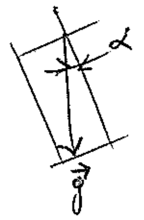
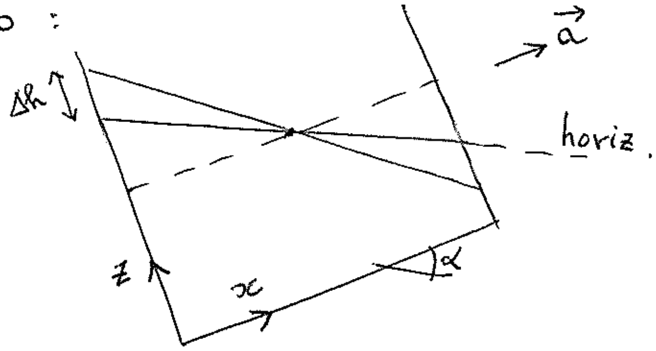
COMPITINO 16 Aprile 2012 — SOLUZIONE FILA A

ESERCIZIO 1

In un sistema solidale con il carrello:





$$-\vec{\nabla} p + \rho \vec{f} = \rho \vec{a}$$

$$\vec{f} = (-g\sin\alpha, -g\cos\alpha)$$

$$\vec{a} = (a, 0)$$

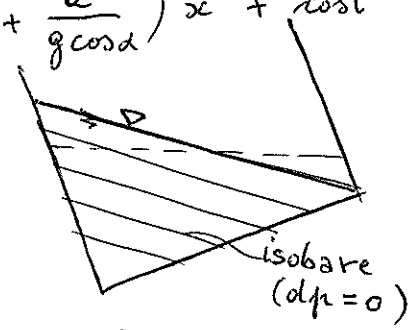
$$\begin{cases} -\dfrac{\partial p}{\partial x} - \rho g \sin\alpha = \rho a & \rightarrow \dfrac{\partial p}{\partial x} = -\rho(g\sin\alpha + a) \\ -\dfrac{\partial p}{\partial z} - \rho g \cos\alpha = 0 & \rightarrow \dfrac{\partial p}{\partial z} = -\rho g \cos\alpha \end{cases}$$

$$dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial z} dz = -\rho(g\sin\alpha + a)\,dx - \rho g \cos\alpha\, dz$$

La superficie libera sarà dunque:

$$dp = 0 \qquad -\rho(g\sin\alpha + a)\,dx - \rho g\cos\alpha\, dz_i = 0$$

$$\frac{dz_i}{dx} = -\operatorname{tg}\alpha - \frac{a}{g\cos\alpha} \Rightarrow z_i = -\left(\operatorname{tg}\alpha + \frac{a}{g\cos\alpha}\right)x + cost$$



La conservazione della massa impone:

$$\left(z_i\big|_{x=0} + z_i\big|_{x=L}\right)\frac{Lb}{2} = V \qquad \text{dunque}$$

$$\left[-\left(\operatorname{tg}\alpha + \frac{a}{g\cos\alpha}\right)\cdot 0 - \left(\operatorname{tg}\alpha + \frac{a}{g\cos\alpha}\right)L + 2cost\right]\frac{Lb}{2} = V \Rightarrow cost = \frac{V}{Lb} + \left(\operatorname{tg}\alpha + \frac{a}{g\cos\alpha}\right)\frac{L}{2}$$

Dunque:

$$z_i = \frac{V}{Lb} + \left(\operatorname{tg}\alpha + \frac{a}{g\cos\alpha}\right)\left(\frac{L}{2} - x\right) = \underbrace{\frac{V}{Lb} + (\operatorname{tg}\alpha)\left(\frac{L}{2} - x\right)}_{\text{livello di quiete}} + \underbrace{\frac{a}{g\cos\alpha}\left(\frac{L}{2} - x\right)}_{\text{variazione con accelerazione}}$$

$$\Delta h = z_i\big|_{\substack{a\neq 0\\ x=0}} - z_i\big|_{\substack{a=0\\ x=0}} = \frac{a}{g\cos\alpha}\frac{L}{2} \Rightarrow a = g\, 2\cos\alpha \frac{\Delta h}{L} = 1{,}35 \text{ m/s}^2$$

La pressione sarà minima in $x = L$ e pari a:

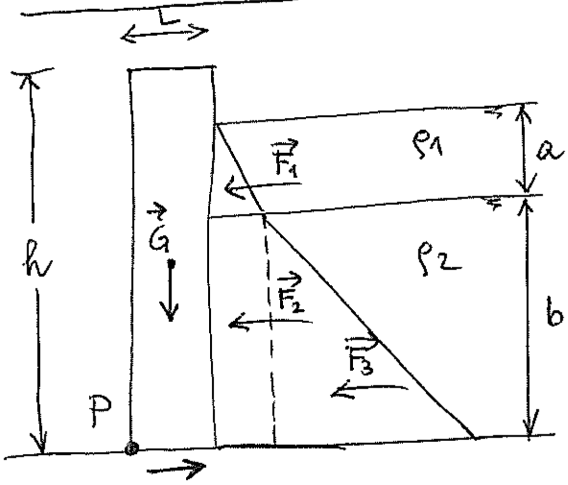
$$p\big|_{x=L} = -\rho g \cos\alpha\, z + cost \qquad \begin{matrix} z = h_0 - \Delta h \\ p = 0 \end{matrix} \Rightarrow p\big|_{x=L} = \rho g \cos\alpha\,(h_0 - \Delta h - z)$$

$$p\big|_{\substack{x=L\\ z=0}} = \rho g \cos\alpha\,(h_0 - \Delta h) = 10{,}93 \text{ kPa}$$

ESERCIZIO 2

si vedano le dispense del Prof. Blondeaux

ESERCIZIO 3



$$|F_1| = \rho_1 g \frac{a^2}{2} = 3973\ N \qquad b_1 = b + \frac{1}{3}a = 1{,}6\ m$$

$$|F_2| = \rho_1 g a b = 11478\ N \qquad b_2 = \frac{b}{2} = 0{,}65\ m$$

$$|F_3| = \rho_2 g \frac{b^2}{2} = 14921\ N \qquad b_3 = \frac{b}{3} = 0{,}43\ m$$

La spinta orizzontale totale sarà

$$|F| = |F_1| + |F_2| + |F_3|$$

Per l'equilibrio allo slittamento deve risultare:

$$\mu G = F$$

$$\mu \rho_c h L g = |F| \Rightarrow L = \frac{|F|}{\mu \rho_c h g} = 0{,}96\ m$$

Per l'equilibrio allo ribaltamento deve risultare:

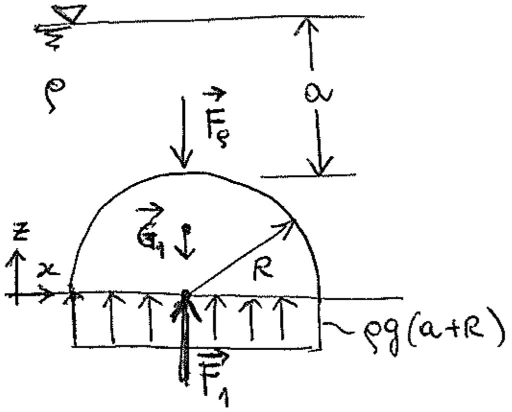
$$G\frac{L}{2} = F_1 b_1 + F_2 b_2 + F_3 b_3$$

$$L = \sqrt{\frac{2}{\rho_c h g}\left(F_1 b_1 + F_2 b_2 + F_3 b_3\right)} \Rightarrow L = 0{,}71\ m$$

Bisognerà dunque dimensionare il muro con uno spessore $L \geq 0{,}96\ m$

ESERCIZIO 4

Calcoliamo la spinta esercitata dal fluido esterno (ρ) sulla cupola:



Considero l'equilibrio della cupola

$$\vec{F}_\rho + \vec{G}_1 + \vec{F}_1 = 0$$

$$\vec{F}_\rho = (0, -F_\rho) \qquad \vec{F}_1 = (0, F_1)$$

$$\vec{G}_1 = (0, -G_1)$$

$$G_1 = \rho g \frac{2}{3}\pi R^3 = 554,7 \text{ kN}$$

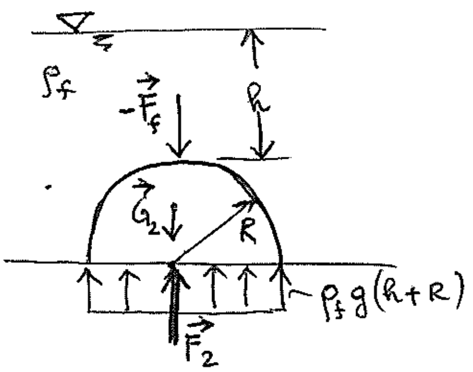
$$F_1 = \rho g (a+R)\pi R^2 = 2912,4 \text{ kN}$$

Dunque per l'equilibrio lungo z deve risultare:

$$-F_\rho - G_1 + F_1 = 0$$

$$F_\rho = F_1 - G_1 = \rho g (a+R)\pi R^2 - \rho g \frac{2}{3}\pi R^3 = \rho g a \pi R^2 + \rho g \frac{\pi R^3}{3}$$

