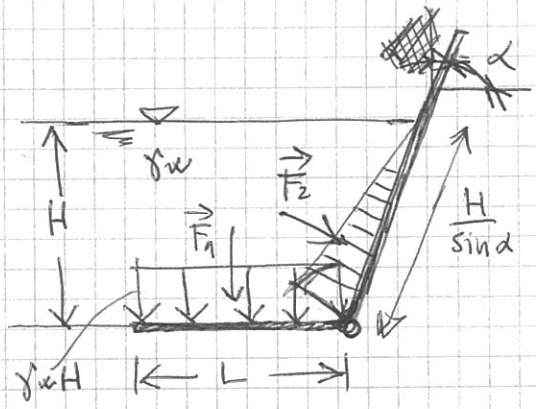
SOLUZIONE ESAME 2 OTTOBRE 2012

ESERCIZIO 1



$|F_1| = \gamma_w H L b \qquad b_1 = \frac{L}{2}$

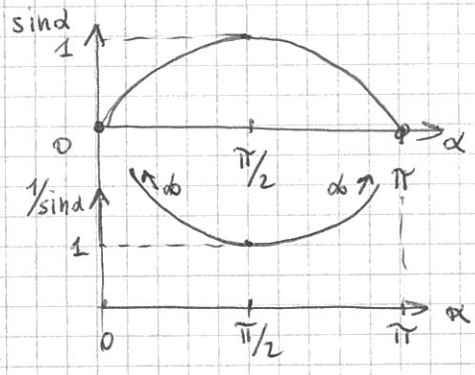
$|F_2| = \gamma_w H \frac{H}{\sin\alpha} b \frac{1}{2} \qquad b_2 = \frac{1}{3} H/\sin\alpha$

$|F_2| b_2 \leq |F_1| b_1$

$\frac{1}{2} \gamma_w \frac{H^2}{\sin\alpha} b \frac{1}{3} \frac{H}{\sin\alpha} \leq \gamma_w H L b \frac{L}{2}$

$L^2 \geq \frac{1}{3} \frac{H^2}{(\sin\alpha)^2} \Rightarrow \boxed{L \leq \sqrt{\frac{1}{3}} \frac{H}{\sin\alpha}}$

Minimo quando $\sin\alpha = 1 \Rightarrow \alpha = \pi/2$



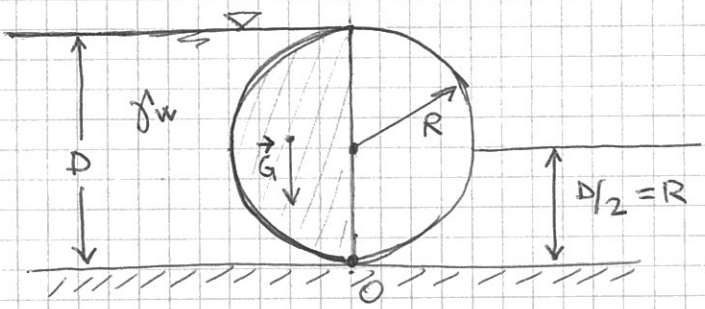
ESERCIZIO 2

$\vec{F} + \vec{G} + \vec{F}_1 = 0$

$\vec{F} = (F_x, F_y)$

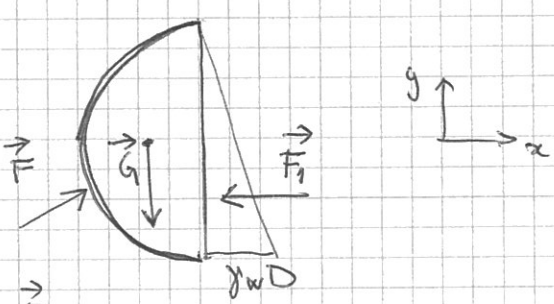
$\vec{G} = (0, -G)$

$\vec{F}_1 = (-F_1, 0)$

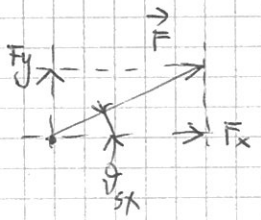


$|\vec{F}_1| = \gamma_w D \frac{D}{2} b \qquad b_1 = \frac{1}{3} D$

$G = \frac{\pi D^2}{8} \gamma_w b \qquad b_2 = \frac{4R}{3\pi}$



$$\begin{cases} F_x - F_1 = 0 \rightarrow F_x = \gamma_w \frac{D^2}{2} b \\ F_y - G = 0 \rightarrow F_y = \frac{\pi D^2}{8} \gamma_w b \end{cases}$$

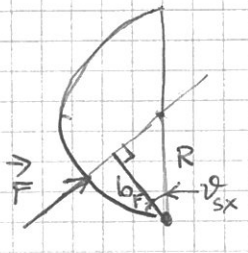


$tg\vartheta_{sx} = \frac{F_y}{F_x} = \frac{\frac{\pi D^2}{8} \gamma_w b}{\gamma_w b \frac{D^2}{2}} = \frac{\pi}{4}$

