Transition from the laminar regime to turbulence within the boundary layer at the bottom of finite amplitude surface waves

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The flow field generated by sea waves is usually assumed to be irrotational and the wave steepness to be much smaller than one, such that the Airy wave theory provides a fair description of the flow. However, to satisfy the no-slip condition at the bottom, it is necessary to introduce a boundary layer where the flow is described by the so-called 'Stokes solution'. The Airy theory was extended by Stokes to consider larger wave amplitudes. In this case, the flow within the boundary layer turns out to be characterized by the presence of a steady streaming (Longuet-Higgins (1953)) and a second harmonic component, which are superimposed to the main velocity oscillations. However, these results are valid when the flow regime is laminar. Laboratory measurements show that, when increasing values of R_δ are considered, perturbations of the laminar flow start to appear when R_δ is larger than about 100 but the average flow does not significantly differ from the laminar solution (herein the Reynolds number is defined using by the amplitude U_0^* of the velocity oscillations outside the boundary layer and the viscous length $\delta^* = \sqrt{2\nu^*/\omega^*}$, ω^* being the angular frequency of the oscillations and ν^* the kinematic viscosity of the fluid). Only when R_δ becomes larger that a value ranging between 500 and 600, turbulence appears. Then, turbulence pervades the whole wave cycle for values of R_δ larger than a third critical value which laboratory measurements show to be smaller than 750. Of course turbulence presence enhances any transport process and in particular sediment transport. In this study we investigate the stability of the boundary layer generated at the bottom of sea waves characterized by small but finite amplitudes such that both the second harmonic component and the steady streaming assume significant values. A linear stability analysis of the laminar flow is made to determine the conditions leading to the appearance of perturbations of the laminar flow and, then, to turbulence. The Reynolds number of the phenomenon is assumed to be large and a 'momentary' criterion of stability is used. The results show that the laminar flow becomes unstable when the irrotational velocity close to the bottom reverses from the onshore to the offshore direction and R_δ exceeds a first critical value $R_{\delta,c1}$ which depends on the local water depth and the period and amplitude of the surface wave. However, for Reynolds numbers close to the critical value, the flow is found to relaminarize during the other phases of the cycle. For larger values of R_δ, the flow is unstable also after the passage of the wave trough and, if R_δ is further increased, the growth rate of the perturbations becomes positive over the entire wave cycle and turbulence is predicted to be always present. The obtained results, along with the results described in Blondeaux and Vittori (2021), show theoretically the existence of the four different flow regimes observed in the laboratory experiments.

References

P. Blondeaux and G. Vittori. (2021) *J. Fluid Mech.* **919**, A36

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