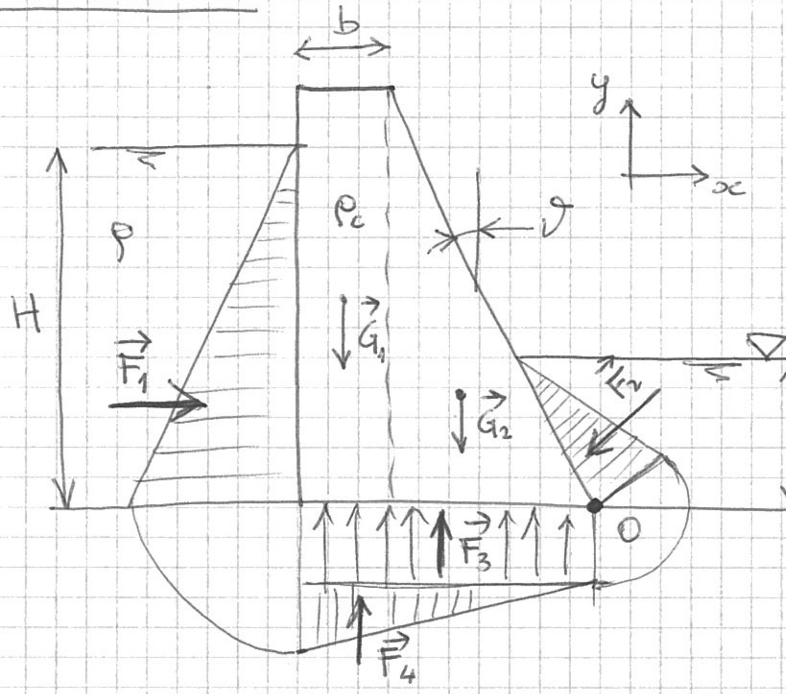
SOLUZIONE ESAME M.d.F. – 5 Sett 2012

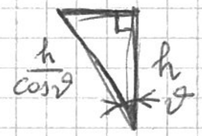
ESERCIZIO 1



$\vec{F}_1 = (F_1, 0)$
$\vec{F}_2 = (F_{2x}, F_{2y})$
$\vec{F}_3 = (0, F_3)$
$\vec{F}_4 = (0, F_4)$
$\vec{G}_1 = (0, -G_1)$
$\vec{G}_2 = (0, -G_2)$

$F_1 = \gamma \frac{H^2}{2}$ $\qquad b_1 = \frac{H}{3}$ (bracci rispetto al polo O)

$|F_2| = \gamma \frac{h^2}{\cos\vartheta} \frac{1}{2}$ $\qquad b_2 = \frac{h}{3\cos\vartheta}$ $\qquad tg\,\vartheta = \frac{B-b}{H+\Delta}$



$F_3 = \gamma h B$ $\qquad b_3 = \frac{B}{2}$

$F_4 = \gamma \frac{(H-h)B}{2}$ $\qquad b_4 = \frac{2}{3} B$

$G_1 = \rho_c g\, b\,(H+\Delta)$ $\qquad b_{G_1} = B - \frac{b}{2}$

$G_2 = \rho_c g\,(B-b)(H+\Delta)$ $\qquad b_{G_2} = (B-b)\frac{2}{3}$

La diga sarà stabile al ribaltamento attorno al polo O se:

$$\underbrace{G_1 b_{G_1} + G_2 b_{G_2} + |F_2|\, b_2}_{\text{Momento stabilizzante}} \geq \underbrace{F_1 b_1 + F_3 b_3 + F_4 b_4}_{\text{Momento destabilizzante}}$$