$b_F = R \cos\vartheta_{sx}$

$M_{sx} = |F| b = \sqrt{F_x^2 + F_y^2}\, b =$

$= \gamma_w b D^2 R \cos\vartheta_{sx} \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\pi}{8}\right)^2}$

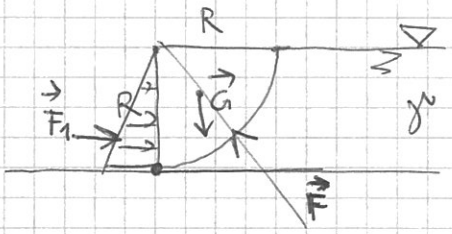


$$\vec{F}_1 + \vec{F} + \vec{G} = 0$$

$$\vec{F} = (F_x, F_y)$$

$$\vec{F}_1 = (F_1, 0)$$

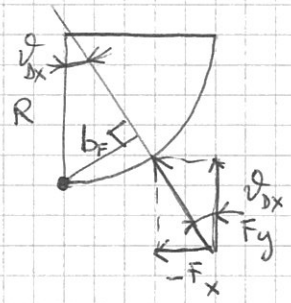
$$\vec{G} = (0, -G)$$



$$F_1 = \gamma R \frac{R}{2} b \qquad b_1 = \frac{1}{3} R$$

$$G = \gamma \frac{\pi R^2}{4} b \qquad b_2 = \frac{2R}{3\pi}$$

$$\begin{cases} F_x + F_1 = 0 & \rightarrow F_x = -F_1 = -\gamma \frac{R^2}{2} b \\ F_y - G = 0 & \rightarrow F_y = G = \gamma \frac{\pi R^2}{4} b \end{cases}$$



$$tg\,\vartheta_{DX} = \frac{-F_x}{F_y} = \frac{\gamma R^2 b}{2} \frac{4}{\gamma \pi R^2 b} = \frac{2}{\pi}$$

$$b_F = R \sin \vartheta_{DX}$$

$$M_{DX} = |F| b_F = \sqrt{F_x^2 + F_y^2}\, b_F = \gamma R^2 b \sqrt{\frac{1}{4} + \left(\frac{\pi}{4}\right)^2} (R \sin \vartheta_{DX}) =$$

$$= \gamma R^2 b\, R \sin \vartheta_{DX} \cdot \sqrt{1 + \frac{\pi^2}{4}} \cdot \frac{1}{2}$$

$$\Rightarrow M_{SX} = M_{DX}$$

$$\gamma_w b D^2 R \cos \vartheta_{SX} \frac{1}{2} \sqrt{1 + \pi^2/16} = \gamma R^2 b R \sin \vartheta_{DX} \frac{1}{2} \sqrt{1 + \pi^2/4}$$

da cui

$$\gamma = \gamma_w \frac{4R^2 \cos \vartheta_{DX}}{R^2 \sin \vartheta_{DX}} \frac{\sqrt{1 + \pi^2/16}}{\sqrt{1 + \pi^2/4}} = \gamma_w \frac{4 \cos \vartheta_{SX}}{\sin \vartheta_{DX}} \frac{\sqrt{1 + \pi^2/16}}{\sqrt{1 + \pi^2/4}} = 4000 \ kg/m^3$$

ESERCIZIO 3

$$H_1 - \frac{U^2}{2g}\left(\xi_{IM} + \xi_{GOM} + \frac{\lambda}{D}L\right) = H_2$$

$H_1 = H$

$H_2 = \frac{U_u^2}{2g}$

$$U = \frac{Q}{\Omega} = \frac{Q}{\pi D^2/4} = 3{,}183 \ m/s$$

$$Q = \Omega U = \Omega_u U_u \Rightarrow U_u = U\left(\frac{D}{D_u}\right)^2$$

$$Re = \frac{UD}{\nu} = 6{,}37 \cdot 10^5$$

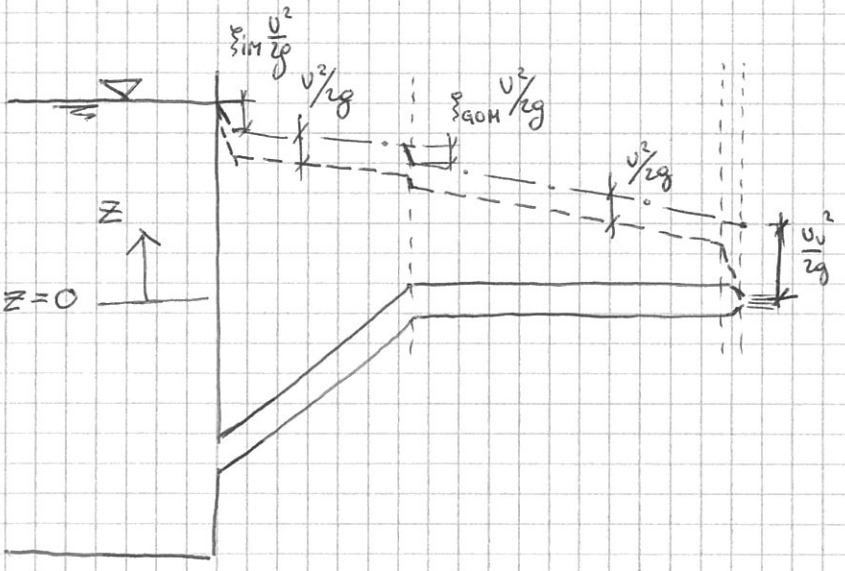
$$\epsilon = \frac{\epsilon_k}{D} = 10^{-3}$$

