Fluid Dynamics

Exercises and questions for the course

January 15, 2014

Problem

(Kinematics)

A two dimensional flow field characterised by the following velocity components in polar coordinates is called a *free vortex*:

$$u_r = 0, \qquad u_\theta = \frac{k}{2\pi r}.$$

In the above expression u_r is the radial velocity component and u_θ the azimuthal one.

- Is the flow rotational, i.e. is the vorticity different from zero?
- In the fluid subject to deformation? If so give an estimate of it.
- What is the equation of particle trajectories? Do trajectories and streamlines coincide?
- What is the acceleration of a material particle at the generic point (x_0, y_0)?

Problem

(Dimensional analysis)

Consider the flow of an incompressible fluid in a pipe with constant and circular cross-section. Suppose that the pipe has a fairly sharp bend. Consider a cross-section 1 upstream of the bend and a cross-section 2 downstream of it. Experience shows that there is a pressure loss Δp through the bend. Identify what quantities Δp might depend on. Use the Π theorem to reduce the above quantities to a smaller number of dimensionless groups.

Problem

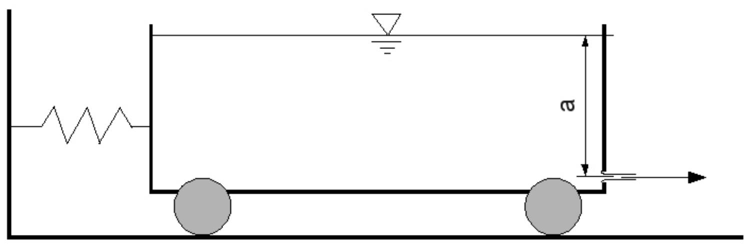
(Integral form of the momentum equation and Bernoulli's theorem)

Water flows through a pipe forming an elbow and exits with a jet in the atmosphere. The flux Q is fixed. The pipe diameter is d_1 and the jet diameter (in a section where streamlines are parallel to each other) is d_2. The elbow lays on a horizontal plane. The angle between the pipe and the jet direction is θ. Neglecting dissipation effects, determine the force on the flange bolts.

Problem

(Integral form of the momentum equation and Bernoulli's theorem)

Consider the cart on wheels depicted in the figure below. The cart contains a liquid with density ρ in contact with the atmosphere and is attached on the left side to a vertical wall by a spring. On the other side, at a depth a from the free surface, there is an orifice on the vertical wall, from which a jet of fluid exits the cart. The orifice is circular with diameter d and it is assumed that the orifice area is much smaller than the free surface area, so that the flow can be considered steady.



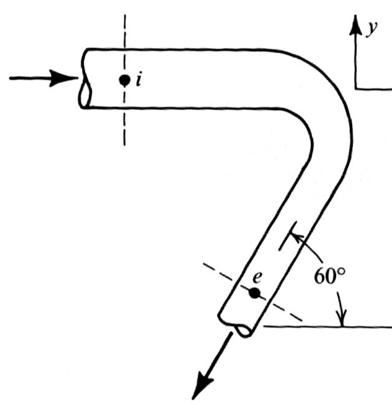
Neglecting friction on the wheels, compute the force on the spring at equilibrium. Is the spring stretched or compressed?

Problem

(Integral form of the momentum equation)

A pipe forms an elbow with a 120° deflection of the flow direction, as shown in the figure below. The cross section of the pipe is Ω_i in i (just upstream of the elbow) and Ω_2 in e (downstream of the elbow). Compute the direction and magnitude of the force exerted by the fluid on the elbow, adopting the following assumptions:

- neglect the pressure drop across the bend;
- assume that the velocity profile in the cross-sections i and e is flat (this is fairly accurate for turbulent flows).



Problem

(Kinematics)

Consider the following steady three-dimensional flow field:

$$u = ax,$$
$$v = ay,$$
$$w = -2az.$$

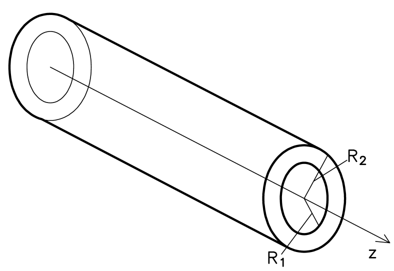
In the above equations (u, v, w) represent the three velocity components with respect to a Cartesian system of coordinates (x, y, z), with z vertically directed coordinate. Let ρ be the fluid density. Assume that the fluid is incompressible and Newtonian, with kinematic viscosity ν.