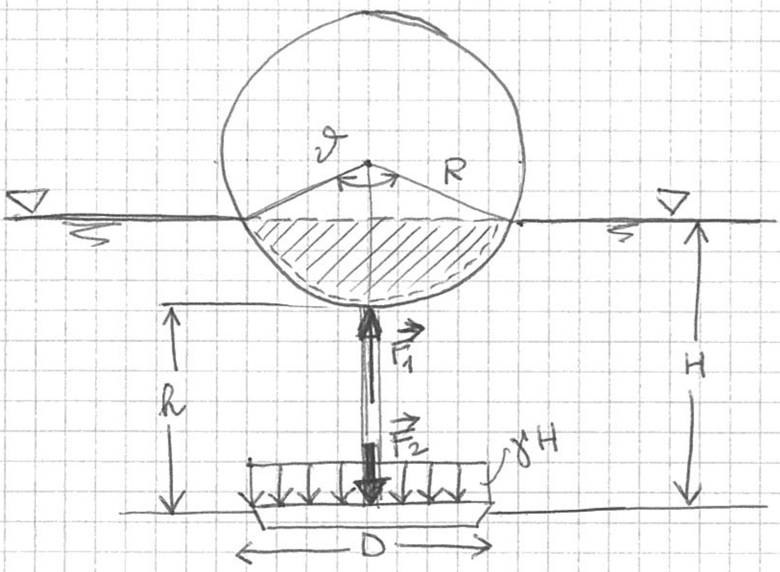
Risulta

$M_{STAB} = 14408{,}9 + 17077{,}2 + 110{,}4 = 31596{,}6$ kN·m/m

$M_{DEST} = 44145 + 1962 + 8502 = 54609$ kN·m/m

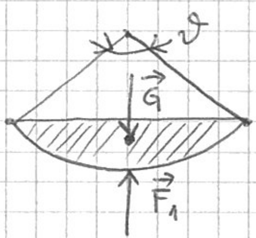
$\Rightarrow$ $M_{DEST} > M_{STAB}$ $\Rightarrow$ INSTABILE

ESERCIZIO 2



Per valutare la spinta esercitata dal fluido sul cilindro si consideri il volume tratteggiato. Per l'equilibrio risulta:

$$\vec{G} + \vec{F}_1 = 0$$



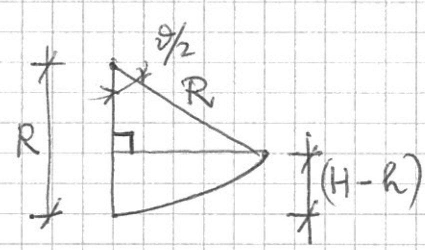
$$\vec{G} = (0, -G)$$

$$\vec{F}_1 = (0, F_1)$$

$$F_1 - G = 0$$

$$F_1 = G = \gamma\left[R^2\frac{\vartheta}{2} - R^2\frac{\cos(\vartheta/2)\sin(\vartheta/2)}{2}\right]L = \gamma R^2\left[\frac{\vartheta}{2} - \frac{\sin\vartheta}{2}\right]L$$

$$\cos(\vartheta/2) = \frac{R-(H-h)}{R} = 1 - \frac{(H-h)}{R}$$



La spinta esercitata sulla valvola risulta:

$$\vec{F}_2 = (0, -F_2)$$

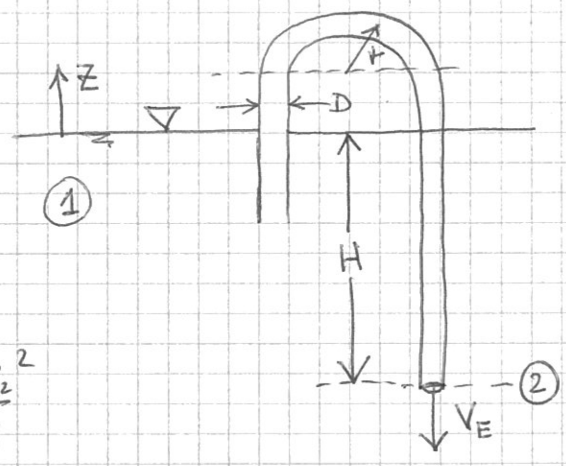
$$F_2 = \gamma H \frac{\pi D^2}{4}$$

Il cilindro si solleverà quando:

$$F_1 - F_2 \geqslant P$$

Procedendo per tentativi si trova: $H = 2{,}34$ m

ESERCIZIO 3



a) Perdite trascurabili

$$H_1 = H_2$$

$$H_1 = \cancel{z_1}^{\,0} + \cancel{\frac{p_1}{\gamma}}^{\,0} + \cancel{\frac{v_1^2}{2g}}^{\,0} = 0$$