Analogamente, la spinta esercitata dal fluido interno (ρ_f) sulla cupola:



l'equilibrio della cupola impone:

$$-\vec{F}_f + \vec{G}_2 + \vec{F}_2 = 0$$

$$\vec{F}_f = \vec{G}_2 + \vec{F}_2$$

$$\vec{F}_f = (0, F_f) \qquad \vec{F}_2 = (0, F_2)$$

$$\vec{G}_2 = (0, -G_2)$$

$$G_2 = \rho_f g \frac{2}{3}\pi R^3 = 998,5 \text{ kN}$$

$$F_2 = \rho_f g (h+R)\pi R^2$$

Da cui per l'equilibrio lungo z deve risultare:

$$F_f = -G_2 + F_2 = \rho_f g (h+R)\pi R^2 - \rho_f g \frac{2}{3}\pi R^3 = \rho_f g h \pi R^2 + \rho_f g \frac{\pi R^3}{3}$$

La cupola si solleverà quando:

$$P + F_\rho = F_f \Rightarrow \rho_f g h \pi R^2 + \rho_f g \frac{\pi R^3}{3} = P + F_\rho \Rightarrow h = \frac{P + F_\rho - \rho_f g \frac{\pi R^3}{3}}{\rho_f g \pi R^2} = 4,07$$

ESERCIZIO 5

$$P = \rho_s g V = 30 \ N$$

$$P - A = 28,42 \ N$$

$$A = \rho g V = P - 28,42 \Rightarrow V = \frac{P - 28,42}{\rho g}$$

$$\rho_s g V = P \Rightarrow \rho_s = \frac{P}{gV} = \frac{P \rho \cancel{g}}{\cancel{g}(P - 28,42)} = \frac{30 \cdot 1000}{(30 - 28,42)} = 18987 \ kg/m^3$$

ESERCIZIO 6

$$V = f\left(\frac{dp}{dx}, h, \mu, y\right)$$

$[V] = LT^{-1}$ $\quad [h] = L$ $\quad [y] = L$

$\left[\frac{dp}{dx}\right] = ML^{-2}T^{-2}$ $\quad [\mu] = ML^{-1}T^{-1}$

Ho 3 grandezze dimensionalmente indipendenti

Scelgo dp/dx, h, μ come variabili indipendenti.

Verifico l'indipendenza:

$$\frac{dp}{dx}^{\alpha} h^{\beta} \mu^{\gamma} = M^{\alpha} L^{-2\alpha} T^{-2\alpha} L^{\beta} M^{\gamma} L^{-\gamma} T^{-\gamma} = M^0 L^0 T^0$$

$$\begin{cases} \alpha + \gamma = 0 \\ -2\alpha + \beta - \gamma = 0 \\ -2\alpha - \gamma = 0 \end{cases} \quad \begin{array}{l} \rightarrow \alpha = -\gamma \\ \\ \rightarrow 2\gamma - \gamma = 0 \rightarrow \gamma = 0 \end{array} \quad \begin{array}{l} \alpha = 0 \\ \beta = 0 \\ \gamma = 0 \end{array}$$

c.v.d.

Adimensionalizzo V:

$$\Pi_1 = \frac{V}{\frac{dp}{dx}^{\alpha} h^{\beta} \mu^{\gamma}} \Rightarrow M^{\alpha} L^{-2\alpha} T^{-2\alpha} L^{\beta} M^{\gamma} L^{-\gamma} T^{-\gamma} = LT^{-1}$$

$$\begin{cases} \alpha + \gamma = 0 \\ -2\alpha + \beta - \gamma = 1 \\ -2\alpha - \gamma = -1 \end{cases} \quad \begin{cases} \alpha = -\gamma \\ \\ 2\gamma - \gamma = -1 \rightarrow \gamma = -1 \end{cases} \quad \begin{array}{l} \alpha = 1 \\ \beta = 1 + 2\alpha + \gamma = 2 \end{array} \Rightarrow \boxed{\Pi_1 = \frac{V}{\frac{dp}{dx} h^2 \mu^{-1}}}$$

Adimensionalizzo y:

$$\Pi_2 = \frac{y}{\frac{dp}{dx}^{\alpha} h^{\beta} \mu^{\gamma}} \Rightarrow \boxed{\Pi_2 = \frac{y}{h}} \qquad \Rightarrow \Pi_1 = f(\Pi_2)$$

Dunque $V = \frac{dp/dx \ h^2}{\mu} f(y/h)$