1. Does this flow field satisfy the continuity equation?
2. Determine the pressure distribution $p(x, y, z)$.
3. Evaluate the stress tensor $\boldsymbol{\sigma}$.
4. Is the flow rotational or irrotational?

Problem

(Kinematics)

Let us consider the longitudinal fully-developed flow in an annular horizontal pipe, as shown in the figure. Let z be the longitudinal direction of the pipe. The fluid flows in the z direction in the region $R_1 < r < R_2$. Flow is steady and is driven by a known pressure gradient $\partial p/\partial z$.



In cylindrical coordinates (z, r, ϕ), the z component of the Navier-Stokes equation reads

$$\frac{\partial u_z}{\partial t} + u_z \frac{\partial u_z}{\partial z} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\varphi}{r} \frac{\partial u_z}{\partial \varphi} + \frac{1}{\rho} \frac{\partial p}{\partial z} - \nu \left[\frac{\partial^2 u_z}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \varphi^2} \right] = 0.$$

Simplify the above equation according to the assumption of fully developed flow in the z direction and axial symmetry and solve it to determine the velocity profile on a cross-section.

Problem

(Kinematics)

Let us consider the following flow field, given in Cartesian coordinates (x, y, z):

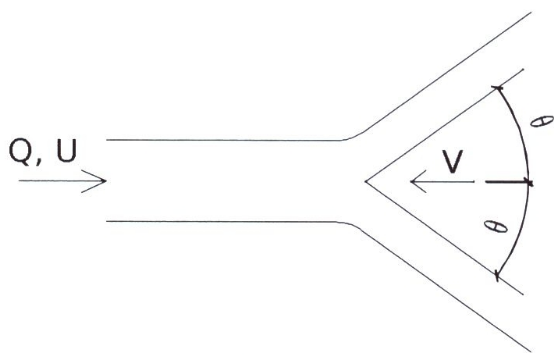
$$\mathbf{u} = (u, v, 0) = (2t + 5y - 5, 7x, 0).$$

- Is the flow two-dimensional?
- Is the flow steady?
- Is the flow incompressible?
- Is the flow rotational?
- Compute the fluid acceleration.
- Compute the rate of strain tensor.

Problem

(Integral form of the momentum equation and Bernoulli's theorem)

Compute the force per unit length that the two-dimensional jet shown in the figure below exerts on the wedge. The wedge is moving with velocity V towards the jet and it is disposed symmetrically with respect to the jet direction. Assume that the flow is steady for an observer moving with the wedge. Let U be the uniform velocity within the jet and Q the corresponding volume flux per unit length.



Problem

(Dimensional analysis)

In a physical model of a river the length scale is $L_m/L_p = \lambda_L = 1/20$ (with L_m any length in the model and L_p the corresponding length in the prototype). The model preserves the Froude number, i.e. $Fr_m = Fr_p$ (we recall that the Froude number is defined as $Fr = U/\sqrt{gL}$).

What is the scale for the flux (volume per unit time)? If the flux in the prototype is $Q = 2500$ m^3/s, what discharge should be used in the model?

Theoretical question

1. Describe the difference between the Eulerian (spatial) and Lagrangian (material) approaches to study the kinematics of fluids.
2. Show how it is possible to compute a material derivative making use of spatial coordinates.

Theoretical question

The Reynolds transport theorem can be stated as follows

$$\frac{D}{Dt}\iiint\limits_{V(t)} \mathcal{F}(\mathbf{x},t)dV = \iiint\limits_{V(t)} \left[\frac{\partial \mathcal{F}}{\partial t} + \nabla\cdot(\mathcal{F}\mathbf{u})\right] dV,$$

with $\mathcal{F}$ any fluid property.

- Derive the above expression;
- discuss its physical meaning;
- show an example of its application in fluid dynamics.

Theoretical question

State and prove Buckingham Π theorem. Discuss its importance in dealing with problems in fluid mechanics.

Theoretical question

Derive and discuss the equations governing an incompressible, irrotational flow.