$$H_2 = z_2 + \cancel{\frac{p_2}{\gamma}}^{\,0} + \frac{v_2^2}{2g} = -H + \frac{v_2^2}{2g}$$

$$\Rightarrow \quad -H + \frac{v_2^2}{2g} = 0 \quad \rightarrow \quad v_2 = \sqrt{2gH} = 4{,}43 \ m/s$$

$$Q = \Omega v = \frac{\pi D^2}{4} v = 0{,}00139 \ m^3/s = 1{,}39 \ l/s$$

b) Con dissipazioni

$$H_1 - \frac{U^2}{2g}\left(\xi_{IMBOCCO} + \xi_{CURVA} + \lambda L/D\right) = H_2$$

$$\xi_{IMBOCCO} \simeq 1$$

$$\xi_{CURVA} \longrightarrow \frac{r}{D} = \frac{0{,}25}{0{,}02} = 12{,}5 \qquad \vartheta = 180^0 \qquad \rightarrow \xi \simeq 0{,}4$$

$$\begin{cases} Re = \dfrac{UD}{\nu} = \\ \epsilon = \epsilon_s/D = \end{cases} \quad \rightarrow \lambda \text{ from Moody's diagram}$$

$$H_1 = 0$$

$$H_2 = -H + \frac{U^2}{2g}$$

$$\Rightarrow \quad H_2 - H_1 + \frac{U^2}{2g}\left(\xi_{IMB} + \xi_{CURV} + \frac{\lambda L}{D}\right) = 0$$

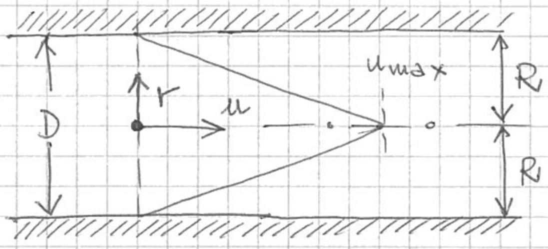
$$-H + \frac{U^2}{2g}\left(\xi_{IMB} + \xi_{CURV} + \frac{\lambda L}{D} + 1\right) = 0$$

Procedendo per tentativi si trova:

U [m/s]	Q [l/s]	Re	λ	Bilancio Energetico $\overset{?}{=} 0$	OUTPUT
4,43	1,392	$2{,}18 \cdot 10^5$	0,0187	4,0	$U \downarrow$
1,0	0,314	$4{,}92 \cdot 10^4$	0,0226	−0,717	$U \uparrow$
2,0	0,628	$9{,}85 \cdot 10^4$	0,0204	0,067	$U \downarrow$
ok $\boxed{1{,}93}$	0,607	$9{,}52 \cdot 10^4$	0,0204	−0,0001	$\boxed{ok}$

ESERCIZIO 4

a) $u(r) = \left(1 - \frac{r}{R}\right) u_{max}$



$$Q = \int_{\Omega} (\vec{u} \cdot \vec{n}) \, d\Omega = \int_0^{2\pi}\int_0^R u(r)\, r \, dr \, d\vartheta =$$

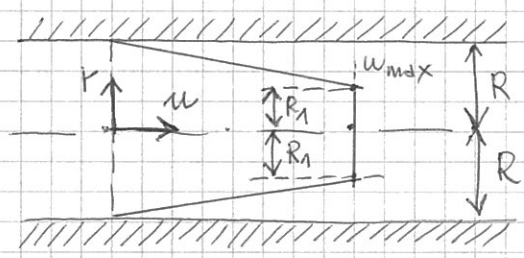
$$= u_{max}\, 2\pi \int_0^R \left(1 - \frac{r}{R}\right) r \, dr = u_{max}\, 2\pi \left[\frac{r^2}{2} - \frac{r^3}{3R}\right]_0^R =$$

$$= u_{max}\, 2\pi \left(\frac{R^2}{2} - \frac{R^2}{3}\right) = u_{max}\, \cancel{2}\pi \frac{R^2}{\cancel{6}\,3} = u_{max} \frac{\pi R^2}{3} = 75{,}4 \cdot 10^{-3}\ m^3/s$$