$\Rightarrow \lambda = 0{,}0201$

$\xi_{IM} \simeq 0.5$

$\xi_{GOM} \simeq 0.4$

$$\Rightarrow H = \frac{U^2}{2g}\left(\xi_{IM} + \xi_{GOM} + \frac{\lambda}{D}L\right) + \frac{U_u^2}{2g} = 143{,}05 \ m$$



ESERCIZIO 4

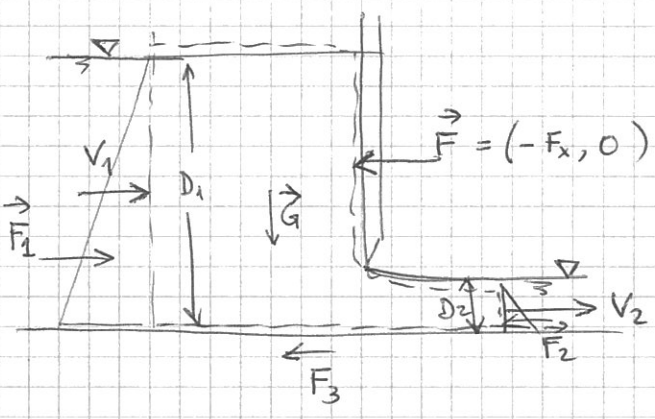
$$Q_1 + Q_2 = Q_3$$

$$U_1 \frac{\pi D^2}{4} + U_2 \frac{\pi D^2}{4} = U_3 \frac{\pi D_3^2}{4} \Rightarrow D_3^2 = D^2\left(\frac{U_1 + U_2}{U_3}\right)$$

$$D_3 = D\sqrt{\frac{U_1 + U_2}{U_3}} = 0{,}053 \ m$$

ESERCIZIO 5

a) $\vec{G} + \vec{\Pi} = \vec{I} + \vec{M}_u - \vec{M}_i$



lungo x

$$F_1 - F_2 - F_x = \rho V_2^2 D_2 - \rho V_1^2 D_1$$

$$F_1 = \rho g \frac{D_1^2}{2}$$

$$F_2 = \rho g \frac{D_2^2}{2}$$

$$F_x = \rho g \left(\frac{D_1^2 - D_2^2}{2} \right) + \rho V_1^2 D_1 - \rho V_2^2 D_2 = 40819{,}8 \text{ N}$$

b) $$F_x^{\text{statico}} = F_1 = \rho g \frac{D_1^2}{2} = 44145 \text{ N} > F_x^{\text{dinamico}}$$

ESERCIZIO 6

$p V = m R T$ per un gas perfetto

$$p = \rho R T$$

$$\frac{p}{\rho} = RT \overset{T=\text{cost}}{=} \frac{p_0}{\rho_0}$$

$$\frac{dp}{dz} = -\rho g = -\frac{p \rho_0}{p_0} g \quad \rightarrow \quad \frac{dp}{p} = -\frac{\rho_0 g}{p_0} dz$$

$$\ln \frac{p}{p_0} = -\frac{\rho_0 g}{p_0} (z - z_0)$$

$$p = p_0 \exp\left[-\frac{\rho_0 g}{p_0} (z - z_0) \right] \quad \text{o} \quad \rho = \rho_0 \exp\left[-\frac{\rho_0 g}{p_0} (z - z_0) \right]$$

ESERCIZIO 7

$$Re_p = Re_m$$

$$\frac{\rho_p U_p L_p}{\mu_p} = \frac{\rho_m U_m L_m}{\mu_m} \rightarrow U_m = U_p \left(\frac{L_p}{L_m}\right)\left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right) = U_p \cdot 4 = 400\ km/h$$

($\mu_m/\mu_p = 1$, $\rho_p/\rho_m = 1$)

$$C_{D_m} = C_{D_p}$$

$$\frac{F_{D_m}}{\rho_m U_m^2 L_m^2} = \frac{F_{D_p}}{\rho_p U_p^2 L_p^2} \Rightarrow F_{D_p} = F_{D_m}\left(\frac{L_p}{L_m}\right)^2\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{U_p}{U_m}\right)^2 = F_{D_m}\left(\frac{L_p}{L_m}\right)^2\left(\frac{L_m}{L_p}\right)^2 = F_{D_m}$$

($\rho_p/\rho_m = 1$)

$$= 16.5\ N$$

ESERCIZIO 8

$$-P + A = F$$

$$-Mg + \rho g \frac{4\pi}{3} R^3 = F \rightarrow F = 2838{,}1\ N$$