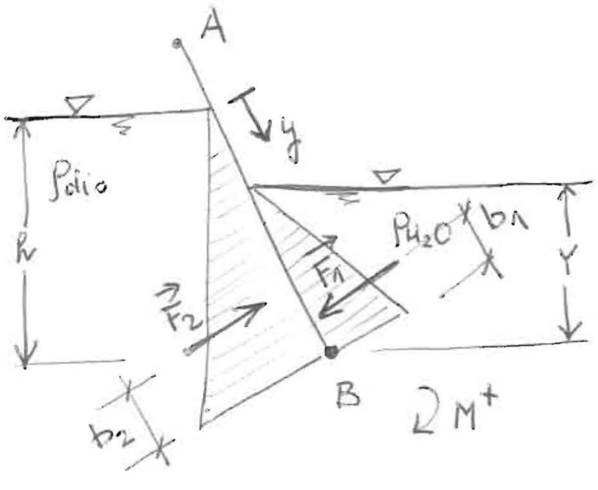
$$|\vec{P}| = |\vec{A}|$$

$$\rho_{olio} \cancel{g} \left[Lh + h^2 \operatorname{tg}\vartheta\right] b = \rho_{H_2O} V_{H_2O} \cancel{g} + M\cancel{g}$$

$$h^2 \operatorname{tg}\vartheta + Lh - \left(\frac{\rho_{H_2O}}{\rho_{olio}}\frac{V_{H_2O}}{b} + \frac{M}{\rho_{olio}\, b}\right) = 0$$

$$h = \frac{-L \pm \sqrt{L^2 + 4\operatorname{tg}\vartheta\left(\frac{\rho_{H_2O}}{\rho_{olio}}\frac{V_{H_2O}}{b} + \frac{M}{\rho_{olio}\, b}\right)}}{2\operatorname{tg}\vartheta} = \begin{cases} h_1 > 0 \\ h_2 < 0 \quad \text{(non accettabile)} \end{cases}$$

$$\Rightarrow h = h_1 > 0$$



$$|F_1| = \rho_{H_2O}\, g\, Y \frac{Y}{2\cos\vartheta}\, b \qquad b_1 = \frac{1}{3}\frac{Y}{\cos\vartheta}$$

$$|F_2| = \rho_{olio}\, g\, h \frac{h}{2\cos\vartheta}\, b \qquad b_2 = \frac{1}{3}\frac{h}{\cos\vartheta}$$

Per calcolare Y basta risolvere :

$$V_{H_2O} = \left[L\cancel{h} + Y^2 \operatorname{tg}\vartheta\right] b$$

$$Y^2 \operatorname{tg}\vartheta + LY - \frac{V_{H_2O}}{b} = 0$$

$$Y = \frac{-L + \sqrt{L^2 + 4\operatorname{tg}\vartheta\, V_{H_2O}/b}}{2\operatorname{tg}\vartheta} \qquad \text{(soluzione > 0 !)}$$

Il momento rispetto al polo B :

$$F_2 b_2 - F_1 b_1 = b\,(F_2 - F_1) \longrightarrow b = \frac{F_2 b_2 - F_1 b_1}{F_2 - F_1} \Rightarrow y = \frac{h}{\cos\vartheta} - b$$

FILA A

$h = 0.143$ m

$y = 0.084$ m

FILA B

$h = 0.121$ m

$y = 0.075$ m

ESERCIZIO 6

$$Y_2 = f(Y_1, V, g)$$

$[Y_2] = L \qquad [V] = L T^{-1}$

$[Y_1] = L \qquad [g] = L T^{-2}$

Ho soltanto due grandezze dimensionalmente indipendenti.
Scelgo Y_1 e g e verifico l'indipendenza dimensionale:

$$Y_1^{\alpha} g^{\beta} = L^0 T^0$$

$$L^{\alpha} L^{\beta} T^{-2\beta} = L^0 T^0 \rightarrow L^{\alpha+\beta} T^{-2\beta} = L^0 T^0 \rightarrow \begin{cases} \alpha+\beta = 0 \\ -2\beta = 0 \end{cases} \rightarrow \begin{cases} \alpha = 0 \\ \beta = 0 \end{cases}$$

c.v.d.

Adimensionalizzo Y_2 e V:

$$[Y_2] = L \rightarrow \Pi_0 = \frac{Y_2}{Y_1}$$

$$[V] = L T^{-1} \rightarrow L^{\alpha+\beta} T^{-2\beta} = L T^{-1} \rightarrow \begin{cases} \alpha+\beta = 1 \\ -2\beta = -1 \end{cases} \rightarrow \begin{cases} \alpha = 1/2 \\ \beta = 1/2 \end{cases}$$

$$\Pi_1 = \frac{V}{(gY_1)^{1/2}}$$

$$\Rightarrow \Pi_0 = f_0\left(\frac{V}{\sqrt{gY_1}}\right)$$