$$Q_m = \rho Q = 75{,}4\ kg/s$$

$$V = \frac{Q}{\Omega} = \frac{Q}{\pi R^2} = 6{,}667\ m/s$$

b) $\begin{cases} u(r) = \left[1 - \frac{(r - R_1)}{(R - R_1)}\right] u_{max} & R_1 < r < R \\ u(r) = u_{max} & r < R_1 \end{cases}$



$$Q = u_{max}\, \pi R_1^2 + \int_0^{2\pi}\int_{R_1}^R u(r)\, r\, dr\, d\vartheta = u_{max}\, \pi R_1^2 + 2\pi u_{max} \left[\frac{r^2}{2} - \frac{(r^3)}{3(R-R_1)} + \frac{r^2 R_1}{2(R-R_1)}\right]_{R_1}^R$$

$$= u_{max}\, \pi R_1^2 + 2\pi u_{max} \left[\frac{R^2 - R_1^2}{2} - \frac{R^3 - R_1^3}{3(R-R_1)} + \frac{(R^2 - R_1^2) R_1}{2(R - R_1)}\right] =$$

$$= u_{max}\, \pi R_1^2 + 2\pi u_{max} \left[\frac{R^2 - \cancel{R_1^2}}{2} - \frac{R^2 + R R_1 + R_1^2}{3} + \frac{(R + \cancel{R_1}) R_1}{2}\right] =$$

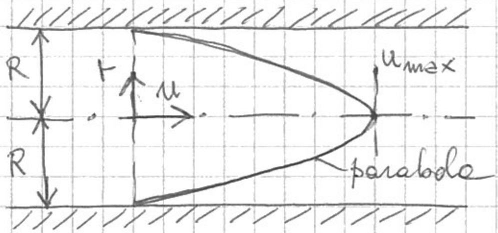
$$= u_{max}\, \pi R_1^2 + 2\pi u_{max} \left[\frac{R^2}{6} + \frac{R R_1}{6} - \frac{R_1^2}{3}\right] = \quad R_1 = R/2$$

$$= u_{max}\, \pi R^2/4 + 2\pi u_{max} \left[\frac{R^2}{6} + \cancel{\frac{R^2}{12}} - \cancel{\frac{R^2}{12}}\right] = u_{max} \frac{7\pi R^2}{12} = 131{,}9 \cdot 10^{-3}\ m^3/s$$

$$Q_m = \rho Q = 131{,}9\ kg/s$$

$$V = \frac{Q}{\Omega} = \frac{Q}{\pi R^2} = 11{,}67\ m/s$$

c) $u(r) = \left[1 - \left(\frac{r}{R}\right)^2\right] u_{max}$



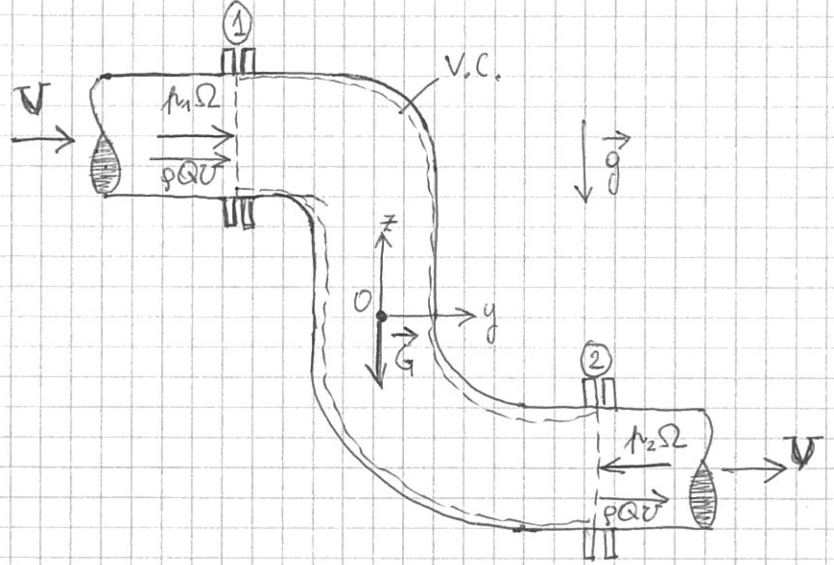
$$Q = \int_0^{2\pi}\int_0^R u(r)\, r\, dr\, d\vartheta = 2\pi u_{max} \int_0^R \left[1 - \left(\frac{r}{R}\right)^2\right] r\, dr =$$

$$= 2\pi u_{max} \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R = 2\pi u_{max} \left(\frac{R^2}{2} - \frac{R^2}{4}\right) = u_{max} \frac{\pi R^2}{2} = 113 \cdot 10^{-3}\, m^3/s$$

$$Q_m = \rho Q = 113{,}1 \; kg/s$$

$$v = \frac{Q}{\Omega} = \frac{Q}{\pi R^2} = 10 \; m/s$$

ESERCIZIO 5



Moto staz. ⇒ $Q = \text{cost}$

$Q = \Omega V =$

$\Omega = \text{cost} \Rightarrow V = \text{cost} = 3\ m/s$

$p_2 = p_1 - 50 = 225\ kPa$

$$\vec{G} + \vec{\Pi} = \vec{I} + \vec{M}_u - \vec{M}_i$$

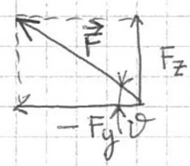
$\vec{G} = (0, -G)$ $\qquad$ $\vec{M}_u = (\rho QV, 0)$ $\qquad$ $\vec{I} = 0$ moto staz.

$\vec{\Pi} = (p_1\Omega - p_2\Omega + F_y, F_z)$ $\qquad$ $\vec{M}_i = (\rho QV, 0)$

$\vec{F} = (F_y, F_z)$ forze esercitate dalla condotta sul fluido

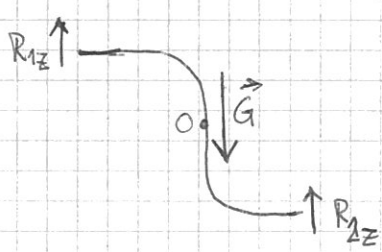
$$\begin{cases} \boxed{y} & (p_1 - p_2)\Omega + F_y = \rho QV - \rho QV = 0 \\ \boxed{z} & -G + F_z = 0 \end{cases} \qquad \begin{cases} F_y = -(p_1 - p_2)\Omega & < 0 \\ F_z = G & > 0 \end{cases}$$

$$\text{tg}\,\vartheta = \frac{F_z}{-F_y} = \frac{G}{(p_1 - p_2)\Omega}$$



La componente verticale F_z della forza esercitata dalla condotta sul fluido sarà ripartita equamente tra le due flange (come suggerito nel testo). Risulta dunque:

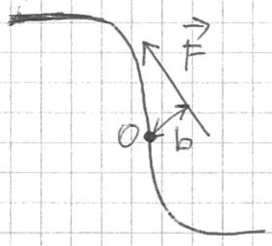
$$R_{1z} = R_{2z} = \frac{F_z}{2} = \frac{G}{2}$$



Per calcolare le componenti orizzontali delle forze esercitate dalle flange bisogna applicare il principio del momento delle quantità di moto (↻ M^+): polo O)

$$p_1\Omega \frac{L}{2} + p_2\Omega \frac{L}{2} - b|F| = -\rho QV \frac{L}{2} - \rho QV \frac{L}{2}$$

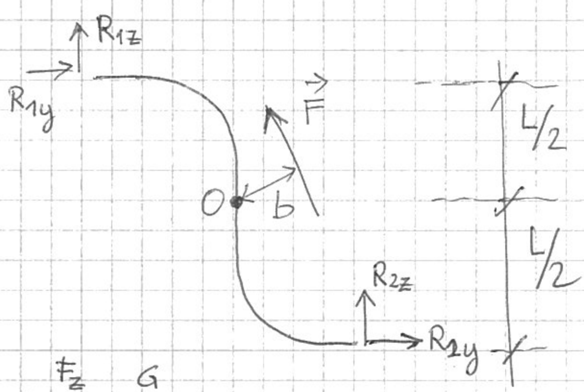
dove b è il braccio della forza $\vec{F}$ esercitata dalla condotta sul fluido



Il momento esercitato dalla condotta sarà dunque:

$$b|F| = (p_1 + p_2)\Omega \frac{L}{2} + \rho Q U L$$

e sarà uguale e contrario rispetto a quello generato dal fluido sulla condotta.



Per l'equilibrio della condotta dovrà dunque risultare:

$$\begin{array}{l} \boxed{y} \\ \boxed{z} \\ \boxed{\text{Mom.}} \end{array} \begin{cases} R_{1y} + R_{2y} = F_y \\ R_{1z} + R_{2z} = F_z \xrightarrow{\text{hp } R_{1z} = R_{2z}} R_{1z} = R_{2z} = \dfrac{F_z}{2} = \dfrac{G}{2} \\ R_{1y}\dfrac{L}{2} - R_{2y}\dfrac{L}{2} = -b|F| \end{cases}$$

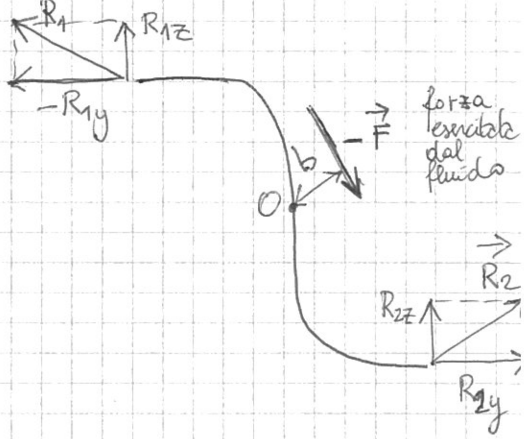
Dalla eq.ne lungo y: $R_{1y} = F_y - R_{2y} = -(p_1 - p_2)\Omega - R_{2y}$

Sostituendo nell'eq.ne dei momenti:

$$-(\cancel{p_1} - p_2)\Omega\frac{\cancel{L}}{2} - R_{2y}\frac{\cancel{L}}{2} - R_2\frac{\cancel{L}}{2} = -(\cancel{p_1} + p_2)\Omega\frac{\cancel{L}}{2} - \rho Q U \cancel{L}$$

$$\rightarrow R_{2y} = p_2\Omega + \rho Q U$$

$$R_{1y} = -(p_1 - \cancel{p_2})\Omega - \cancel{p_2}\Omega - \rho Q U = -p_1\Omega - \rho Q U$$



Utilizzando i valori numerici:

$$G = \gamma\frac{\pi D^2}{4}(L + 2H) = 0{,}770 \text{ kN}$$

$$R_{1z} = R_{2z} = \frac{G}{2} = 0{,}385 \text{ kN}$$

$$R_{1y} = -p_1\Omega - \rho Q U = -2{,}231 \text{ kN}$$

$$R_{2y} = p_2\Omega + \rho Q U = 1{,}838 \text{ kN}$$

ESERCIZIO 6

$$\tau = \mu \left.\frac{du}{dy}\right|_{y=0}$$

$$\frac{du}{dy} = u_{max}\left[\frac{3}{2}\frac{1}{h} - \frac{3}{2}\left(\frac{y}{h}\right)^2\frac{1}{h^2}\right]$$

$$\left.\frac{du}{dy}\right|_{y=0} = u_{max}\frac{3}{2h}$$

$$\Rightarrow \quad \tau = \mu \left.\frac{du}{dy}\right|_{y=0} = \mu\, u_{max}\frac{3}{2h}$$

$$\Rightarrow \quad h = \frac{\mu\, u_{max}}{\tau}\frac{3}{2} = \frac{1.5 \cdot 4 \cdot 10^{-2}}{20}\cdot\frac{3}{2} = 4{,}5 \cdot 10^{-3}\ m$$

ESERCIZIO 7

Similitudine di Froude $\longrightarrow$ $Fr_m = Fr_p$

$$Fr_m = \frac{U_m}{\sqrt{g L_m}} = \frac{U_p}{\sqrt{g L_p}} = Fr_p$$

$$\Rightarrow U_p = U_m\sqrt{\frac{L_p}{L_m}} = U_m\sqrt{20} = 3{,}6\sqrt{20} = 16{,}1\ m/s$$

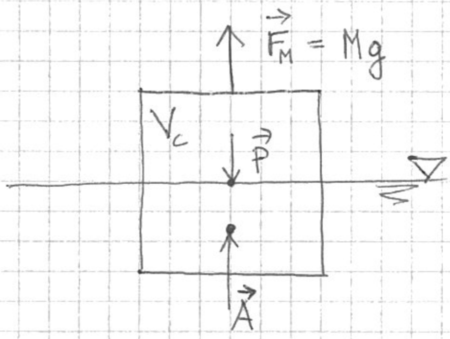
$$C_{D_m} = \frac{F_{Dm}}{\rho_m U_m^2 D_m^2} = \frac{F_{Dp}}{\rho_p U_p^2 D_p^2} = C_{Dp}$$

hp $\rho_p \equiv \rho_m$

$$\Rightarrow F_{D_p} = F_{D_m}\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{U_p}{U_m}\right)^2\left(\frac{D_p}{D_m}\right)^2 = F_{D_m}\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right)^3 = F_{D_m}\left(\frac{L_p}{L_m}\right)^3 =$$

$$= 12{,}2 \cdot 20^3 = 97{,}6\ kN$$

ESERCIZIO 8



V_c = Volume cubo = $1\ m^3$

a) $F_M + A = P$

$F_M = Mg$

$A = \rho g \dfrac{V_c}{2}$

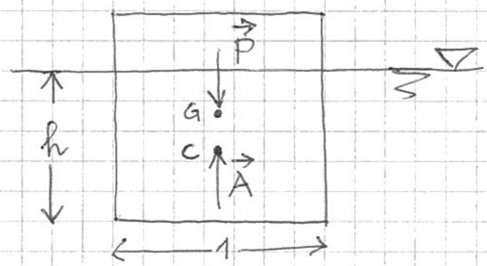
$P = \rho_s g V_c$

da cui $\quad M\cancel{g} + \rho \cancel{g} \dfrac{V_c}{2} = \rho_s \cancel{g} V_c$

$$\rho_s = \frac{M}{V_c} + \frac{\rho}{\cancel{\rho_s}} \frac{1}{2} = \frac{300}{1} + \frac{1000}{2} = 800\ kg/m^3$$

b) $\rho_s < \rho \Rightarrow$ galleggia

Calcolo dell'affondamento h:

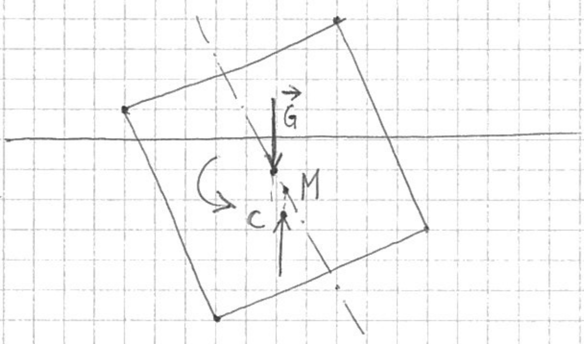


$|P| = |A|$

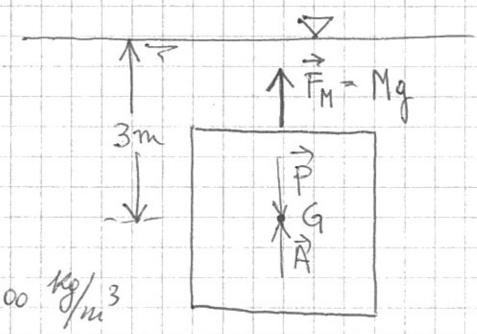
$\rho_s \cancel{g} V_c = \rho \cancel{g}\, h \cdot 1$

$$\Rightarrow h = V_c \frac{\rho_s}{\rho} = 1 \cdot \frac{800}{1000} = 0{,}8\ m$$

Presumibilmente il cubo è in equilibrio instabile perché con una piccola oscillazione si realizza una coppia destabilizzante.



c)



$|P| = |A| + F_M$

$\rho_c V_c g = \rho V_c g + Mg$

$\rho_c V_c \cancel{g} = \rho V_c \cancel{g} + M\cancel{g}$

$$\Rightarrow \rho_c = \rho + \frac{M}{V_c} = 1000 + \frac{300}{1} = 1300\ kg/m^3$$